MATHEMATICS

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STATIONARY CHARACTERISTICS OF THE SINGLE-SERVER CLOSED QUEUEING SYSTEMS WITHOUT INTERRUPTIONS AND WITH THE OPPORTUNITY OF INTERRUPTION OF SERVICE

Abstract

For the single-server closed queueing system with exponentially distributed time of non-failure operation of the technical device and arbitraryly distributed service time probabilities of states of limiting stationary process are defined. The case when there is an opportunity of interruption of process of service is separately examined.

1. Introduction. In case of arbitraryly distributed service time and exponentially distributed interarrival time existence of limiting stationary process for the single-server open queueing system with unlimited queue is proved by a method of embedded Markov chains [1, p. 98]. Research of ergodic properties of the single-server closed queueing system is complicated because for it the input stream of customers never can be the stationary Poisson. This intensity changes together with arrival of each new customer and even there are equal to zero after all technical devices have arrived on service.

However the stochastic process proceeding in closed queueing system with the stationary Poisson streams, is the stationary Poisson process of Birth-and-Death and consequently has ergodic property. Formulas for stationary probabilities of states for such queueing system are known [2, p. 282].

In this work stationary characteristics are defined for single-server closed queueing system with exponentially distributed time of non-failure operation of technical devices and arbitrarily distributed service time. Queueing system for which there is an opportunity of interruption of process of service is separately examined.

2. The general formulas for stationary characteristics. We shall consider single-server closed queueing system, in which each of m $(m \geq 2)$ technical devices can require service at some random moment of time. Let a stream of refusals of each technical device are an elementary (stationary Poisson) with intensity λ. It means, that time T_λ of non-failure operation is exponential. An expectation of a random variable T_λ we shall designate through e_λ: $E(T_\lambda) = e_\lambda = 1/\lambda$. The technical device which requires service and finds the server of service free, immediately is accepted on service. In case of employment of the server the technical device waits for service in queue.

We shall assume that service times are independent equally distributed random variables T_μ with arbitrary distribution function $F_\mu(t)$. Then intensity of a stream of service $\mu = 1/e_\mu$, where $e_\mu = E(T_\mu) = \int\limits_0^\infty t\, dF_\mu(t)$.

We shall introduce numbering of states of queueing system according to number of the technical devices which require service: s_i $(i = \overline{0, m}\,)$. Let $p_i(t)$ is a probability of stay of system in a state s_i at the moment of time t.

We shall assume, that at an initial moment of time the system is in a state s_0, and we shall designate as t_k $(k = 1,2,\ldots)$ the moment of end of service of k-th customer, and as N_k number of the technical devices which require service at such moment when k-th served customer leaves the server of service after the termination of service. Then the event $\{N_k = j\}$ denotes, that at a moment $t_k + 0$ the system passes in a state s_j.

Under the formula of a total probability

$$\begin{gathered} P\{\, N_{k+1} = j\,\} = \sum_{s=0}^{m-1} P\{\, N_{k+1} = j \,|\, N_k = s\,\} P\{\, N_k = s\,\}, \\ j = \overline{0,\, m-1}\,; \qquad k = 1,\, 2, \ldots . \end{gathered} \tag{1}$$

Transitive probabilities $p_{js}^{(k)} = p_{js} = P\{\, N_{k+1} = j \,|\, N_k = s\,\}$ do not depend on number k of the served customer, and these probabilities are determined according to number of the customers arriving on service during service time of one customer T_μ:

$$p_{js} = \begin{cases} \Pi_{j,\, m-1}, & s = 0;\ 1; \\ \Pi_{j-s+1,\, m-s}, & s = \overline{2,\, j+1},\ j = \overline{0,\, m-2};\ s = \overline{2,\, m-1},\ j = m-1; \\ 0, & s > j+1. \end{cases} \tag{2}$$

Here Π_{kl} is the probability of failure of k technical devices during service time T_μ provided that l technical devices were potential customers at the moment of the beginning of service.

