MECHANICS

Vagif D. GADJIEV, Husamaddin M. GASYMOV,
Nizami Z. GULIYEV

AXISYMMETRIC FORM OF STABILITY LOSS OF AN INHOMOGENEOUS CYLINDRICAL SHELL OF ANNULAR CROSS SECTION

Abstract

In the paper we consider an axisymmetric form of stability loss of a cylindrical shell inhomogeneous in length, inhomogeneous in thickness under axial compression. In the first case Bubnov-Galerkin method is applicd and a formula for determining critical value of contractive load is obtained. Calculation is conducted and a graph of dependence between critical load and imhomogeneity parameter is constructed.

It is known that cylindrical shell's of annular cross section made of different materials are the most distributed structural elements that are widely used in engineering practice. One of these materials is continuously inhomogeneous elastic and inelastic material. When a shell is under the action of contractive loads, stability analysis is important [1,2].

Assume that a cylindrical shell of annular cross section is under the action of contractive force applied on the end faces of an annular closed shell. Suppose that elasticity modulus E is a continuous function of length coordinate with its derivatives up to fourth order and Poisson ratio is a constant quantity

$$E = E_0 f(x), \qquad \nu = const \tag{1}$$

here E_0 corresponds to homogeneous case. Assume that the shell loses its stability by axisymmetric form. Then stability equation and strain compatibility condition have the following form:

$$D_0 \frac{d^2}{dx^2}\left[f(x)\frac{d^2W}{dx^2}\right] - \frac{1}{R}\frac{d^2\Phi}{dx^2} - \sigma_{cr} h \frac{d^2W}{dx^2} = 0$$

$$\frac{1}{E_0 f(x) h}\frac{d^4\Phi}{dx^4} = \frac{1}{R}\frac{d^2W}{dx^2} \tag{2}$$

here D_0 is cylindrical rigidity of homogeneous shell, Φ is a stress function, R is a radius, h is shell's thickness, W is flexure.

We can write system (2) in the form:

$$D_0\left[f(x)\frac{d^4W}{dx^4}+2\frac{df(x)}{dx}\frac{d^3W}{dx^3}+\frac{d^2f(x)}{dx^2}\frac{d^2W}{dx^2}\right]-\frac{1}{R}\frac{d^2\Phi}{dx^2}-\sigma_{cr}h\frac{d^2W}{dx^2}=0$$

$$\frac{d^4\Phi}{dx^4}=\frac{E_0f(x)h}{R}\frac{d^2W}{dx^2}. \qquad (3)$$

Twice differentiating the first equation of (3) with respect to X and substituting the expression $\frac{d^4\Phi}{dx^4}$ into the stability equation we get:

$$D_0\left[f(x)\frac{d^6\Phi}{dx^6}+b_1(x)\frac{d^5W}{dx^5}+b_2(x)\frac{d^4W}{dx^4}+b_3(x)\frac{d^3W}{dx^3}+b_4(x)\frac{d^2W}{dx^2}\right]-$$

$$-\frac{E_0f(x)h}{R^2}\frac{d^2W}{dx^2}-\sigma_{cr}h\frac{d^4W}{dx^4}=0. \qquad (4)$$

Here we introduce the following denotation:

$$b_1(x)=4\frac{df(x)}{dx};\quad b_2(x)=6\frac{d^2f(x)}{dx^2};\quad b_3(x)=4\frac{d^3f(x)}{dx^3};\quad b_4(x)=\frac{d^4f(x)}{dx^4}. \qquad (5)$$

To the equation we can give the following form:

$$L(W)-\sigma_{cr}h\frac{d^4W}{dx^4}=0 \qquad (6)$$

where

$$L(W)=D_0\left[f(x)\frac{d^6W}{dx^6}+b_1(x)\frac{d^5W}{dx^5}+b_2(x)\frac{d^4W}{dx^4}+b_3(x)\frac{d^3W}{dx^3}+b_4(x)\frac{d^2W}{dx^2}\right]-$$

$$-\frac{E_0h}{R^2}f(x)\frac{d^2W}{dx^2}. \qquad (7)$$

As is seen, equation (6) is complicated and its complexity depends on the function $f(x)$. For homogeneous boundary conditions we can apply Bubnov-Galerkin's method to the equation, choosing $W(x)$ in the following form [3]:

$$W=\sum_{i=1}^{n}A_iV_i(x) \qquad (8)$$

here each of $V_i(x)$ should satisfy the appropriate boundary conditions.

