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BEARING CAPACITY OF A COMPOSITE ANNULAR PLATE WITH DIFFERENT FIXING CONDITIONS, SITUATED UNDER THE ACTION OF UNIFORMLY DISTRIBUTED LOAD

Abstract

In the paper we solve a problem an definition of an ultimate load (bearing capacity) for perfect rigidly plastic annular composite plates simply supported at intermal and builtin at external edges and situated under the action of uniformly distributed lateral load. It is shown that the plate's surface is divided into five annular zones, at each of these different plastic states are relaized. Static fields of moments and kinematic fields of velocities of flexions are determined, the equations for the unknown radii separating different plastic zones, and also the equations for determining support reaction and ultimate load, are found.

Introduction 1. A system of loads at which for the first time plastic flow arises in a body made of perfectly plastic material is called ultimate. Finding ultimate loads is the subject of the theory of ultimate equilibrium. Materials with plastic properties are widely used in different fields of up to date engineering. At present, composites occupy a special place among these materials. Thin-shelled constructions made of such materials found wide application in cosmic and aviation engineering, shipbuilding, machinebuilding, construction and etc. The problems on determination of bearing capacity of such constructions are very urgent.

Ultimate state of flexural plates has been studied in numerous papers [1-6, 8]. Plastic behavior of composite materials and structural elements of these has been studied not enough; here we note the papers [8-6, 10, 11], where bearing capacity of round and annular plates made of fibrous composite material under different fixing conditions, situated under the action of lateral uniform and non-uniform loads, are researched.

Problem Statement. Let's consider a plastic (tensionless) flexion of an annular composite plate occupiging the domain $A \leq R \leq B$, $-\dfrac{H}{2} \leq z \leq \dfrac{H}{2}$, $0 \leq \varphi \leq 2\pi$ for axisymmetric load of intensity $q = q(R)$ (fig.1), cylindric system of coordinates R, φ, z, where z is a downwards directed axis, the plane $R\varphi$ coincides with median surface of the plate, and origin of coordinates coincides with the centers of concentric contour circles. Assume that the load q is downwards directed and the thickness H of the plate is constant. We'll procced from the scheme of rigidly-plastic material. Then the plate remains non-deformable till it achieves the ultimate load (characterizing the bearing capacity). The composite consists of perfectly plastic matrix with different yield points for compression σ_0 and tension $k\sigma_0$ where $0 \leq k \leq 1$ made of more strength perfectly plastic reinforcing thin fibers. Let S_{0i}^{+} and $S_{0i}^{-} = \mu_i S_{0i}^{+}$ be ultimate forces for fibers at tension and compressin, respectively; $S_{0i}^{+} = F_i^{+} \sigma_{0i}^{+}$,

$S_{0i}^{-} = F_i^{-}\sigma_{0i}^{-} : F_i^{-},\ F_i^{+}$ be cross-section areas of fibers; $\sigma_{0i}^{+},\ \sigma_{0i}^{-}$ be yield points for fibers at tension and compression; $i = 1, 2$ be orthogonal directions coinciding with axes of principal bending moments. The matrix is reinforced by fibers in two orthogonal directions coinciding with the axes of principal bending moments. The fibers are presented in each direction in two layers, not symmetric with respect to median plane. Their amount is different in each direction. Approximate yield condition in the plane of principal bending moments has the form of different irregular hexagons depending on mechanical properties of the matrix and fibers and typical geometrial parameters. We'll investigate the problems of ultimate equilibrium for annular plates obtained in such a way, from macroscopic isotropic homogeneous composite materials Remaining with the engineering theory of flexion of plates we assume that Kirchhoffs conjecture is fulfilled. By M_1 and M_2 we denote principal bending moments in redial and peripheral directions referred to unit lenght. Then the equilibrium equation will be of the form

$$(rM_1)' - M_2 = -\int_A^R q\,(R)\,RdR + TA, \tag{1}$$

where the prime means the derivative with respect to R, T - is an unknown support reaction on an internal contour (that is determined in the course of solution) referred to the unit lenght, $q\,(R)$ is an axisymmetric lateral load. The first term in the right hand side of the equality expresses the intersecting force per unit length of a cylindric section of radius R.