Taking into account, that the presence in system of i potential clients means, that the rest of time till the moment of arrival of the next client is exponential with parameter $i\lambda$ (we shall designate this random variable as $T_{i\lambda}$ $(i = \overline{1,\, m-1}\,)$), for probabilities Π_{kl} we shall receive relations:

$$\begin{gathered} \Pi_{0l} = P\{\, T_\mu < T_{l\lambda}\,\}, \qquad l = \overline{1,\, m-2}; \\ \Pi_{ll} = P\{\, T_{l\lambda} + T_{(l-1)\lambda} + \cdots + T_\lambda < T_\mu\,\}, \qquad l = \overline{1,\, m-1}; \\ \Pi_{kl} = P\{\, T_{l\lambda} + T_{(l-1)\lambda} + \cdots + T_{(l-k+1)\lambda} < T_\mu < \\ < T_{l\lambda} + T_{(l-1)\lambda} + \cdots + T_{(l-k)\lambda}\,\}, \qquad l = \overline{2,\, m-1}, \quad k < l. \end{gathered} \tag{3}$$

We shall be convinced that the sequence $\{N_k\}$ forms Markov chain.

If Δ_k is the number of the customers who have arrived to system in service time T_μ of k-th customer, then

$$N_k = \begin{cases} N_{k-1} + \Delta_k - 1, & N_{k-1} > 0; \\ \Delta_k, & N_{k-1} = 0. \end{cases} \quad (4)$$

The rest of time from the moment of end of service of k-th customer till the moment of arrival of next customer is exponential (with parameter, divisible by λ) and, therefore does not depend on behaviour of system till the moment t_k. So that the random variable Δ_k does not depend on the past. So, equality (4) proves Markov property of sequence $\{N_k\}$. Equalities (2) prove uniformity of Markov chain $\{N_k\}$.

Using the ergodic theorem for homogeneous nonreducible Markov chain with finite set of states ([2, p. 61] or [3, p. 67]), we can prove existence of limits

$$\pi_j = \lim_{k\to\infty} P\{\, N_k = j \,\} \qquad (\, j = \overline{0,\, m-1}\,).$$

Having passed to a limit in equality (1) and and considering the relations (2), we shall receive system of the algebraic equations for definition of probabilities π_j:

$$\pi_j = \sum_{s=0}^{j+1} p_{js}\pi_s \quad (j = \overline{0,\, m-2}); \qquad \sum_{s=0}^{m-1} \pi_s = 1. \quad (5)$$

Let's analyse work of system on very big time interval T for reception of relation between probabilities π_j $(j = \overline{0,\, m-1})$ and stationary distribution of probabilities of states of system $p_j = \lim\limits_{t\to\infty} p_j(t)$ $(j = \overline{0,\, m})$.

Time intervals of stay of system in a state s_0 are exponentially distributed with parameter $m\lambda$ $(\, E(T_{m\lambda}) = e_{m\lambda} = 1/(m\lambda)\,)$. Let $N(T)$ is the number of the customers served in system in time T, then

$$T \approx (e_{m\lambda}\pi_0 + e_\mu)N(T). \quad (6)$$

Equality (6) is carried out with greater accuracy when the time interval T increases. Taking into account, that $T_0(T) \approx e_{m\lambda}\pi_0 N(T)$ is total time (in time T) of stay of system in a state s_0, we shall define stationary probability of stay of system in this state by means of limiting transition

$$p_0 = \lim_{T\to\infty} \frac{T_0(T)}{T} = \frac{e_{m\lambda}\pi_0}{e_\mu + e_{m\lambda}\pi_0}.$$

At the moment when the served customer leaves the server of service, the state s_j comes with probability π_j and lasts during random time $T_{(m-j)\lambda}$. Therefore $T_j(T) \approx\ e_{(m-j)\lambda}\pi_j N(T)$ is total time (in time T) of stay of system in a state s_j, and

$$p_j = \lim_{T\to\infty} \frac{T_j(T)}{T} = \frac{e_{(m-j)\lambda}\pi_j}{e_\mu + e_{m\lambda}\pi_0} \qquad (j = \overline{0,\, m-1}). \quad (7)$$

We shall define probability p_m by means of a normalization condition $\sum\limits_{j=0}^{m} p_j = 1$:

$$p_m = \frac{e_\mu - \sum\limits_{j=1}^{m-1} e_{(m-j)\lambda}\pi_j}{e_\mu + e_{m\lambda}\pi_0}. \tag{8}$$