Taking into account (8) in (6), by means of Bubnov-Galerkin orthogonalization we can write:

$$\sum_{i=1}^{n}A_i\int_0^l\left[L(V_i)-\sigma_{cr}h\frac{d^4V_i}{dx^4}\right]V_k(x)\,dx=0 \qquad \left(k=\overline{1,n}\right). \qquad (9)$$

As is known, in general case σ_{kp} is determined from the condition of equality to zero of the main determinant of a system of linear algebraic equations composed of coefficients A_i. As it is rightly noticed in [3], for determining enjineering desing formula it suffices to determine the value of σ_{cr} that corresponds to the first approximation, i.e.

$$\int_0^l \left[L(V_1) - \sigma_{cr} h \frac{d^4 V_1}{dx^4} \right] V_1(x)\, dx = 0. \tag{10}$$

Hence we can establish:

$$\sigma_{cr} h = \frac{\int_0^l L(V_1) V_1(x)\, dx}{\int_0^l \frac{d^4 V_1}{dx^4} V_1(x)\, dx}. \tag{11}$$

For determining $\sigma_{cr} h$ we use a simple form (hinged joint) of approximation:

$$V_1 = \sin\ m\pi\overline{x}, inhomogeneity\ \ functions\ \ f(\overline{x}) = 1 + \mu\overline{x}. \tag{12}$$

Allowing for (12) formula (11) has the following form:

$$\sigma_{cr} h = \frac{\int_0^1 L(\sin\ m\pi\overline{x}) \sin\ m\pi\overline{x} d\overline{x}}{\left(\frac{m\pi}{l}\right)^4 \int_0^1 \sin^2\ m\pi\overline{x} d\overline{x}}, \quad \left(\overline{x} = x l^{-1}\right). \tag{13}$$

Now, let's calculate $\int_0^1 L(\sin\ m\pi\overline{x}) \sin\ m\pi\overline{x} d\overline{x}$ when $b_1(x) = 4\mu l^{-1}$; $b_2(x) = b_3(x) = b_4(x) = 0.$

$$L(V_1) = D_0 \left[-\left(\frac{m\pi}{l}\right)^6 (1 + \mu\overline{x}) \sin\ m\pi\overline{x} + 4\mu l^{-1} \left(\frac{m\pi}{l}\right)^5 \cos m\pi\overline{x} \right] +$$

$$+ \frac{E_0 h}{R^2} \left(\frac{m\pi}{l}\right)^2 (1 + \mu\overline{x}) \sin\ m\pi\overline{x}. \tag{14}$$

Substituting (14) into the formula (13) we get:

$$\sigma_{cr} h = \frac{D_0 \left(\frac{m\pi}{l}\right)^6 \int_0^1 (1 + \mu\overline{x}) \sin^2\ m\pi\overline{x} d\overline{x} + \frac{E_0 h}{R^2} \left(\frac{m\pi}{l}\right)^2 \int_0^1 (1 + \mu\overline{x}) \sin^2\ m\pi\overline{x} d\overline{x}}{\left(\frac{m\pi}{l}\right)^4 \int_0^1 \sin^2\ m\pi\overline{x} d\overline{x}}$$

or

$$\sigma_{cr} h = \left(1 + \frac{1}{2}\mu\right) \left(-D_0 \lambda^2 + \frac{E_0 h}{R^2 \lambda^2}\right), \quad \lambda^2 = \left(\frac{m\pi}{l}\right)^2$$

For $\mu = 0$ we get the solution of the problem for a homogeneous shell

$$\sigma_{cr}^{0} h = -D_0 \lambda^2 + \frac{E_0 h}{R^2 \lambda^2}$$

Hence we get:

$$\overline{\sigma}_{cr} = \frac{\sigma_{cr}^{0}}{\sigma_{cr}} = (1 + 0,5\mu)^{-1}$$

The $\overline{\sigma}_{cr}$ are calculated for different values of μ and represented on the form of a table and graph of dependence $\overline{\sigma}_{cr} \sim \mu$.