We accept the following dimensionless quantities

$$r = \frac{R}{H},\quad a = \frac{A}{H},\quad b = \frac{B}{H},\quad P = \frac{q}{4\sigma_0},\quad m_i = \frac{4M_i}{\sigma_0 H^2}. \tag{2}$$

Then (1) is transformed into the form

$$(rm_1)' - m_2 = -T^{ar} + Ta \quad \left(T^{ar} = \int_a^r P\,(\eta)\,\eta d\eta\right). \tag{3}$$

The velocities of alternation of curvature in radial and peripheral directions χ_1 and χ_2 are expressed by the derivatives of the flexion w:

$$\chi_1 = -w'',\quad \chi_2 = -\left(w'/r\right). \tag{4}$$

The term "velocity" here is understood in conditionol sense: it is a derivative from w with respect to any monotonically increasing parameter.

The equation (3) is an ordinary differential equation with two unknows m_1 and m_2. The missing equation between these quantities is given by the plastic flow condition. The solution of the obtained equation is related with known difficulties. However, the problems are essentially simplified if we accept a pieccewise constant hexagon of plastic flow in the plane of moments m_1, m_2. Then the plate is divided into annular zones, at each of these the yield condition is linear and integration is easily realized.

We'll assume that the plate is subjected to the yield condition, that in the plane m_1 m_2 is an irregular hexagon $ABCDEF$ (fig. 2). On the circle separating the annular domains of different solutions, by the equilibirium conditions the bending moment m_1 and intersecting force should be continuous, but the bending moment m_2 should be discontinuous.

For the sides AB and AF of the hexagon we have [10]

$$m_i = m_{0i}^{+} = c_0 + c_{1i}^{+} s_{0i}^{+} + c_{2i} \left(s_{0i}^{+} \right)^2 , \tag{5}$$

for he sides CD and DE

$$m_i = -m_{0i}^{-} = -\left[c_0 + c_{1i}^{-} s_{0i}^{+} + c_{2i} \left(s_{0i}^{+} \right)^2 \right] . \tag{6}$$

Here m_{0i}^{+} and m_{0i}^{-} are the limiting values of positive and negative bending moments. For the sides EF and BC we have

$$m_2 = \alpha m_1 + b_1, \quad m_2 = \alpha m_1 + b_2, \tag{7}$$

Here we accept the following denotation for the coefficients that are positive quantities:

$$c_0 = \frac{2k}{k+1}, \quad c_{1i}^{+} = 4\left[d_i' - \mu_i d_i'' + \frac{(1-k)(1-\mu_i)}{2(1+k)}\right], \quad c_{2i} = -\frac{2(1-\mu_i)^2}{1+k},$$

$$c_{1i}^{-} = 4\left[d_i' - \mu_i d_i'' - \frac{(1-k)(1-\mu_i)}{2(1+k)}\right], \quad \alpha = \frac{(1-k)(1-\mu_1)s_{01}^{+} + k}{(1-k)(1-\mu_2)s_{02}^{+} + k}, \quad i = 1,2;$$

$$b_1 = a_2 - \alpha a_1, \quad b_2 = a_4 - \alpha a_3, \quad s_{0i}^{+} = \frac{S_{0i}^{+}}{\sigma_0 H^2},$$

$$a_1 = \frac{1}{1-k^2}\left\{k(1-k) + \left(1+k^2\right)(1-\mu_1)s_{01}^{+} - 2k(1-\mu_2)s_{02}^{+}\right\} + 4\left(d_1'' - \mu_2 d_2''\right)s_{01}^{+},$$

$$a_2 = \frac{1}{1-k^2}\left\{k(k-1) - \left(1+k^2\right)(1-\mu_2)s_{02}^{+} + 2k(1-\mu_1)s_{01}^{+}\right\} + 4\left(d_2' - \mu_2 d_2''\right)s_{02}^{+},$$

$$a_3 = -a_1 + 4(1-\mu_1)\left(d_1' + d_1''\right)s_{01}^{+}, \qquad a_4 = -a_2 + 4(1-\mu_2)\left(d_2' + d_2''\right)s_{02}^{+},$$

d_i' and d_i'' are dimensionaless distances (referred to the thickness H) from the median plane up to upper and lower fibre-reinforced layers.

