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COMPLETENESS OF ELEMENTARY SOLUTIONS OF DIFFERENTIAL-OPERATOR EQUATIONS

Abstract

In the paper the Cauchy problem is considered for parabolic differential-operator equation. Here we investigate the problem, when the solution of Cauchy problem for differential-operator equations can be approximated by the linear combinations of elementary solutions. In the present paper the considered boundary conditions contains the time differentiation. The obtained abstract results have numerious applications.

The role and importance of Fourier method in mathematical physics is known. Unfortunately, basically, it applicable in those cases, when the corresponding spectral problem is self-adjoined. In this paper the Caucy problem is investigated for differential-operator equations of parabolic type.

The Cauchy problem for parabolic differential-operator equations and mixed problem for partial parabolic equations is studied in the S.Ya.Yakubov paper [1]. In the present paper the considered boundary conditions contain the time differentiation. This reduces to the consideration of system of operator bundles in boundary conditions.

Let's E and E^{ν}, $\nu = 1, ..., m$, be a Banach spaces. Let's consider in Banach space E the system of operator bundles

$$L(\lambda) u = \lambda^n u + \lambda^{n-1} A_1 u + ... + A_n u = 0 \text{ in } E$$

$$L_{\nu}(\lambda) u = \lambda^{n_{\nu}} A_{\nu 0} u + \lambda^{n_{\nu}-1} A_{\nu 1} u + ... + A_{\nu n} u = 0 \text{ in } E^{\nu}$$

where $n \geq 1,\ m \geq 0,\ A_n$ are linear operators in E, and $A_{\nu k}$ are linear operators from E in E^{ν}.

The elements $u_1, ..., u_k$ connected with eigen vector u_0 with the relations

$$L(\lambda_0) u_p + \frac{1}{1!} L'(\lambda_0) u_{p-1} + ... + \frac{1}{p!} L^{(p)}(\lambda_0) u_0 = 0,$$

$$L_{\nu}(\lambda_0) u_p + \frac{1}{1!} L'_{\nu}(\lambda_0) u_{p-1} + ... + \frac{1}{p!} L_{\nu}^{(p)}(\lambda_0) u_0 = 0,$$

$p = 0, 1, ..., k,\ \nu = 1, 2, ..., m$ is called adjoined to eigen vector u_0 of the bundle $L(\lambda)$.

The number $k+1$ is called length of chain $u_0, u_1, ..., u_k$. It can be both finite, and infinite. u_0 is called eigen vector of rank r, if the greatest by length chain, answering the eigen vector u_0 has the length equal r. The eigen-value λ_0 is called eigen-value of finite algebraic multiplicity of the bundle $L(\lambda)$ if the maximum number of linear independent eigen-vectors, corresponding to λ_0 is finite.

The function of the form

$$u(t) = e^{\lambda_0 t} \left(\frac{t^k}{k!} u_0 + \frac{t^{k-1}}{(k-1)!} u_1 + ... + \frac{t}{1!} u_{k-1} + u_k \right)$$

is called a solution of a system of equations

$$L(D_t) u = u^{(n)}(t) + A_1 u^{(n-1)}(t) + A_2 u^{(n-2)}(t) + ... + A_n u(t) = 0$$

$$L_\nu (D_t) u = A_{\nu_0} u^{(n_\nu)}(t) + A_{\nu_1} u^{(n_\nu -1)}(t) + ... + A_{\nu_{n_\nu}} u(t) = 0$$

iff $u_0, u_1, ..., u_k$ is a chain of root vectors, corresponding to eigen value of system of bundles $L(\lambda), L_\nu(\lambda)$ and is called elementary solution of a system of equations $L(D)u = 0, \ L_\nu (D) u = 0$. It is easy to see that each of collections of vectors

$$u_0^{(j)}(0), u_1^{(j)}(0), ..., u_k^{(j)}(0), \quad j = 0, 1, ..., n-1$$

is a chain from eigen and adjoint to it vectors, answering to the number λ_0.

