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INVESTIGATION OF INFLUENCE OF INITIAL DEFLECTION OF MEDIUM-FILLED CYLINDRICAL SHELL REINFORCED BY A REGULAR SYSTEM OF CROSS RIBS ON CRITICAL STRESSES OF GENERAL STABILITY LOSS

Abstract

In the paper we investigate influence of initial deflection of medium-filled shell reinforced by a regular system of cross ribs on critical stresses of general stability loss. The statement of the problem using mixed energetic method and nonlinear equation of joint deformation is on basis of investigations.

Introduction. Ribbed cylindrical shells are important structural elements of rockets, submarines, motor cars and etc. Investigation of behavior of such constructions allowing for external factors has special value in the field of contact problems of the theory of ribbed shells.

Problem statement. Total energy of the system is written in the form [1]:

$$\Pi = \ni + A \tag{1}$$

Here

$$\ni = \frac{Eh^3}{24\left(1-\nu^2\right)R^2}\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left\{\left(\frac{\partial^2 w}{\partial\xi^2}+\frac{\partial^2 w}{\partial\theta^2}\right)^2 - 2\left(1+\nu\right)\left[\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\theta^2} - \right.\right.$$

$$\left.\left. - \left(\frac{\partial^2 w}{\partial\xi\partial\theta}\right)^2\right]\right\}d\xi d\theta + \frac{h}{2r^2}\left\{\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left(\frac{\partial^2\varphi}{\partial\xi^2}\frac{\partial^2\varphi}{\partial\theta^2}\right)^2 - \right.$$

$$\left. -2\left(1-\nu\right)\frac{\partial^2\varphi}{\partial\xi^2}\frac{\partial^2\varphi}{\partial\theta^2} - \left(\frac{\partial^2\varphi}{\partial\xi\partial\theta}\right)^2\right\}d\xi d\theta +$$

$$+\sigma_x h\int\limits_0^{\xi_1}\int\limits_0^{2\pi}\left\{\frac{1}{E}\left(\frac{\partial^2\varphi}{\partial\theta^2} - \nu\frac{\partial^2\varphi}{\partial\xi^2}\right) - \frac{1}{2}\left[\left(\frac{\partial w}{\partial\xi}\right)^2 + 2\frac{\partial w}{\partial\xi}\frac{\partial w_0}{\partial\xi}\right]\right\}d\xi d\theta + \frac{1}{2R^3}\times$$

$$\times\sum_{j=1}^{k_1}\int\limits_0^{2\pi}\left[E_s I_{xs}\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)^2 + G_s I_{cr.s}\frac{\partial^2 w}{\partial\xi\partial\theta}\right]\Bigg|_{\xi=\xi_j} d\theta$$

Here $\xi = \frac{x}{r}$, $\theta = \frac{y}{r}$; E_s, G_s are elasticity and shear modulus of the material of longitudinal ribs; k is the quantity of longitudinal ribs; σ_x are axial contracting stresses u, v, w are components of displacement vector of the shell; h and R are thickness and radius of the shell, respectively; E and ν are the Young modulus and

Poisson ratio of the shell's material; $\xi_1 = \frac{L_1}{r}, L_1$ is the length of the shell, $I_{cr.s}$ is inertia moment in torsion, w_0 is initial deflection.

Influence of medium on the shell is determined as of external surface loads applied to the shell and is calculated as work done by these loads when changing over the system from strain state to initial unstrained one and is represented in the form:

$$A = -R^2 \int_0^{\xi_1} \int_0^{2_\pi} q_z w d\xi d\theta. \tag{2}$$

In order to determine q_z the Pasternak model is used [2]. The essence of this method is that the influence of medium on the shell on the contact surface is determined by the relation

$$q_z = \left(\widetilde{q} + \widetilde{q}_0 \nabla^2 \right) w = kw \tag{3}$$

where ∇^2 is Laplace two dimensional operator on the contact surface.

Strain continuity equation is written in the form [1] :

$$\Delta\Delta\varphi = E \left\{ \left[\frac{\partial^2 (w + w_0)}{\partial\xi\partial\theta} \right]^2 - \left(\frac{\partial^2 w_0}{\partial\xi\partial\theta} \right)^2 - \frac{\partial^2 (w + w_0)}{\partial\xi^2} \frac{\partial^2 (w + w_0)}{\partial\theta^2} + \right.$$

$$\left. + \frac{\partial^2 w_0}{\partial\xi^2} \frac{\partial^2 w_0}{\partial\theta^2} - r \frac{\partial^2 w}{\partial\xi^2} \right\}. \tag{4}$$