Using the found probabilities p_j $(j = \overline{0,\, m})$, we can define parameters of efficiency of single-server closed queueing system, in particular an average of customers in queue

$$\begin{gathered} \overline{r} = \sum_{j=2}^{m} (j-1)p_j = \\ = \frac{1}{e_\mu + e_{m\lambda}\pi_0}\left(\sum_{j=2}^{m-1}(j-1)e_{(m-j)\lambda}\pi_j + (m-1)\left(e_\mu - \sum_{j=1}^{m-1} e_{(m-j)\lambda}\pi_j\right)\right), \end{gathered} \tag{9}$$

and average time of stay of the technical device in queue

$$\begin{gathered} \overline{t}_r = \lim_{T\to\infty} \frac{\sum\limits_{j=2}^{m}(j-1)T_j(T)}{N(T)} = \lim_{T\to\infty} \frac{T}{N(T)} \sum_{j=2}^{m} \frac{(j-1)T_j(T)}{T} = \\ = \overline{r} \lim_{T\to\infty} \frac{T}{N(T)} = \overline{r}\,(e_\mu + e_{m\lambda}\pi_0) = \\ = \sum_{j=2}^{m-1}(j-1)e_{(m-j)\lambda}\pi_j + (m-1)\left(e_\mu - \sum_{j=1}^{m-1} e_{(m-j)\lambda}\pi_j\right). \end{gathered} \tag{10}$$

For queueing system with two technical devices ($m = 2$) the system (5) consists only of two equations

$$\pi_0 = \Pi_{01}(\pi_0 + \pi_1), \qquad \pi_0 + \pi_1 = 1,$$

where according to (3) $\Pi_{01} = P\{\, T_\mu < T_\lambda \,\}$. So, in this case

$$\pi_0 = \Pi_{01} = P\{\, T_\mu < T_\lambda \,\} = \int_0^\infty e^{-\lambda t}\, dF_\mu(t), \qquad \pi_1 = 1 - \pi_0. \tag{11}$$

For cases $m = 3$ and $m = 4$ solutions of system (5) are convenient for writing down, having entered new designations for probabilities Π_{kl}:

$$\pi_0 = \frac{\beta_1\beta_2}{\beta_1 + \beta_{21}}, \quad \pi_1 = \frac{\beta_1(1-\beta_2)}{\beta_1 + \beta_{21}}, \quad \pi_2 = \frac{\beta_{21}}{\beta_1 + \beta_{21}}; \qquad (\, m = 3 \,) \tag{12}$$

$$\begin{gathered} \pi_0 = (1-\beta_1)\beta_2\beta_3\Delta, \qquad \pi_1 = (1-\beta_1)\beta_2(1-\beta_3)\Delta, \\ \pi_2 = \beta_{32}(1-\beta_1)\Delta, \qquad \pi_3 = (\beta_2\beta_{31} + \beta_{21}\beta_{32})\Delta, \\ \frac{1}{\Delta} = \beta_2(1-\beta_1+\beta_{31}) + \beta_{32}(1-\beta_1+\beta_{21}), \qquad (\, m = 4 \,) \end{gathered} \tag{13}$$

where

$$\beta_i = P\{\,T_\mu < T_{i\lambda}\,\} \quad (\,i = \overline{1,\,3}\,)\,; \qquad \beta_{21} = P\{\,T_{2\lambda} + T_\lambda < T_\mu\,\};$$
$$\beta_{31} = P\{\,T_{3\lambda} + T_{2\lambda} + T_\lambda < T_\mu\,\}; \qquad \beta_{32} = P\{\,T_{3\lambda} + T_{2\lambda} < T_\mu\,\}.$$

3. Results of calculations for typical distributions of a service time. Using the found probabilities π_j $(j = \overline{0,\,m-1}\,)$, we can define stationary distribution of probabilities of states of system p_j $(j = \overline{0,\,m}\,)$, average values $\overline{r}$, $\bar{t}_r$ and other parameters of efficiency of single-server closed queueing system for any distribution of a service time T_μ. In particular, in case of exponential distribution of a service time when $e_\mu = 1/\mu$, from (7), (8) we receive formulas [2, p. 282–283] for single-server closed queueing system with the stationary Poisson streams.

