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FREE VIBRATIONS OF LIQUID-FILLED CYLINDRICAL SHELLS REINFORCED BY ANNULAR RIBS, UNDER AXIAL COMPRESSION AND WITH REGARD TO DISCRETE ALLOCATION OF RIBS

Abstract

The paper in devoted to the investigation of liquid-filled cylindrical shells under axial compression reinforced by discretely distributed cross ribs. It is assumed that the ribs are uniformly distributed on the surface of the shell. The problem is solved by energetic method. Using the Hamilton-Ostrogradskii principle, freqnency equations are found and its least root is found. Analysis of influence of external medium parametrs of contractive force on parameter of eigen vibrations frequency of the system in carried out.

Indroduction. One of the reasons that impel desiners to reinforce thin shells by ribs is stipulated by necessity of protection of their reliability under the action of different type loads, calling appearance of contracting stresses. In working conditions the reinforced cylindrical shells are in contact with different media. Different capacities and pipelines, special purpose constructions and others are reduced to design model of reinforced, liquid-filled cylindrical shells. Therefore, elaboration of theory and calculation methods for vibrations of reinforced cylindrical shells with regard to external actions and under axial compression is an urgent problem of great practical value. The solutions given in references belong mainly to liquidless cylindrical shells [1]. Problems of vibrations of such constructions with medium have not been studied practically. Notice that vibrations of smooth cylindrical shells with filler are sufficiently studied in the paper [2]. Accepting a shell to be structurally-orthotropic, the vibrations of elastic medium-filled cylindrical shells reinforced by longitudinal ribs are researched in [3].

The present paper is devoted to the investigations of virations of liquid-filled cylindrical shells under axial compression reinforced by discretely allocated cross ribs. The influence of parametrs of external medium on the parameter of eigen vibrations frequency of the system is analyzed.

Problem Statement. The problem is solved by energetic method. Potential energy of a shell loaded by axial contracting forces is of the form [1]:

$$\ni \frac{Eh}{2\left(1-v^2\right)}\int_0^{\xi_1}\int_0^{2\pi}\left\{\left(\frac{\partial u}{\partial \xi}+\frac{\partial v}{\partial \theta}-w\right)^2+2\left(1-\nu\right)\left[\frac{\partial u}{\partial \xi}\left(\frac{\partial v}{\partial \theta}-w\right)-\right.\right.$$

$$-\frac{1}{4}\left(\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial\xi}\right)^2\Bigg]\Bigg\}d\xi d\theta+\frac{Eh^3}{24\left(1-\nu^2\right)R^2}\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left\{\left(\frac{\partial^2 w}{\partial\xi^2}+\frac{\partial^2 w}{\partial\theta^2}+\frac{\partial v}{\partial\theta}\right)^2-\right.$$

$$\left.-2\left(1-\nu\right)\left[\frac{\partial^2 w}{\partial\xi^2}\left(\frac{\partial^2 w}{\partial\theta^2}+\frac{\partial v}{\partial\theta}\right)-\left(\frac{\partial^2 w}{\partial\xi\partial\theta}+\frac{\partial v}{\partial\xi}\right)^2\right]\right\}d\xi d\theta+$$

$$+\frac{E_h}{2R}\sum_{j=1}^{k_1}\int\limits_0^{2\pi}\left[F_h\left(\frac{\partial v}{\partial\theta}-w-\frac{h_h\partial^2 w}{R\partial\theta^2}\right)^2+\frac{I_{xh}}{R^2}\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)^2+\frac{G_h}{R^2E_h}I_{cr.h}\times\right. \quad (1)$$

$$\left.\times\left(\frac{\partial^2 w}{\partial\xi\partial\theta}+\frac{\partial u}{\partial\theta}\right)^2\right]\Bigg|_{\xi=\xi_j}d\theta-\frac{\sigma_x h}{2}\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left(\frac{\partial w}{\partial\xi}\right)^2 d\xi d\theta.$$

Here $\xi=\frac{x}{r}$, $\theta=\frac{y}{r}$; E_h,G_h are elasticity and shear modulus of the material of longitudinal ribs, respectively; k_1 is the quantity of cross ribs; σ_x are axial contractive stresses; u,v,w are components of displacament vector of the shell; h and r are thickness and radius of the shell, respectively; E,ν are Young modulus and Poisson ratio of the shell; $F_h,\ I_{xh},\ I_{cr.h}$ are area and inertia moment of cross sectian of the longitudinal shell with respect to the axis ox and oz and also inertia moment in torsian.