We give initial deflection as

$$w_0 = \sum_{m_0=1}^{M\ /\ 2} \sum_{n_0=2}^{N\ /\ 2} a_{m_0 n_0} \cos 2d_{m_0}\xi \cos n_0\theta + b_{m_0 n_0} \sin 2d_{m_0}\xi \sin n_0\theta \tag{5}$$

The coefficients $a_{m_0 n_0}$, $b_{m_0 n_0}$ are calculated by the formula

$$a_{m_0 n_0} = \frac{4}{MN} \sum_{i_1=1}^{M} \sum_{j_1=1}^{N} f_{i_1 j_1} \cos \frac{2\pi m_0}{M} i_1 \cos \frac{2\pi n_0}{N} j_1,$$

$$b_{m_0 n_0} = \frac{4}{MN} \sum_{i_1=1}^{M} \sum_{j_1=1}^{N} f_{i_1 j_1} \sin \frac{2\pi m_0}{M} i_1 \sin \frac{2\pi n_0}{N} j_1,$$

where M and N are the amount of intervals of partitioning of shell's surface in longitudinal and annular directions, respectively, $f_{i_1 j_1}$ is the measured initial deflection at the point with coordinates $\xi_{i_1} = \dfrac{\xi_1 i_1}{M}$, $\theta_{j_1} = \dfrac{2\pi}{N} j_1$.

Solution method. The and faces of the shell are assumed to be simply supported. Deflection of the shell under load is approximated by the expression

$$w = f_1 \sin d_m\xi \sin n\theta + f_2 \sin^4 d_m\xi \sin^2 n\theta, \tag{6}$$

where f_1 and f_2 are variable parameters, m is the number of half waves in longitudinal direction, n is the number of waves in peripheral direction. The expression accepted for w satisfies the boundary conditions $\overline{w}=\overline{M_x}=0$.Putting (6) in (4) and integrating we can obtain an expression for φ, that integrally satisfies the boundary conditons $\dfrac{\partial^2\varphi}{\partial\theta^2}=0,\ \dfrac{\partial^2\varphi}{\partial\xi\partial\theta}=0$.

After subsititution of expressions for w, w_0 and φ in (1) and (2), use of necessary conditions of extremality $\ni$ with respect to f_1 and f_2 the problem is reduced to the solutions of the following system of nonlinear algebraic equations:

$$f_1\frac{h^{*2}}{32}\sum_{m_0=1}^{M\ /\ 2}\sum_{n_0=2}^{N\ /\ 2}\left(\overline{a}^2_{m_0n_0}+\overline{b}^2_{m_0n_0}\right)\widetilde{C}_1+4f_1^3\widetilde{C}_9+f_1\left(2\widetilde{C}_6+\pi\xi_1k\right)+$$

$$+2f_1f_2^2\widetilde{C}_{10}+2f_1f_2\widetilde{C}_{11}-\frac{1}{2}nf_1\overline{\varphi_1}=0 \qquad (7)$$

$$f_2\frac{h^{*2}}{32}\sum_{m_0=1}^{M\ /\ 2}\sum_{n_0=2}^{N\ /\ 2}\left(\overline{a}^2_{m_0n_0}+\overline{b}^2_{m_0n_0}\right)\left(\widetilde{C}_2+\widetilde{C}_3+\frac{9}{4}n^4\widetilde{C}_4+4d_m^4n_0^4\widetilde{C}_5\right)+4f_2^3\widetilde{C}_7+$$

$$f_2\left[2\widetilde{C}_8+\frac{3\pi}{32}\left(1+\frac{27}{8}\xi_1\right)k\right]+2f_1f_1^2\widetilde{C}_{10}+2f_1f_1\widetilde{C}_{11}-\frac{15}{32}\eta f_2\overline{\varphi}_2=0 \qquad (8)$$

where $\overline{\varphi}_2=h^*d_m^2, \eta=\dfrac{\sigma_xR}{Eh}, \overline{a}_{m_0n_0}=\dfrac{a_{m_0n_0}}{h}, \overline{b}_{m_0n_0}=\dfrac{b_{m_0n_0}}{h}$. The constants $\widetilde{C}_i$ $(i=1,2,3...,11)$ have the form:

$$\widetilde{C}_1=(d_{m_0}n-d_mn_0)^4\left\{\left[(d_m+d_{m_0})^2+(n+n_0)^2\right]^{-1}+\right.$$

$$\left.+\left[(d_m-d_{m_0})^2+(n-n_0)^2\right]\right\}+(d_{m_0}n+d_mn_0)^4\times$$