Let's demonstrate the results received under formulas (7) – (13) for some typical distributions of a random variable T_μ.

Example 1. *The service time is determined:* $T_\mu = T = const$, $e_\mu = T$.

Case when $m = 2$.

$$p_0 = \alpha_0 e^{-\lambda T}, \quad p_1 = 2\alpha_0(1 - e^{-\lambda T}), \quad p_2 = 2\alpha_0(\lambda T + e^{-\lambda T} - 1),$$
$$\frac{1}{\alpha_0} = 2\lambda T + e^{-\lambda T}; \qquad \bar{t}_r = (\lambda T + e^{-\lambda T} - 1)/\lambda.$$

Case when $m = 3$.

$$p_0 = \beta_0 e^{-3\lambda T}, \quad p_1 = 1,5\beta_0 e^{-\lambda T}(1 - e^{-2\lambda T}), \quad p_2 = 3\beta_0(1 - 2e^{-\lambda T} + e^{-2\lambda T});$$
$$p_3 = 1,5\beta_0\left(2\lambda T(1 - e^{-\lambda T} + e^{-2\lambda T}) - 2(1 + e^{-2\lambda T}) + 3e^{-\lambda T} + e^{-3\lambda T}\right);$$
$$\frac{1}{\beta_0} = e^{-3\lambda T} + 3\lambda T(1 - e^{-\lambda T} + e^{-2\lambda T});$$
$$\bar{t}_r = 2T + \frac{e^{-3\lambda T}}{\lambda(1 - e^{-\lambda T} + e^{-2\lambda T})} - \frac{1}{\lambda}.$$

Case when $m = 4$.

$$\frac{1}{p_0} = 1 + 4\lambda T e^{\lambda T}(1 - 2e^{\lambda T} + 3e^{2\lambda T} - e^{3\lambda T} - e^{4\lambda T} + e^{5\lambda T});$$
$$p_1 = \frac{4}{3}(e^{3\lambda T} - 1)p_0; \qquad p_2 = 2e^{2\lambda T}(e^{3\lambda T} - 3e^{\lambda T} + 2)p_0;$$
$$p_3 = 4e^{\lambda T}(e^{5\lambda T} - 2e^{4\lambda T} - e^{3\lambda T} + 5e^{2\lambda T} - 4e^{\lambda T} + 1)p_0;$$
$$p_4 = \frac{p_0}{3}\left(7 + 12\lambda T e^{\lambda T}(1 - 2e^{\lambda T} + 3e^{2\lambda T} - e^{3\lambda T} - e^{4\lambda T} + e^{5\lambda T}) - \right.$$
$$\left. -12e^{\lambda T}(e^{5\lambda T} - e^{3\lambda T} - 4e^{\lambda T} + 1) - 38e^{3\lambda T} + 18e^{5\lambda T}\right);$$
$$\overline{r} = p_2 + 2p_3 + 3p_4; \qquad \bar{t}_r = \frac{1 - p_0 + \overline{r}}{\lambda(3 + p_0 - \overline{r})} - T.$$

Example 2. *The service time is uniform distributed on the interval* $(\,a,\,b\,)$: $e_\mu = (a + b)/2$.

Case when $m = 2$.

$$p_0 = \gamma_0(e^{-\lambda a} - e^{-\lambda b});\ p_1 = 2\gamma_0\left(\lambda(b-a) + e^{-\lambda b} - e^{-\lambda a}\right);$$

$$p_2 = \gamma_0\left(\lambda^2(b^2 - a^2) - 2\left(\lambda(b-a) + e^{-\lambda b} - e^{-\lambda a}\right)\right);$$

$$\frac{1}{\gamma_0} = \lambda^2(b^2 - a^2) + e^{-\lambda a} - e^{-\lambda b};$$

$$\bar{t}_r = \frac{a+b}{2} - \frac{\lambda(b-a) - e^{-\lambda a} + e^{-\lambda b}}{\lambda^2(b-a)}.$$

Case when $m = 3$.