It easy to observe that

$$\left(\frac{d}{dt} - \lambda_0 \right) u_0(t) = 0, \ \left(\frac{d}{dt} - \lambda_0 \right) u_k(t) = u_{k-1}(t) \quad (k \geq 1)$$

Hence it follows that

$$u_0^{(j)}(t) = \lambda_0 u_0^{(j-1)}(t),$$

$$u_k^{(j)}(t) = \lambda_k u_k^{(j-1)}(t) + u_{k-1}^{(j-1)}(t), \quad (k \geq 1)$$

Let E and F be a Banach spaces. We'll denote by $E\dot{+}F$ the space, consisting of elements (u, ν), where $u \in E, \ \nu \in F$ with the norm

$$\|(u,\nu)\|_{E+F} = \left(\|u\|_E^2 + \|\nu\|_F^2 \right)^{\frac{1}{2}}$$

$E\dot{+}F$ is Banach space and is called straight sum of Banach spaces E and F.

Let's determine the following Banach spaces:

$$C_\mu ((0,T]; E) = \Big\{ f / f \in C((0,T]; E), \|f\|_{C_\mu((0,T];E)} =$$

$$= \sup_{(0,T]} \|t^\mu f(t)\| < \infty, \mu \geq 0, t \in (0,T] \Big\};$$

$$C_\mu^\theta ((0,T]; E) = \Big\{ f / f \in C((0,T]; E), \|f\|_{C_\mu^\theta((0,T];E)} =$$

$$= \sup_{(0,T]} \|t^\mu f(t)\| + \sup \|f(t+h) - f(t)\| h^{-\theta} t^\mu < \infty,$$

$$\mu \geq 0, t \in (0,T], \ 0 < t < t+h \leq T, \ \theta \in (0,1] \}$$

Let E and $E^\nu, \ \nu = 1, ..., s$ be Banach spaces. The Cauchy problem is considered for a system of diferential-operator equations

$$L(D_t) u = u'(t) + Au(t) = f(t), \tag{1}$$

$$L_\nu (D_t) u = A_{\nu_0} u'(t) + A_{\nu_1} u(t) = f_\nu (t), \ \nu = 1, ..., s, \tag{2}$$

$$u(0) = u_0, \tag{3}$$

where $t \in [0,T]$ is an $A-$operator in E; $A_{\nu 0}, A_{\nu 1}$, and the operators from E in $E^\nu, f(t) u f_\nu (t)$, are given functions, corresponding from $[0,T]$ in E and from $[0,T]$ in $E^\nu, u(t)$, respectively, it is unknown function from $[0,T]$ in E. The operators $A, A_{\nu 0}, A_{\nu 1}$, generally, are unbounded.

Let's consider the characteristic operator bundles of the system of equations (1)-(2)

$$L(\lambda)=\lambda I+A$$

$$L_\nu(\lambda)=\lambda A_{\nu 0}+A_{\nu 1},\ \nu=1,...,S,$$

where λ is complex number.

Theorem 1. *Let the following conditions be fulfilled:*

1. A is compactly determined, closed operator in E, $D(A)=E_1$;

2. $A_{\nu 0}, \nu=1,...,s$ and $A_\nu, \nu=1,...,s$ from E_1 in E^ν are bounded operators;

3. for some $\eta\in(0,1]$, $\alpha>0$ all complex numbers from the angle $|\arg\lambda|\le\frac{\pi}{2}+\alpha$ at sufficiently large modulus are regular numbers of the operator bundle $Z(\lambda): u\to Z(\lambda)u=(L(\lambda)u, L_1(\lambda)u,...,L_s(\lambda)u)$ boundedly operating from E_1 to $E\dot{+}E_1\dot{+}...\dot{+}E^s$ and for $|\arg\lambda|\le\frac{\pi}{2}+\alpha$, $|\lambda|\to\infty$.

4. for some $\theta\in(1-\eta,1]$, $\mu\in[0,\eta)$, $f\in C_\mu^\theta((0,T];E)$, $f_\nu\in C_\mu^\theta((0,T];E^\nu)$;