3. Definition of static moments field. Let a plate be simply supported at internal and builtin at external contours and situated under the action of axisymmetric lateral load applied on the surface. It is seen from the form of the curve of the plate (fig. 3) that the velocity of the curvature χ_2 changes its sign.

For the mentioned form of the load and boundary conditions a radial bending moment will have a positive value (tension of lower compression of upper layers) up to the domain adjoining to the builtin external contour, where radial bending moment has a negative value. In this case a plastic state of the plate is determined by the side E_1E of the yield hexagon near the internal edge, on which $m_2 = -m_{20}^{-}$. Considering that the load q of the plate is donwards directed, for boundary conditions $m_1 = 0$ for $r = a$ and $m_1 = -m_{01}^{-}$, for $r = b$ we can look for the solution of the problem for the states $E_1E - EF - FA - AB - BC$, since in this case it is possible to determine the static field satisfying the appropriate continuity conditions.

Let $p = const$. We'll need the following integrals

$$T^{ar} = \frac{p}{2}\left(r^2 - a^2\right),$$

$$\int_a^r \frac{1}{\xi} T^{a\xi} d\xi = \frac{p}{4}\left(r^2 - a^2 - 2a^2 In\frac{r}{a}\right), \quad \int_a^r T^{a\xi} d\xi = \frac{p}{6}\left(r^3 - 3a^2 r + 2a^3\right), \tag{8}$$

Then, from (3) we'll get

$$(rm_1)' - m_2 = Ta - 2p\left(r^2 - a^2\right), \quad a \le r \le b. \tag{9}$$

The equation (9) will be solved under the following boundary conditions at the supported edge $m_1 = 0,\ w = 0$; along the fixed edge $w = 0,\ dw/dr = 0$ or $m_1 = -m_{10}^{-}$.

The equation (9) is an ordinary differential equation with two unknowns m_1 and m_2. A missing eqution between these quantities is given by the plastic flow condition. The solution of the obtained equation is related with known difficulties. However, the problem is essentially simplified if we accept a piecewise constant hexagon of plastic flow in the plane of moments m_1 and m_2. Then, the plane is divided into annular zones, in each of these the yield condition is linear and integration is easily relaized. On the area $a \le r \le \rho_1$ the acceptable will be the condition E_1E by which $m_2 = -m_{20}^{-}$. Substituting this into the equation (9) and integrating it we get

$$rm_1 = -m_{20}^{-}r + Tar - \frac{p}{6}\left(r^3 - 3a^2 r\right) + C$$

Fig 4. —$k = 1,\ \mu = 1; - - - - k = 0,8,\ \mu = 1 - . - . - .k = 1,\ \mu = 0,8;$

Defining the integration constant C from the condition $m_1(a) = 0$, we find

$$rm_1 = \left(m_{10}^{+} - m_{20}^{+} - m_{20}^{-}\right)(r - a) + \frac{P}{6}\left(3\rho_3^2 r - 3\rho_3^2 a - r^3 + a^3\right),$$

$$m_2 = -m_{20}^{-}, \qquad a \le r \le \rho_1. \tag{10}$$

Defining $m_1(\rho_1)$ from (10) and substituting into the formula $m_2 = \alpha m_1 + b_1$, as a result we get $-m_{20}^{-}$ that will give

$$m_{20}^{-}\left(\alpha - 1 - \alpha\frac{a}{\rho_1}\right) = b_1 + Ta\alpha\frac{\rho_1 - a}{\rho_1} - \frac{\alpha p}{6\rho_1}\left(\rho_1^3 - 3a^2\rho_1 + 2a^3\right). \tag{11}$$