Kinetic energy of the shell is as follows;

$$K=\frac{Eh}{2\left(1-\nu^2\right)}\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left[\left(\frac{\partial u}{\partial t_1}\right)^2+\left(\frac{\partial v}{\partial t_1}\right)^2+\left(\frac{\partial w}{\partial t_1}\right)^2\right]d\xi d\theta+ \quad (2)$$

$$+\frac{\overline{\rho}_h E_h F_h}{2R\left(1-\nu^2\right)}\sum_{j=1}^{k_1}\int\limits_0^{2\pi}\left[\left(\frac{\partial\vartheta}{\partial t_1}\right)^2+\left(\frac{\partial w}{\partial t_1}\right)^2\right]\Bigg|_{\xi=\xi_j}d\theta$$

Here $\overline{\rho}_h=\frac{\rho_h}{\rho_0}$ where ρ_0 and ρ_h are densities of shell and cross bar materials, respectively, $\theta_i=\frac{2\pi}{k_1}i$.

Influence of liquid on the shell is determined as of external surface loads applicd to the shell and is calculated as a work done by, these loads when changing over the system from strained state to initial unstrained one and is represented as:

$$A_0=-\int\limits_0^{\xi_1}\int\limits_0^{2\pi}q_z w d\xi d\theta, \quad (3)$$

where q_z is the pressure of liquid on the shell.

The total energy of the system is as follows:

$$\Pi=\ni+K+A_0 \quad (4)$$

Lineariazed wave equation describing small perturbations propagation in ideal compressible liquid is of the from [3]:

$$\Delta\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 0, \qquad (5)$$

where Φ is a potential, c is sound propagation velocity in liquid.

Shell motion equation (1) and liquid motion equation (2) are complemented by contact conditions. On the contact surface "shell-liquid" continuity of radial velocities and presures

$$\vartheta_r = \frac{\partial w}{\partial t}, \quad q_z = -p \qquad (6)$$

is observed.

Hydrodynamic pressure p and radial velocity ϑ_r in liquid are determined in the following way [3]:

$$p = -\rho\frac{\partial\Phi}{\partial t}, \quad \vartheta_r = \frac{\partial\Phi}{\partial r}, \qquad (7)$$

where ρ is density of liquid.

Velocity potential Φ is represented as

$$\Phi = AJ_n(\gamma r)\cos\chi\xi\cos n\theta\sin\omega_1 t_1, \qquad (8)$$

where $\gamma^2 = -\chi^2 + \frac{\omega^2}{c^2}$, J_n are Bessel functions of first kind n-th order, A is an integration constant.

Using formula (4) and contact conditions (3) for q_z we can obtain:

$$q_z = \frac{\rho E\omega_1 J_n(\gamma R)}{(1-\nu^2)\rho_0\gamma J'_n(\gamma R)} w_0 \sin\chi\xi\cos n\theta\exp(i\omega_1 t_1). \qquad (9)$$

Solution method. We'll look for the displacements of the shell in the form:

$$u = u_0\cos\chi\xi\cos n\theta\exp(i\omega_1 t_1),$$
$$\vartheta = \vartheta_0\sin\chi\xi\sin n\theta\exp(i\omega_1 t_1), \qquad (10)$$
$$w = w_0\sin\chi\xi\cos n\theta\exp(i\omega_1 t_1).$$

where u_0, ϑ_0, w_0 are the unknown constants; $\chi = \frac{m\pi}{L}$ $(m = 1, 2, ...)$ are the wave numbers in longitudinal and peripheral directions, respectively L is the length of the shell, $\omega_1\frac{\omega}{\omega_0}$, $t_1 = \omega_0 t$, $\omega_0 = \sqrt{\frac{E}{(1-\nu^2)\rho_0 R^2}}$, $\omega_1 = \sqrt{\frac{(1-\nu^2)\rho_0 R^2\omega^2}{E}}$.

After substituting (9) into (3) and integrating with respect to ξ and θ for the work of external pressures from the side of liquid applied to the shell we get

$$A_0 = -\pi\frac{\rho E\omega_1^2 J_n(\gamma R)}{(1-\nu^2)\rho_0\gamma J'_n(\gamma R)}\left(\frac{1}{2}\xi_1\frac{\sin 2\chi\xi_1}{4\chi}\right) w_0^2\exp(2i\omega_1 t_1) = \qquad (11)$$

$$= \omega_1^2 q_z^{(0)}\omega_0^2\exp(2i\omega_1 t_1); \; q_z^{(0)} = -\pi\frac{\rho E J_n(\gamma R)}{(1-\nu^2)\rho_0\gamma J'_n(\gamma R)}\left(\frac{1}{2}\xi_1\frac{\sin 2\chi\xi_1}{4\chi}\right)$$