5. $u_0\in E_1$

Then there exists the unique solution $u(t)$ of problem (1)-(3), the function $t\to(u(t), A_{10}u(t),...,A_{s0}u(t))$ from $(0,T]$ in $E\dot{+}E_1\dot{+}...\dot{+}E^s$ is continuously differentiable and from $[0,T]$ to $E+E_1+...+E^s$ is continuous, and for $t\in(0,T]$ the following estimations are fulfilled

$$\|u(t)\|_E+\sum_{\nu=1}^{s}\|A_{\nu 0}u(t)\|_{E^\nu}\le$$

$$\le C\left(\|A_{\nu 0}\|+\|u_0\|+\|f\|_{C\mu((0,t);E)}+\sum_{\nu=1}^{s}\|f_\nu\|_{C\mu((0,t);E^\nu)}\right),$$

$$\|u(t)\|+\|Au(t)\|+\sum_{\nu=1}^{s}\left\|A_{\nu 0}u(t)'\right\|_{E^\nu}\le$$

$$\le C\left\{t^{\eta-1}\left[\|A_{\nu 0}\|+\|u_0\|\right]+t^{\eta-\mu-1}\left(\|f\|_{C^\theta\mu((0,t);E^\nu)}+\sum_{\nu=1}^{s}\|f_\nu\|_{C_\mu^\theta((0,t);E^\nu)}\right)\right\}.$$

Proof. Let's determine the operator *or* in the space $E\dot{+}E_1\dot{+}...\dot{+}E^s$ in the following way

$$D(or)=\{\nu/\nu=(u, A_{10}u,...,A_{s0}u), u\in E_1\};$$

$$or(u, A_{10}u,...,A_{s0}u)=(-Au,-A_{11}u,...,-A_{s1}u).$$

The Cauchy problem (1)-(3) is equivalent to the following Cauchy problem

$$V'(t)-orv(t)=F(t) \tag{4}$$

$$\nu(0)=V_0$$

where $F(t)=(f(t), f_1(t),...,f_s(t))$, $V_0=(u_0, A_{10}u_0,...,A_{s0}u_0)$.

In its turm the equation

$$\lambda V-orV=F,\ F=(f, f_1,...,f_s) \tag{5}$$

is equivalent to the system

$$L(\lambda)=\lambda u+Au=f$$

$$L_\nu(\lambda)u = \lambda A_{\nu 0} + A_{\nu 1}u = f_\nu,\ \nu = 1,...,s \tag{6}$$

is equivalent

According to condition 3 of the theorem problem (6) has a unique solution

$$u = Z^{-1}(\lambda)(f, f_1, ..., f_s),$$

for all λ from the angle $|\arg\lambda| \le \frac{\pi}{2} + \alpha$ at sufficiently large modules. Therefore for solution of problem (5) we'll get

$$V = \left(Z^{-1}(\lambda)(f, f_1, ..., f_s), A_{10}Z^{-1}(\lambda)(f, f_1, ..., f_s), ..., A_{s0}Z^{-1}(\lambda)(f, f_1, ..., f_s)\right).$$

According to the condition 3

$$\|R(\lambda, or)\| \le C|\lambda|^{-\eta},\ |\arg\lambda| \le \frac{\pi}{2} + \alpha,\ |\lambda| \to \infty.$$

Thus the operator *or* satisfies the condition 1 of theorem 3.2. from paper [1] (see [1], p.109). Conditions 2 and 3 of theorem 3.2 follow from conditions 4 and 5. So all conditions of theorem 3.2 from S.Yakubov's book are checked. (see [1]). Theorem 1 is proved.

Let's consider in Hilbert space H the Cauchy problem for perturbed homogeneous equation of the first order

$$u'(t) = Au(t) + Bu(t) \tag{7}$$

$$u(0) = u_0. \tag{8}$$

Let's find the conditions, which provides the approximation of Cauchy problem (7)-(8) by the linear combinations of elementary solutions of equation (7).

Denote by λ the eigen-values of the operator $A+B$ subject to the algebraic multiplicity. If $u_{j0}, ..., u_{jkj}$ is a chain of root vectors of the operator $A+B$ corresponding to λ_j, then the function

$$u_j(t) = e^{\lambda_j t}\left(\frac{t^{k_j}}{k_j!}u_{j0} + \frac{t^{k_j-1}}{(k_j-1)!}u_{j1} + ... + \frac{t}{1!}u_{j,k_j-1} + u_{jk_j}\right) \tag{9}$$

is an elementary solution of equation (7).