$$\times\left\{\left[(d_m-d_{m_0})^2+(n+n_0)^2\right]^{-1}+\left[(d_m+d_{m_0})^2+(n-n_0)^2\right]^{-1}\right\}$$

$$\widetilde{C}_2=(d_{m_0}n-d_mn_0)^4\left\{\left[(2d_m+d_{m_0})^2+(2n+n_0)^2\right]^{-1}+\right.$$

$$\left.+\left[(2d_m-d_{m_0})^2+(2n-n_0)^2\right]\right\}+(d_{m_0}n+d_mn_0)^4\times$$

$$\times\left\{\left[(2d_m-d_{m_0})^2+(2n+n_0)^2\right]^{-1}+\left[(2d_m+d_{m_0})^2+(2n-n_0)^2\right]^{-1}\right\}$$

$$\widetilde{C}_3=\frac{1}{16}(d_{m_0}n-2d_mn_0)^4\left\{\left[(4d_m+d_{m_0})^2+(2n+n_0)^2\right]^{-1}+\right.$$

$$\left.+\left[(4d_m-d_{m_0})^2+(2n-n_0)^2\right]^{-1}\right\}+\frac{1}{16}(d_{m_0}n+d_mn_0)^4\times$$

$$\times\left\{\left[(4d_m-d_{m_0})^2+(2n+n_0)^2\right]^{-1}+\left[(4d_m+d_{m_0})^2+(2n-n_0)^2\right]^{-1}\right\}$$

$$\widetilde{C}_4=d_{m_0}^4\left\{\left[d_{m_0}^2+(2n+n_0)^2\right]^{-1}+\left[d_{m_0}^2+(2n-n_0)^2\right]^{-1}\right\}$$

$$\widetilde{C}_5=\left[(4d_m+d_{m_0})^2+n_0^2\right]^{-1}+\left[(2d_m+d_{m_0})^2+n_0^2\right]^{-1}+$$

$$+\left[(4d_m-d_{m_0})^2+n_0^2\right]^{-1}+\left[(2d_m-d_{m_0})^2+n_0^2\right]^{-1};$$

$$\widetilde{C}_6=\frac{d_m^4}{4\left(d_m^2+n^2\right)^2}+\frac{a^2\left(d_m^2+n^2\right)^2}{h^*\left(1-v\right)}+\frac{\pi}{2R^3}\left(E_sI_{xs}\left(n^2-1\right)^2S_1+G_sI_{kp.s}n^2d_m^2S_2\right)$$

$$\widetilde{C}_7=\frac{\left(h^*\right)^2n^4}{32}\left[\frac{d_m^4}{512\left(9d_m^2+4n^2\right)^2}+0,243+\frac{1945d_m^4}{2048\left(d_m^2+n^2\right)^2}+\frac{25d_m^4}{32\left(4d_m^2+n^2\right)^2}+\right.$$

$$\left.+\frac{d_m^4}{128\left(16d_m^2+n^2\right)^2}+\frac{25d_m^4}{128\left(9d_m^2+n^2\right)^2}+\frac{225d_m^4}{512\left(d_m^2+4n^2\right)^2}+\frac{425d_m^4}{1024n^4}\right]$$

$$\widetilde{C}_8=\frac{a^2}{h^*\left(1-v^2\right)}\left(\frac{5}{2}n^2+6d_m^2+\frac{35n^4}{16d_m^2}\right)+\frac{\pi}{2R^3}\left(E_sI_{xs}\left(\frac{3}{4}-4n^2+16n^4\right)S_3+\right.$$

$$\left.+64G_sI_{kp.s}n^2d_m^2S_4;\qquad \widetilde{C}_9=\frac{\left(h^*\right)^2}{128}\left(d_m^4+n^4\right);\right.$$

$$\widetilde{C}_{10}=\frac{\left(h^*\right)^2}{512}\left(10d_m^4+7n^4\right)+\frac{\left(h^*\right)^2d_m^4n^4}{1024}\left[\frac{197}{\left(d_m^2+n^2\right)^2}+\frac{1089d_m^4}{\left(9d_m^2+n^2\right)^2}+\right.$$

$$\left.+\frac{289}{\left(25d_m^2+n^2\right)^2}+\frac{484}{\left(d_m^2+9n^2\right)^2}+\frac{1}{\left(25d_m^2+n^2\right)^2}\right],$$

$$\widetilde{C}_{11}=-\frac{h^*n^2}{32}\left(1+\frac{14d_m^2}{\left(d_m^2+n^2\right)^2}\right),\ \widetilde{\varphi}_1=\left(h^*\right)^2d_m^2$$

$$S_1=\sum_{j=1}^{k_1}\sin^2\frac{d_m\xi_1}{M}j=2k_1-\frac{\sin k_1x\cos\left(k_1+1\right)x}{2\sin x};\ S_3=\sum_{j=1}^{k}\sin^8 d_m\xi_j;\ x=\frac{d_m\xi_1}{M}$$

$$S_2=\sum_{j=1}^{k_1}\cos^2\frac{d_m\xi_1}{M}j=2k_1+\frac{\sin k_1x\cos\left(k_1+1\right)x}{2\sin x};\ S_4=\sum_{j=1}^{k_1}\sin^6 d_m\xi_j\cos^2\ d_m\xi_j.$$