Using (1),(2) and (11) for total energy of the system we get a second order polynomial for the parameters of constants A, B, C :

$$\Pi = \left(\breve{\varphi}_{11} - \psi_{11}\omega_1^2\right) A^2 + \left(\breve{\varphi}_{22} - \psi_{22}\omega_1^2\right) B^2 + \left(\breve{\varphi}_{33} - \psi_{33}\omega_1^2 + l_1\sigma_x + l_2 q_z^{(0)}\right) C^2 +$$

$$+\breve{\varphi}_{44}AB + \breve{\varphi}_{55}AC + \breve{\varphi}_{66}BC$$

Notice that the quantities $\breve{\varphi}_{ii}\,(i = 1,2,...,6)$, $\psi_{ii}\,(i = 1,2,3)$, $l_i\,(i = 1,2)$ have a bulky from and we don't city them here.

Extremum conditions Π by the parametrs A, B, C reduce the solution of the problem on vibrations of longitudinally reinforced, liquid-filled shells subjected to longitudional compression to homogeneous systems of linear algebrain equations of third order whose non-trivial solutions are possible only in the case when determinant of this system equats zero. Further, equating the determinants of the indicated systems to zero, we get the following frequance equations:

$$\begin{cases} 2\left(\breve{\varphi}_{11} - \psi_{11}\omega_1^2\right) A + \breve{\varphi}_{44}B + \breve{\varphi}_{55}C = 0 \\ \breve{\varphi}_{44}A + 2\left(\breve{\varphi}_{22} - \psi_{22}\omega_1^2\right) B + \breve{\varphi}_{66}C = 0 \\ \breve{\varphi}_{55}A + \breve{\varphi}_{66}B + 2\left[\breve{\varphi}_{33} + \left(l_2 q_z^{(0)} - \psi_{33}\right)\omega_1^2 + l_1\sigma_x\right] C = 0 \end{cases} \qquad (12)$$

It is easy to see that in the case of incompressible liquid the system of eqations (12) is reduced to cubic equation with respect to ω_1^2 , otherwise it is transcendental. Since in future wi'll be interested only in low frequencies of flexural vibrations, in the case of incompressible liquid we can simplify this equation having rejected the terms with ω_1^4 and ω_1^6. As a resuet we get $\left(\omega_1^2 = \lambda\right)$:

$$\lambda = \frac{\left(\breve{\varphi}_{44}^2 - 4\breve{\varphi}_{11}\breve{\varphi}_{22}\right)\left(\breve{\varphi}_{33} + l_1\sigma_x\right) + \breve{\varphi}_{44}\breve{\varphi}_{55}\breve{\varphi}_{66} + \breve{\varphi}_{55}^2\psi_{22} + \breve{\varphi}_{66}^2\psi_{11}}{4\breve{\varphi}_{11}\breve{\varphi}_{22}\left(l_2 q_z^{(0)} - \psi_{33}\right) + \breve{\varphi}_{55}^2\psi_{11} + \breve{\varphi}_{66}^2\psi_{11} + \breve{\varphi}_{44}^2\left(\breve{\varphi}_{33} + l_1\sigma_x\right)}$$

Analysis of calculation result. We cite the results of investigations on influence of the number of ribs and mediumrigidity on critical stress of axial compression. Calculations are carried out for a shell, medium and ribs with the following parameters:

$$E = E_h = 6,67 \cdot 10^9 n/m^2;\ \nu = 0,3;\ \chi = 1;\ n = 8;\ h_h = 1,39\ mm;\ R = 160\ mm;$$

$$L_1 = 800mm;\ h = 0,45mm;\ F_h = 5,75mm^2;\ I_{xh} = 19,9\ mm^4; I_{kp.h} = 0,48mm^4.$$

Fig. 1. Dependence of ω_1 on the number of n waves in peripheral direction.

Fig. 2. Dependence of ω_1 on contractive stress.

The results of calculations are represented in figures 1 and 2. Here we give dependence of ω_1 on the number of n waves in peripheral direction and on contractive force, respectively $k_1 = 4$ corresponds to solid lines, $k_1 = 6$ to dotted lines. It is seen from figure 1 that with increase of n at first ω_1 decreases and then attains minimum and begins to increase. Besides, eigen frequencies of vibrations of the considered sysrem also increase according to the number of longitudinal ribs. It is see from figure 2 that when contractive force increases, the frenquencies of the system fall off.

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