Notice that calculation is not difficult for the cases of other kinds of inhomogeneity. The following functions are examples: $f(\overline{x}) = 1 + \mu \overline{x}^2$; $f(\overline{x}) = e^{\beta \overline{x}}$; $f(\overline{x}) = \left(1 + \mu e^{\beta \overline{x}}\right)$; $f(\overline{x}) = 1 + \mu \cos 2m\pi \overline{x}$ and others.

As we see, when μ increases, the true solution sharply differs from the appropriate inhomogeneous problem.

Similtaneously we notice that the problem is sufficiently simplified when elasticity modulus is only a thickness coordinate function.

$$E = E_0 \psi(z), \qquad \nu = const.$$

In this case, a system of stability equations and continuity equations are written in the following form:

$$\overline{D} \frac{d^4 W}{dx^4} - \frac{1}{R} \frac{d^2 \Phi}{dx^2} - \sigma_{cr} h \frac{d^2 W}{dx^2} = 0$$

$$\frac{1}{\overline{\overline{E}} h} \frac{d^4 \Phi}{dx^4} = \frac{1}{R} \frac{d^2 W}{dx^2}, \tag{15}$$

here $\overline{E} = \frac{2E_0}{h} \int\limits_{-h/2}^{h/2} \psi(z)\, dz$ or

$$\overline{E} = E_0 \int\limits_{-1/2}^{1/2} \psi(\eta)\, d\eta, \quad \overline{D} = D_0 A,$$

$$D_0 = \frac{E_0 h^3}{12(1-\nu^2)} \quad A = 12 \int\limits_{-1/2}^{1/2} \psi(\eta)\, \eta^2 d\eta, \quad \eta = zh^{-1}.$$

Here we can accept:

$$\Phi = A_m \sin \frac{m\pi}{l} x; \quad W = B_m \sin \frac{m\pi}{l} x \tag{16}$$

Substituting (16) into (15) we get:

$$B_m \left[\overline{D} \left(\frac{m\pi}{l} \right)^4 + \sigma_{cr} h \left(\frac{m\pi}{l} \right)^2 \right] + A_m \frac{1}{R} \left(\frac{m\pi}{l} \right)^2 = 0$$

$$B_m \frac{1}{R} \left(\frac{m\pi}{l} \right)^2 + A_m \frac{1}{\overline{E}h} \left(\frac{m\pi}{l} \right)^4 = 0 \tag{17}$$

For the existence of non-trivial solution, the main determinant should vanish. Hence we find

$$\sigma_{cr} = \frac{\overline{E}}{R^2 \lambda^2} - \frac{\overline{D} \lambda^2}{h}$$

Giving the values of $\psi(z)$ we can calculate σ_{cr}.

References

[1]. Lomakin V.A. *Theory of elasticity of inhomogeneous bodies.* Izd. MGU, 1976, 376 p. (Russian).

[2]. Volmir A.S. *Stability of deformable systems.* M., 1976, 890 p. (Russian).

[3]. Alfutov A.A. *Stability analysis of elastic systems.* M., 1991. 334 p. (Russian).

[4]. Gasymov G.M. S*tability of inhomogeneous cylindrical shells and right-angled plates allowing for geometrical nonlinearity and resistence of enviroument.* The auther's thesis for Ph. *degree.* Baku, 1991, 15 p. (Russian).

Vagif D. Gadjiev, Husamaddin M. Gasymov
Institute of Mathematics and Mechanics of NAS of Azerbaijan.
9, F.Agayev str., AZ1141, Baku, Azerbaijan.
Tel.: (99412) 439 47 20 (off.)

Nizami Z. Guliyev
Nakhicivan State University,
University campus, AZ 7000, Nakcivan, Azerbaijan.
Tel.: (99412) 439 4720 (off.).

Received January 15, 2008; Revised April 23, 2008.