Theorem 2. *Let:*

1. the operator A in H have dense domain of determination;

2. at some $\rho > 0$ and $\lambda_0 \in \rho(A)$ the operator $R(\lambda_0, A) \in \sigma_p(H)$;

3. there exist the rays $l_k(a)$ with angles between the neighbouring rays not bigger $\frac{\pi}{p}$ and the number $\alpha > 0$, $\beta \in (0,1]$ such that

$$R(\lambda, A) \le c|\lambda|^{-\beta},\ |\arg\lambda| \le \frac{\pi}{2} + \alpha,\ \lambda \in l_k(a);$$

4. B is an operator in H: $D(B) \supset D(A)$ and for any $\varepsilon > 0$

$$\|Bu\| \le \varepsilon\|Au\|^\beta \|u\|^{1-\beta} + C(\varepsilon)\|u\|,\ u \in D(A);$$

5. $u_0 \in D(A)$.

Then problem (7)-(8) has a unique solution

$u(t)$ $u \in C((0,T],H) \cap C'((0,T],H(A),H)$ and there exist the numbers c_{jn} such that

$$\lim \max_{[0,T]} \left\| u(t) - \sum_{j=1}^{n} c_{jn} u_j(t) \right\|_H = 0, \ n \to \infty \ t \in [0,T]$$

$$\lim \sup t \left(\left\| u'(t) - \sum_{j=1}^{n} c_{jn} u'_j(t) \right\|_H + \left\| Au(t) - \sum_{j=1}^{n} c_{jn} Au_j(t) \right\|_H \right) = 0,$$

$$n \to \infty \ t \in (0,T]$$

where $u(t)$ is a solution of problem (7)-(8), and $u_j(t)$ are elementary solutions (9) of equation (7).

Proof. Under the conditions of the theorem the system of root vectors of the operator $A+B$ is complete in the space $H(A)$.

$\left(H(A) = \left\{ u : u \in D(A), \|u\|_{H(A)} = \|Au\|_H + \|u\|_H \right\} \right)$. (see for example [2], p.277). Consequently, there exist the numbers c_{jn} such that

$$\lim_{n\to\infty} \left(\left\| u_0 - \sum_{j=1}^{n} C_{jn} u_{jk_j} \right\|_H + \left\| Au_0 - \sum_{j=1}^{n} C_{jn} Au_{jk_j} \right\|_H \right) = 0 \tag{10}$$

$\nu(t) = u(t) - \sum_{j=1}^{n} C_{jn} u_j(t)$ is a solution of equation (7) and satisfies the initial condition $\nu(0) = u_0 - \sum_{j=1}^{n} C_{jn} u_{jk_j}$. Then from theorem 1 it follows the estimations

$$\left\| u(t) - \sum_{j=1}^{n} C_{jn} u_j(t) \right\| \le$$

$$\le C \left(\left\| u_0 - \sum_{j=1}^{n} C_{jn} u_{jk_j} \right\| + \left\| Au_0 - \sum_{j=1}^{n} C_{jn} Au_{jk_j} \right\| \right), \tag{11}$$

$$\left\| u'(t) - \sum_{j=1}^{n} C_{jn} u'_j(t) \right\| + \left\| Au(t) - \sum_{j=1}^{n} C_{jn} Au_j(t) \right\| \le$$

$$\le Ct^{-1} \left(\left\| u_0 - \sum_{j=1}^{n} C_{jn} u_{jk_j} \right\| + \left\| Au_0 - \sum_{j=1}^{n} C_{jn} Au_{jk_j} \right\| \right). \tag{12}$$

From (11) and (12) by virtue of (10) it follows the assertion of the theorem.

Remark 1. For $\beta = 1$ problem (7)-(8) has a unique solution $u \in C^{-1}((0,T],H(A),H)$ and there exist the numbers C_{jn} such that

$$\lim \max \left(\left\| u'(t) - \sum_{j=1}^{n} C_{jn} u'_j(t) \right\| + \left\| Au(t) - \sum_{j=1}^{n} C_{jn} Au_j(t) \right\| \right) = 0,$$

$$n\to\infty,\ t\to[0,T].$$

Theorem 3. *Let:*

1. the operator A has a domain of determination H, dense in $D(A)$;

2. at some $q>0$, $\lambda_0\in\rho(A)$ the operator $R(\lambda_0,A)\in\sigma_q(H)$;

