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RESEARCH OF VIBRATIONS OF A COMPRESSIBLE LIQUID-FILLED CYLINDRICAL SHELL BY THE INVERSE METHOD

Abstract

We consider free radial-bending vibrations of a compressible liquid-filled cylindrical shell. We research dependence of eigen vibrations of a liquid-free shell expressed by eigen frequencies of the system.

The stectra of curves of eigen frequencies are constracted.

Introduction. Recently in connection with inquiries of engineering the problems related with dynamical behavior of thin-shelled constructions that in working conditions are in contact with environment call a great interest of researchers. The problems on free vibrations of elastic thin shells contacting with elastic rigid medium and liquid occupy an important place among the dynamic contact problems of shells theory. The contact between a shell and medium may be realized in different ways: rigid coupling only along normal without tangent interaction and etc. While solving these problems it is usually assumed that elasticity modulus of medium is smaller for some order than the elasticity of shell's material.

In the paper [1] transverse-radial vibrations of a shell were investigated and explicitly analyzed frequency equation was obtained. The roots of this frequency equation were determined in computer for random numbers n in peripheral direction.

Influence of compressible liquid and medium on the found eigen frequencies for concrete collection of parameters of a shell, medium and liquid was studied in [2].

Problem Statement. In the paper we consider free radial vibrations of compressible fluid containing thin-walled infinite cylindrical shell. Since the finding of eigen frequencies is connected with solution of transcendental equations, here the frequency of vibrations of a liquid-free shell is expressed by the frequency of vibrations in an obvious form, that allows to investigate the frequencies spectra of the system both analytically and graphically.

The equations of technical theory of cylindrical shells have the form [3], [4]

$$\frac{1}{Eh}\nabla^4\psi - \nabla_k^2 W = 0 \tag{1}$$

$$\nabla_k^2\psi - D\nabla^4 W - Z = 0,$$

where

$$\nabla^2 = \frac{1}{AB}\left[\frac{\partial}{\partial\alpha}\left(\frac{B}{A}\frac{\partial}{\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{A}{B}\frac{\partial}{\partial\beta}\right)\right]$$

$$\nabla_k^2 = \frac{1}{AB}\left[\frac{\partial}{\partial\alpha}\left(\frac{B}{A}\frac{1}{R_2}\frac{\partial}{\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{A}{B}\frac{1}{R_1}\frac{\partial}{\partial\beta}\right)\right].$$

W is radial displacement of a part of a shell, A and B are the coefficients of the first quadratic form of median surface of a shell, α and β are orthogonal coordinates, R_1 and R_2 are the radii of curvatures of lines, $\alpha = const$ and $\beta = const$, E is Young's modulus, ν is Poisson's ratio, h is shell's thickness, Z are mass and external forces per volume and area unit, respectively.

In the given case

$$Z = -q\frac{\partial^2 W}{\partial t^2} + p\ , \tag{2}$$

here p is liquid pressure, q is shell's material mass referred to per area unit of medium surface.

Assuming the motion of liquid to be potential and ignoring the convective terms in the liquid motion equations we can accept

$$p = -\rho\frac{\partial\psi}{\partial t}\ . \tag{3}$$

ρ is liquid's density.

Introducing $W = \nabla^4\varphi;\ \psi = Eh\nabla_k^2\Phi$ we can reduce (1), (2) and (3) to the form

$$\nabla^8\Phi + \frac{12(1-\nu^2)}{h^2}\nabla_k^4\Phi + \frac{4q}{Eh}\nabla\frac{\partial^2\Phi}{\partial t^2} + \frac{\rho}{Eh}\frac{\partial\varphi}{\partial t} = 0\ . \tag{4}$$

Expressing the liquid's velocity on the boundary with shell by a potential we have

$$\frac{\partial W}{\partial t} = \frac{\partial\varphi}{\partial r}$$

or

$$\nabla^4\frac{\partial\Phi}{\partial t} = \frac{\partial\varphi}{\partial r}\ . \tag{5}$$