For $\rho_1 \le r \le \rho_2$ we have the state EF for which $m_2 = \alpha m_1 + b_1$; an appropriate equilibrium equation has the form

$$rm_1' + (1-\alpha)\, m_1 = (Ta + b_1) - \frac{p}{6}\left(r^2 - a^2\right).$$

Solving this equation and defining arbitrary integration constant from the continuity condition $m_1(\rho_2) = m_{10}^+$, we find

$$m_1 = m_{10}^+\left(\frac{r}{\rho_2}\right)^{\alpha-1} + \left[\frac{p}{2}\left(\rho_3^2 - a^2\right) + m_{10}^+ - m_{20}^+ + b_1\right]\frac{1}{\alpha-1}\left[1 - \left(\frac{r}{\rho_2}\right)^{\alpha-1}\right] +$$

$$+\frac{p}{r}\left[\frac{1}{3-\alpha}\left(\rho_2^{3-\alpha} r^{\alpha-1} - r^2\right) - \frac{\alpha^2}{1-\alpha}\left(\left(\frac{r}{\rho_2}\right)^{\alpha-1} - 1\right)\right], \qquad (12)$$

$$m_2 = \alpha m_1 + b_1, \qquad \rho_1 \le r \le \rho_2.$$

When the stressed state of the plate corresponds to the side $FA\,(\rho_2 \le r \le \rho_3)$ for the strain velocities we have

$$\chi_1 = -w'' \ge 0, \qquad \chi_{21} = -\frac{1}{r}w' = 0.$$

Natural solution of these equations will be $w = w_0 = const\ \ t$, i.e. an annular part of the plate $\rho_2 \le r \le \rho_3$ remains rigid and commutes in this domain as an absolute rigid body.

The circles $r = \rho_2$ and $r = \rho_3$ are hiinge circles on which the first derivative of flexion velocity undergoes break, the flexure's velocity is continuous and radical bending moment has a maximal value. In the domain $\rho_2 \le r \le \rho_3$ the static moments field will be

$$m_1 = m_{10}^+,$$

$$m_2 = m_{20}^+ - \frac{p}{2}\left(\rho_3^2 - r^2\right), \qquad \rho_2 \le r \le \rho_3.$$

Since

$$m_2(\rho_3) = m_{20}^+, \quad m_2(\rho_2) = m_{20}^+ - \frac{p}{2}\left(\rho_3^2 - \rho_2^2\right) < m_{20}^+,$$

the plastic condition in the rigid domain doesn't increase, vise versa, along the condition FA a moment state of the plate is inside of the yield hexagon.

For $\rho_3 \le r \le \rho_4$ we have the state AB for which $m_2 = m_{20}^+$ and from the equilibrium equation (9) we get

$$rm_1 = \left(m_{20}^+ + Ta\right) r - \frac{p}{6}\left(r^3 - 3a^2 r + 2a^3\right) + C.$$

Here, we define the arbitrary constant C from the condition $m_1(\rho_3) = m_{10}^+$ then

$$rm_1 = m_{10}^+ r + \frac{p}{6}\left(3\rho_3^2 r - 2\rho_3^3 - r^3\right),$$

$$m_2 = m_{20}^+, \qquad \rho_3 \le r \le \rho_4. \qquad (13)$$

For $\rho_4 \leq r \leq b$ we have the state BC for which $m_2 = \alpha m_1 + b_2$. There with, from the equation (9) and condition $m_1(b) = -m_{10}^-$ we get

$$m_{20}^+ = \alpha m_1(\rho_4) + b_2, \qquad m_2(b) = -\alpha m_{10}^- + b_2,$$

$$m_1(r) = -m_{10}^- \left(\frac{r}{b}\right)^{\alpha-1} + \left(m_{10}^+ - m_{20}^+ + b_2\right)\frac{1}{1-\alpha}\left[1 - \left(\frac{r}{b}\right)^{\alpha-1}\right] +$$
$$+\frac{p}{2}\left(\rho_3^2 - a^2\right)\frac{1}{1-\alpha}\left[1 - \left(\frac{r}{b}\right)^{\alpha-1}\right] +$$
$$+\frac{p}{2}\left\{\frac{b^{3-\alpha}r^{\alpha-1}}{3-\alpha} - \frac{r^2}{3-\alpha} + \frac{a^2}{1-\alpha}\left[1 - \left(\frac{r}{b}\right)^{\alpha-1}\right]\right\},$$
$$m_2 = \alpha m_1 + b_2, \qquad \rho_4 \leq r \leq b.$$