$$p_0 = 2\delta_0(e^{-\lambda a} - e^{-\lambda b})(e^{-2\lambda a} - e^{-2\lambda b});$$

$$p_1 = 3\delta_0(e^{-\lambda a} - e^{-\lambda b})\left(2\lambda(b-a) + e^{-2\lambda b} - e^{-2\lambda a}\right);$$

$$p_2 = 6\delta_0\lambda(b-a)\left(2\lambda(b-a) - 4(e^{-\lambda a} - e^{-\lambda b}) + e^{-2\lambda a} - e^{-2\lambda b}\right);$$

$$p_3 = 3\delta_0\left(\lambda^2(b^2 - a^2)\left(2\lambda(b-a) - 2(e^{-\lambda a} - e^{-\lambda b}) + e^{-2\lambda a} - e^{-2\lambda b}\right)\right) -$$

$$-3\lambda(b-a)\left(2\lambda(b-a) - 3(e^{-\lambda a} - e^{-\lambda b}) + e^{-2\lambda a} - e^{-2\lambda b}\right) -$$

$$-(e^{-\lambda a} - e^{-\lambda b})(e^{-2\lambda a} - e^{-2\lambda b})\Big);$$

$$\overline{r} = p_2 + 2p_3; \qquad \bar{t}_r = \frac{\overline{r}}{6\lambda}\left(2\pi_0 + 3\lambda(a+b)\right);$$

$$\pi_0 = \frac{(e^{-\lambda a} - e^{-\lambda b})(e^{-2\lambda a} - e^{-2\lambda b})}{\lambda(b-a)\left(2\lambda(b-a) - 2(e^{-\lambda a} - e^{-\lambda b}) + e^{-2\lambda a} - e^{-2\lambda b}\right)}.$$

Example 3. *The service time is distributed under generalized Erlang law of the second order with parameters $\mu_1,\ \mu_2$; $e_\mu = (\mu_1 + \mu_2)/(\mu_1\mu_2)$.*

Case when $m = 2$.

$$p_0 = (\mu_1\mu_2)^2\eta_0; \qquad p_1 = 2\lambda\mu_1\mu_2\eta_0(\lambda + \mu_1 + \mu_2);$$

$$p_2 = 2\lambda\eta_0\left((\mu_1 + \mu_2)(\lambda + \mu_1)(\lambda + \mu_2) - \mu_1\mu_2(\lambda + \mu_1 + \mu_2)\right);$$

$$\frac{1}{\eta_0} = (\mu_1\mu_2)^2 + 2\lambda(\mu_1 + \mu_2)(\lambda + \mu_1)(\lambda + \mu_2);$$

$$\bar{t}_r = \frac{\mu_1 + \mu_2}{\mu_1\mu_2} - \frac{\lambda + \mu_1 + \mu_2}{(\lambda + \mu_1)(\lambda + \mu_2)}.$$

Case when $m = 3$. We shall enter designations: $\alpha_i = \lambda/\mu_i$ $(i = 1,\ 2)$. Then

$$\frac{1}{p_0} = 1 + 3(\alpha_1 + \alpha_2)\left((1 + \alpha_1)(1 + \alpha_2) +\right.$$

$$\left.+(\alpha_1 + \alpha_2 + \alpha_1\alpha_2)(1 + 2\alpha_1)(1 + 2\alpha_2)\right); p_1 = 3(\alpha_1 + \alpha_2 + 2\alpha_1\alpha_2)p_0;$$

$$p_2 = 6\left(\alpha_1^2 + \alpha_2^2 + \alpha_1\alpha_2 + 2\alpha_1^2\alpha_2^2 + 3\alpha_1\alpha_2(\alpha_1 + \alpha_2)\right)p_0;$$

$$p_3 = 6\left(\alpha_1^2(\alpha_1 + 3\alpha_1\alpha_2 + \alpha_2^2 + 2\alpha_1\alpha_2^2) + \right.$$

$$\left. +\alpha_2\left(\alpha_1^2 + \alpha_2^2 + \alpha_1\alpha_2 + 2\alpha_1^2\alpha_2^2 + 3\alpha_1\alpha_2(\alpha_1 + \alpha_2)\right)\right) p_0;$$

$$\overline{r} = p_2 + 2p_3; \qquad \overline{t}_r = \frac{\overline{r}\,\pi_0}{3\lambda p_0};$$

$$\frac{1}{\pi_0} = (1+\alpha_1)(1+\alpha_2) + (\alpha_1 + \alpha_2 + \alpha_1\alpha_2)(1+2\alpha_1)(1+2\alpha_2).$$