For a cylindrical shell whose displacements are independent of α, i.e. along the axis of the cylinder

$$\frac{\partial\varphi}{\partial\alpha} \equiv 0;\quad \nabla^2 = \frac{1}{R^2}\frac{\partial^2}{\partial\beta^2};\quad \nabla_k^2 = \frac{1}{R^3}\frac{\partial^2}{\partial\beta^2},$$

i.e. the system of equations (4) and (5) has the form

$$\frac{1}{R^8}\frac{\partial^8\Phi}{\partial\beta^8} + \frac{12(1-\nu^2)}{h^2R^6}\frac{\partial^4\Phi}{\partial\beta^4} + \frac{q}{EhR^4}\frac{\partial^6\Phi}{\partial\beta^4\partial t^2} + \frac{\rho\partial\varphi}{Eh\partial t} = 0 \tag{6}$$

$$\frac{1}{R^4}\frac{\partial^5\varphi}{\partial\beta^4\partial t} = \frac{\partial\varphi}{\partial r}.$$

In order to investigate free vibrations we consider the particular solutions of the system (6)

$$\Phi = \Phi_0 e^{i\omega t}\sin n\beta \quad \text{and} \quad \varphi = \varphi_0 e^{i\omega t}\sin n\beta\ . \tag{7}$$

Substituting (7) into (6) we get

$$\frac{n^8}{R^8}\Phi_0 + \frac{12(1-\nu^2)n^4}{h^2R^6}\Phi_0 - \frac{\omega^2 qn^4}{EhR^4}\Phi_0 + \frac{i\omega\rho}{Eh}\varphi_{0n} = 0 \tag{8}$$

$$\frac{in^4\omega}{R^4}\Phi_0 = \frac{d\varphi_{0n}}{dr}.$$

The potential of the liquid, satisfying the equation [3]

$$\frac{\partial^2\varphi_n}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi_n}{\partial r} + \frac{n^2}{r^2}\frac{\partial^2\varphi_n}{\partial\beta^2} = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}$$

in the case (7) interior to the cylinder will have the form

$$\varphi_{0n} = BJ_n\left(\frac{\omega r}{a}\right), \tag{9}$$

where B is a constant, J_1 is Bessel's first order function.

Allowing for (9) the non-triviality condition of the system (8) will be

$$\left[\frac{n^4}{R^4} + \frac{12(1-\nu^2)}{h^2R^2} - \frac{\omega^2 q}{Eh}\right]\frac{J_n'}{a} = \frac{\omega\rho J_n}{Eh}\,. \tag{10}$$

We denote frequency of free vibrations of a liquid-free shell by ω_0.

In the last case equation (10) takes the form

$$n^4 + \frac{12(1-\nu^2)}{h^2}R^2 = \frac{qR^4}{Eh}\omega_0^2\,. \tag{11}$$

Using (11) in (10) we can get

$$q\omega_0^2 - q\omega^2 = a\rho\omega\frac{J_n}{J_n'}$$

or

$$\omega_0 = \sqrt{\omega^2 + \frac{a\rho\omega J_n}{qJ_n'}}\,. \tag{12}$$

Equation (17) connects the frequency of free vibrations of the system with the frequency of free vibrations of a liquid-free shell. Finding of frequencies of free vibrations of the system is wholly connected with the solution of the transcendental equation (10). And very often the authors appeal to approximate methods, especially to asymptotic ones. However, the solution of the inverse problem (12) allows to construct a spectrum of graphs and simplifies the research including definition of frequency.

Fig. 1.

Having multiplied (12) by R/a we get

$$F(z) = \sqrt{z^2 + \frac{kJ_n}{J_n'} z},$$

where

$$Z = \omega R/a;\ \ k = \rho/q.$$

The graphs of dependences $F(z) - z$ for $n = 2$ and $n = 3,\ k = 1/2$ are given in figures 1 and 2.

Fig. 2.

In the second and next modes we observe inflection of graphs, i.e. with increase of $F(z)$ the growth of z increases up to some value, and then it slows down. The points of the least stability of vibrations rest on a straight line

$$F(z) = z.$$

The inflection points of curves are the zeros of the function $J_n\left(\frac{\omega r}{a}\right)$. Near to these points $F(z) \approx z$.

References

[1]. Baltruconis J.H. *Free, transverse vibrations of solid elastic mass in an infinitely long, rigid, circular-cylindrical tank.* –J. Appl. Mech., 1960., vol. 27, No4, pp. 663-668.

[2]. Latifov F.S. *Asymptotic analysis of problems on free non axisymmetric vibrations in infinite elastic ideal liquid-filled cylindrical shell.* DAN Azerb. Republic, 1999, vol. LV, No 5. (Russian)

[3]. Vlasov V.Z. *General theory of shells.* Qostekh. 1949. (Russian)

[4]. Filipkov A.P. *Vibrations of deformable systems.* Mashinostroyeniye. M.1970, 388 p. (Russian)

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