Analysis of calculation results. On the basis of the joint solution of the system of equations (7),(8) we construct a curve of equilibrium states whose maximum corresponds to critical value of the load. The load is calculated as follows. The equations (7) and (8) are written in the form of two relations for the parameter of the load

$$\eta=\widetilde{\eta}_1\left(f_1,f_2\right)=\left[\frac{h^{*2}}{16}\sum_{m_0=1}^{M\ /\ 2}\sum_{n_0=2}^{N\ /\ 2}\left(\overline{a}_{m_0n_0}^2+\overline{b}_{m_0n_0}^2\right)\widetilde{C}_1+8f_1^2\widetilde{C}_9+\right.$$

$$\left.+2\left(2\widetilde{C}_6+\pi\xi_1k\right)+4f_2^2\widetilde{C}_{10}+4f_2\widetilde{C}_{11}\right]\widetilde{\varphi}_1^{-1}$$

$$\eta=\widetilde{\eta}_2\left(f_1,f_2\right)=\left\{\frac{h^{*2}}{16}\sum_{m_0=1}^{M\ /\ 2}\sum_{n_0=2}^{N\ /\ 2}\left(\overline{a}_{m_0n_0}^2+\overline{b}_{m_0n_0}^2\right)\times\right.$$

$$\times\left(\widetilde{C}_2+\widetilde{C}_3+\frac{9}{4}n^4\widetilde{C}_4+4d_m^4n_0^4\widetilde{C}_5\right)+4f_2^2\widetilde{C}_7+$$

$$+\left[2\widetilde{C}_8+\pi\left(\frac{3}{4}d_m^2\left(\frac{\xi_1}{7}-\frac{1}{2}\right)\widetilde{q}_0+\frac{3}{32}\left(1+\frac{27}{8}\xi_1\right)k\right)\right]+$$

$$+2f_1^2\widetilde{C}_{10}+2f_1,f_2\ \widetilde{C}_{11}\bigg\}\frac{32}{15}\widetilde{\varphi}_1^{-1}$$

and are third order equations of two surfaces, and their intersection line is a curve of equilibrium state. The numerical values of the parameters m, n and of variable f_2 are given for its construction and the intersection point of the curves $\widetilde{\eta}_1(f_1)$, $\widetilde{\eta}_2(f)$ is sought by means of variation of the variable f_1. After the intersection point of these lines is found, some increment is given to f_2 and computation procedure is repeated until maximum of the curve of equilibrium states is contructed for each pair. The least value of critical load is determined by maximums of these curves.

Fig 1. Dependence of the critical load parameter μ on relative distance between cross ribs b_1

Analysis shows that mean values of critical loads obtained with regard to initial imperfections for 8 cross ribs will be larger than mean values of smooth shells. This is explained by the fact that in the availability of reinforcement the influence of initial deviation on stability of shells decreases.

Naturally, there arises a question on the choice of such reinforcement of the shell in longitudinal direction at which influence of initial deviations will decrease as a result of rational choice of the number of longitudinal ribs and their rigidity.

The dependencies of the critical load parameter on relative distance between the cross ribs b_1, are given in fig. 1. In calculations we accept

$$E=E_c=6,67\cdot 10^9 H/m^2, \rho=\rho_c=0,26\cdot 10^4 H\cdot c^2/m^4, \nu=0,3, \quad M=18,$$

$$N = 32, m_0 = 9, n_0 = 16, k = 2 \cdot 10^6 H/m^3, R = 20sm, L_1 = 45sm, h =$$
$$= 0,5mm, \widetilde{\gamma}_c^{(1)} = 0,306, \widetilde{q}/\widetilde{q}_0 = 0,25.$$

Perfect and imperfect shells were considered. The curves in the figures with indices p and i relate to perfect and imperfect shells, respectively. Solid lines correspond to vibrations of mediumless shell, the dash line-to vibrations of a shell with medium. Notice that the presence of medium reduces to decrease of the value of critical load parameter.

References

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Received December 18, 2007; Revised March 05, 2008.