Now, let's investigate possibility of continuation of the static field on the domain $\rho_2 \leq r \leq \rho_3$. Accepting that the tangential moment m_2 and intersecting force are continuous functions, from the equilibrium equations we get that the derivative dm_1/dr may not have jumps in the domain of the plate, i.e.

$$\frac{dm_1}{dr} = 0 \quad for \quad r = \rho_2 \quad and \quad r = \rho_3, \tag{14}$$

since $m_1 = m_{10}^+$ on these radii. But when we assume that the moment m_2 step-wisely changes, then from the equilibrium condition (9) we can get:

$$r\left[\frac{dm_1}{dr}\right] = [m_2], \tag{15}$$

where

$$\left[\frac{dm_1}{dr}\right] = \frac{dm_1^+}{dr} - \frac{dm_1^-}{dr}, \quad [m_2] = m_2^+ - m_2^-$$

mean the jumps of appropriate quantities at the considered point.

Since $[m_2]$ for $r = \rho_2$ and $r = \rho_3$ has a positive value, and $[dm_1/dr]$ on these radii may be only negative, then condition (15) may not be fulfilled. From this we conclude that the field of moments m_2 should be continuous in the plate's domain, i.e. $[m_2] = [dm_1/dr] = 0$.

Fulfilling the condition (14) with using the derivative of formula (12) for $r = \rho_2$ and derivative of formula (14) for $r = \rho_3$, we get

$$Ta = \frac{p}{2}\left(\rho_3^2 - a^2\right) + m_{10}^+ - m_{20}^-, \tag{16}$$

$$m_{20}^+ - \left(\alpha m_{10}^+ + b_1\right) = \frac{p}{2}\left(\rho_3^2 - \rho_2^2\right). \tag{17}$$

The formula (16) determines the unknown reaction Ta, (17) defines the ultimate load.

In the paper [9] the formula

$$\varphi = \frac{pb^2}{m_0} = \frac{2}{x_3^2 - x_2^2}$$

where $x_i = \dfrac{\rho_i}{b}$, $i = 2, 3..$ is obtained for the ultimate load.

As is seen, at the expense of reinforcement the ultimate load of the plate increases by

$$1 - \frac{\alpha m_{10}^{+} + b_1}{m_{20}^{+}}$$

times, since $\alpha m_{10}^{+} + b_1 < 0$.

The graphs of dependence of ultimate load pb^2 on s_0 for concrete values of k, μ and γ, where the upper three straight lines refer to the case $\gamma = 0,5$, and the three lower straight lines to $\gamma = 0,25$ are represented in the figure 4. Graph of dependence of dimensionaless radii $x_i = \dfrac{\rho_i}{b}$, $i = \overline{1,4}$ on the relation $k = \dfrac{a}{b}$ is in figure 5. Figure 6 represents dependence of ultimate load on $k = \dfrac{a}{b}$ for composite (curve 2) and ordinary (curve 1) plates.

Fig 5. Dependence of radii $x_i = \dfrac{\rho_i}{b}$, $i = \overline{1,4}$ on the relation $k = a/b$.

Fig 6. Dependences of ultimate load on the relation $k = a/b$ for composite (2) and ordinary plates.

4. Determination of kinematically possible field of flexure velocities. Here we'll determine kinematically possible field of velocities of flexure at the moment when the yield has just occurred, displacements are yet small and alternation of plate's geometry is unessential. Each element of the plate going over into yield condition, is connected with rigid elements. Therefore, the relations between the velocities of deformation of separate elements are connected with each other and this leads to the fact that the velocities are found to within undeterminate factor.