3. there exist the rays $l_k(a)$ with the angles between neigboring rays not large $\frac{\pi}{q}$ such that

$$R(\lambda,A)\le C|\lambda|^{-1},|\arg\lambda|\le\frac{\pi}{2}\text{ or }\lambda\in l_k(a),\ |\lambda|\to\infty$$

4. *B is an operator in H, $D(B)\supset D(A)$ and the operator $BR(\lambda_0,A)$ in H is compact;*

5. *$u_0\in(H,H(A))_{1-\frac{1}{p},p}$ at some $p\in(1,\infty)$. $(H,H(A))_{\theta,p}$is interpolation space between $H(A)$ and H (see [3]).*

Then problem (7)-(8) has a unique solution $u\in W_p^1((0,T),H(A),H)$, where

$$W_p^1((0,1),H(A),H)=\{u:Au,u'\in L_p((0,1),H)\}\ \|u\|^p_{W_p^1((0,1),H(A),H)}=$$

$$=\|Au\|_{L_p((0,1),H)}+\|u'\|_{L_p((0,1),H)}$$

and there exist the numbers C_{jn} such that

$$\lim\int_0^T\left(\left\|u'(t)-\sum_{j=1}^nC_{jn}u_j'(t)\right\|^p+\left\|Au(t)-\sum_{j=1}^nC_{jn}Au_j\right\|^p\right)dt=0,$$

where $u(t)$ is a solution of problem (7)-(8), and $u_j(t)$ are elementary solutions (9) of equation (7).

Proof. From condition 2 and 4 it follows that the linear span of root vectors of the operator $A+B$ is compact in the space $H(A)$. On the other hand, the set $H(A)$ is compact in $(H,H(A))_{\theta,p}$. Consequently, the linear span of root vectors of the operator $A+B$ is compact in the space $(H,H(A))_{\theta,p}$. Then by virtue of theorem 2 we have

$$\left\|Au(t)-\sum_{j=1}^nc_{jn}Au_j(t)\right\|_{L_p((0,T),H(A),H)}+\left\|u'(t)-\sum_{j=1}^nc_{jm}u_j'(t)\right\|_{L_p((0,T),H(A),H)}\le$$

$$\le\left\|u(t)-\sum_{j=1}^nC_{jn}u_j(t)\right\|_{W_p^1((0,T),H(A),H)}+C\left\|u_0-\sum_{j=1}^nC_{jn}u_{jk_j}\right\|_{(H,H(A))_{1-\frac{1}{p}p}}$$

whence the assertion of the theorem follows.

Let H and H^ν, $\nu=1,...,s$ be Hilbert space. Let's consider the Cauchy problem for system of differential –operator equations

$$L(D_t)u=u'(t)+Au(t)=0 \tag{13}$$

$$L_\nu(D_t)u=A_{\nu_0}u'(t)+A_{\nu_1}u(t)=0,\ \nu=1,...,s, \tag{14}$$

$$u(0)=u_0, \tag{15}$$

Theorem 4. *Let:*

1. A be a closed operator with dense domain of determination $D(A) = H_1$ in the space H and at some $q \rhd 0, J \in \sigma_q(H_1; H)$, i.e., the embedding operator from $A(H) = H_1$ in H belongs the Neumann-Shetten class at some $q \rhd 0$;

2. $A_{\nu 0} \in \sigma_q(H_1; H), \ \nu = 1, 2, ..., S,$
$\mathrm{A}_{\nu 1} \in B(H_1; H^{\nu}), \ \nu = 1, 2, ..., S;$

3. linear manifold

$$\{\nu : \nu = (u, A_{10}u, ..., A_{s0}u), u \in H_1\}$$

is compact in Hilbert space $H + H' + ... + H^s$;

4. there exits the rays $l_k = \{\lambda : \lambda \in C, \ \arg \lambda = \varphi_k\}$ with the angles between neighboring rays not larger that $\frac{\pi}{q}$ and the number $\eta \in (0, 1]$ such that

$$\left\| Z^{-1}(\lambda) \right\|_{B\left(H +_{\nu=1}^{s} H^{\nu}, H_1\right)} \le C |\lambda|^{-\eta}, \ |\arg \lambda| \le \frac{\pi}{2} + \alpha \text{ or } \lambda \in l_k(a), \ |\lambda| \to \infty;$$

where $Z(\lambda) = (L(\lambda), L_1(\lambda), ..., L_S(\lambda))$;

5. $u_0 \in H_1$.