4. Single-server closed queueing system with an opportunity of interruption of process of service. Let, as well as earlier, T_λ is exponential time of non-failure operation of each technical device, and let's to consider service time T_μ is also exponentially distributed. We shall assume, that the working server of closed queueing system can interrupt service in random time T_ν for random time T_γ. Time T_ν is counted from the moment of the beginning of service. After end of time T_γ the interrupted service renews and comes to an end in exponential time T_μ.

We assume, that random variables T_λ, T_μ, T_ν and T_γ are independent in aggregate, and T_ν, T_γ are arbitrarily distributed with distribution functions $F_\nu(t)$ and $F_\gamma(t)$ respectively.

We shall consider a random variable

$$T_{\tilde{\mu}} = \begin{cases} T_{\mu\nu}, & \text{if } \; T_\mu < T_\nu; \\ T_{\mu\nu} + T_\gamma + T_\mu, & \text{if } \; T_\nu < T_\mu, \end{cases} \tag{14}$$

where

$$T_{\mu\nu} = \min\{T_\mu,\ T_\nu\}.$$

$T_{\tilde{\mu}}$ is a service time of one technical device with an opportunity of interruption of process of service. In case of interruption of service $T_{\tilde{\mu}}$ includes also time of interruption T_γ.

Let's keep numbering of states of queueing system s_i $(i = \overline{0,\, m})$ according to number of the technical devices which require service. Then all reasonings of item 2 will remain in force if to replace in them random variable T_μ on $T_{\tilde{\mu}}$. So, for stationary characteristics of system with interruptions we shall receive formulas of a kind (7)–(13) but it is necessary to replace in these formulas an expectation $e_\mu = E(T_\mu)$ on

$$e_{\tilde{\mu}} = E(T_{\tilde{\mu}}) = \int\limits_0^\infty t p_{\tilde{\mu}}(t)\, dt \tag{15}$$

($p_{\tilde{\mu}}(t)$ is a distribution density of probabilities of random variable $T_{\tilde{\mu}}$), and $F_\mu(t)$ it is necessary to replace on $F_{\tilde{\mu}}(t)$, that is distribution function of random variable $T_{\tilde{\mu}}$.

Considering (14), distribution density of probabilities of random variable $T_{\tilde{\mu}}$ is determined in the form of

$$p_{\tilde{\mu}}(t) = P\{T_\mu < T_\nu\} p_{\mu\nu}(t) + P\{T_\nu < T_\mu\} p_{(\mu\nu)\gamma\mu}(t).$$

Here $p_{\mu\nu}(t)$, $p_{(\mu\nu)\gamma\mu}(t)$ are distribution densities of random variables $T_{\mu\nu}$ and

$$T_{(\mu\nu)\gamma\mu} = T_{\mu\nu} + T_{\gamma} + T_{\mu}$$

respectively,

$$P\{T_{\nu} < T_{\mu}\} = \int_0^{\infty} e^{-\mu t} dF_{\nu}(t), \qquad p_{\mu\nu}(t) = \mu e^{-\mu t}\left(1 - F_{\nu}(t)\right) + p_{\nu}(t) e^{-\mu t};$$

$$p_{(\mu\nu)\gamma\mu}(t) = \int_0^t \left\{ \int_0^y p_{\mu\nu}(x) p_{\gamma}(y-x)\, dx \right\} \mu e^{-\mu(t-y)} dy,$$

$p_{\gamma}(t)$ there is a distribution density of random variable T_{γ}.

Instead of the formula (15), for calculation $e_{\tilde{\mu}}$ we can use equality

$$e_{\tilde{\mu}} = e_{\mu\nu} + P\{T_{\nu} < T_{\mu}\} \left(e_{\gamma} + \frac{1}{\mu} \right),$$

where

$$e_{\mu\nu} = E(T_{\mu\nu}), \qquad e_{\gamma} = E(T_{\gamma}).$$

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