Using the associated low of plastic flow in principal directions

$$d\xi_i = \lambda_p \frac{\partial f_p}{\partial m_i} \quad (i = 1, 2; \quad p = 1, 2, ..., 6),$$

where in the present case the yield surface equation $f_p = const \quad (p = 1, 2, .., 6)$ is a plastic flow hexagon considered above, and expression (4) for the velocities of curvature change, we get ordinary linear differential equations for flexure velocity for plastic states corresponding to different sides of a hexagon.

For the plastic state E_1F the velocity of the curvature $\chi_1 = -w'' = 0$ should vanish, i.e. $w'' = 0$. Associated low of plastic flow shows that the vector of curvature change velocity is parallel to the normal to the plastic flow surface. The solution of this equation satisfying the boundary condition $w(a) = 0$ is

$$w = C(r - a), \qquad a \le r \le \rho_1, \tag{18}$$

where C is an arbitrary constant.

For the plastic state EF we have $m_2 = \alpha m_1 + b_1$, the vector of the normal of this straightline $\{\alpha, -1\}$ should be parallel to the vector of plastic flow velocities $\{\chi_1, \chi_2\}$, i.e. $\chi_1 : \alpha = \chi_2 : (-1)$, or

$$w'' + \frac{\alpha}{r} w' = 0. \tag{19}$$

For the plastic state FA the curvature velocity $\chi_2 = -\frac{1}{r}w' = 0$, i.e. $w = w_0 = const$ for $\rho_2 \le r \le \rho_3$.

The solution of equation (19) satisfying the continuity condition for $r = \rho_1$ and $r = \rho_2$ is

$$w(r) = C(\rho_1 - a) + [w_0 - C(\rho_1 - a)] \frac{r^{1-\alpha} - \rho_1^{1-\alpha}}{\rho^{1-\alpha} - \rho_1^{1-\alpha}},$$
$$\rho_1 \le r \le \rho_2. \tag{20}$$

Here C and w_0 are the unknown constants. From the continuity condition of the first derivative w' at the point $r = \rho_1$, we determine the constant C:

$$C = w_0 \frac{(1-\alpha)\rho_1^{-\alpha}}{\rho_2^{1-\alpha} - \rho_1^{1-\alpha} + \rho_1^{-\alpha}(\rho_1 - a)(1-\alpha)}.$$

The constant C is positive, both for $\alpha > 1$, and $\alpha < 1$, but for $\alpha = 1$ it has the expression

$$C = w_0 \frac{1}{\rho_1 - a + \rho_1 In \dfrac{\rho_2}{\rho_1}}.$$

For the plastic state AB the curvature velocity $\chi_1 = -w'' = 0$ and we have the solution

$$w(r) = w_0 + C_1(r - \rho_3), \quad \rho_3 \le r \le \rho_4, \tag{21}$$

satisfying the continuity condition for $r = \rho_3$.

Finally, for the plastic state BC we use the equation $m_2 = \alpha m_1 + b_2$ and again get the equation (19). The solution satisfying the condition $w(b) = 0$ is

$$wr = [w_0 + c_1(\rho_4 - \rho_3)]\left(1 - \frac{r^{1-\alpha} - \rho_4^{1-\alpha}}{b^{1-\alpha} - \rho_4^{1-\alpha}}\right), \quad \rho_4 \le r \le b. \tag{22}$$

As the point $r = \rho_4$ the derivative function w', i.e. $[w'(\rho_4)] = 0$, from which we determine the constant C_1:

$$C_1 = \frac{w_0(1-\alpha)\rho_4^{1-\alpha}}{b^{1-\alpha} - \rho_4^{1-\alpha} + \rho_4^{-\alpha}(\rho_4 - \rho_3)(1-\alpha)}$$

should be also continuous.