Then problem (13)-(15) has a unique solution

$u \in C((0, T], H) \cap C'((0, T]; H(A), H)$ and there exist the numbers C_{jn} such that

$$\lim_{n\to\infty} \max_{t\in[0,T]} \left(\left\| u(t) - \sum_{j=1}^{n} C_{jn} u_j \right\|_H + \sum_{j=1}^{S} \left\| A_{\nu 0} u(t) - \sum_{j=1}^{n} C_{jn} A_{\nu 0} u_j(t) \right\|_{H^{\nu}} \right) = 0$$

$$\lim_{n\to\infty} \sup_{t\in[0,T]} \left(\left\| u'(t) - \sum_{j=1}^{n} C_{jn} u_j'(t) \right\|_H + \left\| Au(t) - \sum_{j=1}^{n} C_{jn} A u_j(t) \right\| + \right.$$

$$\left. + \sum_{\nu=1}^{S} \left\| A_{\nu 0} u'(t) - \sum_{j=1}^{n} C_{jn} A_{\nu 0} u_j'(t) \right\|_{H^{\nu}} \right) = 0$$

Proof. Problem (7)-(9) is equivalent to the following Cauchy problem

$$V'(t) = orV(t),$$
$$V(0) = V_0 \qquad (16)$$

where

$$D(or) = \{V/V = (u, A_{10}u, ..., A_{S0}u), \ u \in H_1\},$$
$$or(u, A_{10}u, ..., A_{S0}u) = (-Au, -A_{11}u, ..., -A_{S1}u)$$

Let's show the resolvent of the operator *or* is compact. For this aim instead of equation $(or - \lambda I)\nu = F, \ F = (f, f_1, ..., f_S)$ we'll solve the equivalent to it system

$$-Au - \lambda u = f, \ -A_{\nu 1}u - \lambda A_{\nu 0}u = f_{\nu} \qquad (17)$$

in the space $H(A) = H_1$. If λ is a regular point of the bundle $Z(\lambda) = (L(\lambda), L_1(\lambda), ..., L_S(\lambda))$ from H in $H + H' + ... + H^{S^{\nu}}$, then the system (11) has the unique solution

$$\nu = \left(Z^{-1}(\lambda)(-f, -f_1, ..., f_S), A_{\nu 0} Z^{-1}(\lambda)(-f, -f_1, ..., -f_S)\right) \qquad (18)$$

By the condition of theorem $J \in \sigma_q (H(A), H)$, then by virtue of conditions 1,2 the operator $R(\lambda, or)$ is compact. On the other hand from condition 2,4 it follows the estimation

$$\left\| A_{\nu 0} Z^{-1}(\lambda) \right\|_{B\left(H \overset{S}{\underset{\nu=1}{+}} H^{\nu}, H^{\nu} \right)} \leq C |\lambda|^{\eta}$$

From (18) and conditions 2 it follows that

$$Z^{-1}(\lambda) \in B\left(H \overset{S}{\underset{\nu=1}{+}} H^{\nu}, H(A) \right),$$

$$A_{\nu 0}^{-1} Z^{-1}(\lambda) \in \sigma_q \left(H \overset{S}{\underset{\nu=1}{+}} H^{\nu}, H^{\nu} \right)$$

Consequently, for $\lambda \in I_k(a)$, $|\lambda| \to \infty$

$$R(\lambda, or) \in \sigma_q \left(H \overset{S}{\underset{\nu=1}{+}} H^{\nu} \right),$$

$$\| R(\lambda, or) \| \leq C |\lambda|^{-\eta}$$

Then the system of root vectors of the operator or is complete in the space $H \overset{S}{\underset{\nu=1}{+}} H^{\nu}$. Consequently, there exits the numbers C_{jn} such that

$$\lim_{n\to\infty} \left(\left\| \nu_0 - \sum_{\nu=1}^{n} C_{jn} u_{jk_j} \right\| + \left\| or\nu_0 - \sum_{\nu=1}^{n} C_{jn} or\nu_{jk_j} \right\| \right) = 0$$

Since $\nu_0 = (u_0, A_{10}u_0, ..., A_{S0}u_0)$, then by virtue of theorem 3 the assertion of theorem follows.

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