Then the formula (21) and (22) take the form:

$$w = w_0 - \frac{w_0(1-\alpha)\rho_4^{-\alpha}(r - \rho_3)}{b^{1-\alpha} - \rho_4^{1-\alpha} + \rho_4^{-\alpha}(\rho_4 - \rho_3)(1-\alpha)}, \quad \rho_3 \le r \le \rho_4,$$

$$w = w_0 \frac{b^{1-\alpha} - r^{1-\alpha}}{b^{1-\alpha} - \rho_4^{1-\alpha} + \rho_4^{-\alpha}(\rho_4 - \rho_3)(1-\alpha)}, \quad \rho_4 \le r \le b.$$

5. Example. For certainty of the found solutions in a very special case we compare our solutions with the ones known in references. For an isotropic plate we have:

$$\begin{gathered} m_{10}^{+} = m_{20}^{+} = m_0, \ \alpha = 1, \ b_1 = -m_0, \ b_2 = m_0, \ \ m_1(\rho_1) = 0, \\ m_1(\rho_2) = m_0, \ \ m_1(\rho_3) = m_2(\rho_3) = m_0, \ \ m_1(\rho_4) = 0, \\ m_1(b) = -m_0, \ \ m_2(b) = 0, \ \ m_2(\rho_2) = 0, \ \ m_2(\rho_4) = m_0. \end{gathered} \tag{23}$$

Using these values from formula (10) we get $\rho_1 = a$, i.e. the domain $[a, \rho_1]$ disappears at all. In the formula (12) we calculate undeterminacy as $\alpha \to 1$ by de L'Hospital law and get

$$m_1 = \left(\frac{p}{2}\rho_3^2 - m_0\right)\ln\frac{r}{\rho_2} - \frac{p}{4}\left(r^2 - \rho_2^2\right) + m_0.$$

Since $m_1 = (\rho_1) = 0$, then subtracting the equality

$$0 = \left(\frac{p}{2}\rho_3^2 - m_0\right)\ln\frac{\rho_1}{\rho_2} - \frac{p}{4}\left(\rho_1^2 - \rho_2^2\right),$$

from the previous one, we get

$$m_1 = \left(\frac{p}{2}\rho_3^2 - m_0\right)\ln\frac{r}{\rho_1} - \frac{p}{4}\left(r^2 - \rho_1^2\right), \quad m_2 = m_1 - m_0, \quad \rho_1 \le r \le \rho_2. \tag{24}$$

There will be no changes in the domain $\rho_2 \le r \le \rho_3$

$$rm_1 = m_0 r - \frac{p}{6}\left(r^3 - 3\rho_3^2 r + 2\rho_3^3\right), \quad m_2 = m_0, \ \rho_3 \le r \le \rho_4, \tag{25}$$

in the domain $\rho_4 \le r \le b$ we'll have

$$m_1 = m_0 \ \ln\frac{r}{b} - m_0 + \frac{p}{4}\left(b^2 - r^2 + 2\rho_3^2 \ln\frac{r}{b}\right).$$

Since $m_1(\rho_4) = 0$, then

$$0 = m_0 \ln\frac{\rho_4}{b} - m_0 + \frac{b}{4}\left(b^2 - \rho_4^2 + 2\rho_3^2 \ln\frac{\rho_4}{b}\right).$$

Subtracting this equality from the previous one, we get.

$$m_1 = m_0 \ \ln\frac{\rho_4}{b} - \frac{p}{4}\left(r^2 - \rho_4^2 - 2\rho_3^2 \ln\frac{r}{b}\right), \ m_2 = m_1 + m_0, \ \rho_4 \le r \le b.$$

If we redenote $\rho_1 \to a, \ \ \rho_2 \to \rho_1, \ \ \rho_3 \to \rho_2, \ \ \rho_4 \to \rho_3$, then the obtained formulae will coincide with the formulae (3.1) of the paper [9].

Bearing capacity is determined by the formula

$$\varphi = \frac{pb^2}{m_0} = \frac{2}{x_2^2 - x_1^2} \qquad \left(x_i = \frac{\rho_i}{b}, \quad i = 1, 2\right).$$

The inequality $\rho_2 > \rho_1$, follows from the last formula, since $p/m_0 > 0$.

As $m_1(\rho_4) = 0$, then

$$m_0 \rho_4 = \frac{p}{6}\left(\rho_3 - \rho_4\right)\left(\rho_4^2 + \rho_4\rho_3 - 2\rho_3^2\right).$$

Hence, it is seen that $\rho_4 > \rho_3$.

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