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MODEL OF JOINT FILTRATION OF TWO-COMPONENT LIQUID

Abstract

In the paper we investigate a problem on elaboration of methods for solving a problem on optimal choice of effect method on oil stratum and bottom-hole zone to increase oil recovery and maximal oil extraction from the entrails of the earth, on the basis of two-velocity theory of filtrational flow.

Motion of phases with respect to each other because of different densities and viscosity of phase materials that differently perceive effect on mixture, is characteristic to many flows of non one-phase media.

Let's consider joint filtration of two liquids-oil and water when inertial forces because of acceleration of material particles $d_i\nu_i/dt = \partial\nu_i/\partial t + \nu_i\partial\nu_i/\partial x$, $i =$ oil, water should be taken into account such flows are realized, in particular, when imposed perturbance in the form of injected water is not smooth, but pertaining to shock.

Let oil and water in a porous medium be monodispersive mixture consisting of bearing liquid (let it be water) and dispersive liquid (oil) in the form of spherical drops of radius R with number concentration n. Oil and water parameters will be supplied by lower indices p ("petroleum") and w ("water"). In particular,S_p and S_w is oil saturation and water saturation, respectively. Here volumetric fractions of liquids in the saturated porous medium equal $\alpha_p = mS_p$ and $\alpha_w = mS_w$ respectively, and the equalities:

$$S_p + S_w = 1 \tag{1}$$

hold.

We expresse the reduced densities ρ_i (phase masses in medium volume unit) by the true ρ_i^0:

$$\rho_i = \rho_i^0\alpha_i, \quad \alpha_i = mS_i, \quad i = p, w \tag{2}$$

For describing two-velocity flows we use the scheme of interpenetrating and interacting continuums [2,3]. At each point for each i-th continuum the velocity ν_i is also introduced aside another parameters. Difference of $\overrightarrow{\nu_1}$ from $\overrightarrow{\nu_2}$ and etc. corresponds to velocity disbalance of the mixture.

A closed system of two-velocity motion equations in one-dimensional variant for mixture of two incompressible liquids is of the form [3]:

$$\frac{\partial(mS_i\rho_i^0)}{\partial t} + \frac{\partial(mS_i\rho_i^0\nu_i)}{\partial x} = 0, \;\; i = p, w \tag{3}$$

$$mS_p\rho_p^0\frac{d_p\nu_p}{dt} = -\alpha_p\frac{\partial p}{\partial x} + F_\mu + F_e \tag{4}$$

$$mS_w\rho_w^0\frac{d_w\nu_w}{dt} = -\alpha_w\frac{\partial p}{\partial x} - F_\mu + F_e \tag{5}$$

$$F_\mu = \alpha_p\alpha_w K_\mu(\nu_p - \nu_w)$$

Here mass and impulse preservation equation of each liquid, friction force F_μ with coefficient K_μ, p is mass pressure, F_e are external mass forces, are represented.

Assume that we can neglect convective terms in equations (4) and (5) in comparison with local derivatives. Besides, we'll assume that phase slips are small. The one that has been said allows to consider a linear approximation in whose scope we get:

$$\rho_p^0 \frac{\partial \nu_p}{\partial t} = -\frac{\partial p}{\partial x} + \lambda_1(\nu_p - \nu_w) + G_e \tag{6}$$

$$\rho_p^0 \rho \frac{\partial \nu_w}{\partial t} = -\frac{\partial p}{\partial x} - \lambda_1 \alpha(\nu_p - \nu_w) + \alpha G_e \tag{7}$$

$$\rho_p^0, \rho_w^0 = const.$$

Here $\lambda_1 = \alpha_{w0} K_\mu$, $\alpha = \alpha_{p0}/\alpha_{w0}$, $\rho = \rho_w^0/\rho_p^0$ are positive constants of the number, G_e are the functions that take effect of external mass forces into account.

It we average equations (6) and (7) according to the length of a stratum and pass to dimensionless parameters and variables, we get the following system of equations:

$$\begin{cases} \dot{x}_p = -\mu(\tau) + \sigma(x_p - x_w) + y(\tau) \\ \rho \dot{x}_w = -\mu(\tau) + \alpha\sigma(x_w - x_p) + ay(\tau) \end{cases} \tag{8}$$

$$\sigma = const.$$

Such model was used in [1].

In [1] the attempt was taken to use a heat convection model in atmosphere that Edward Norton Lorentz suggested for modeling the process of oil displacement by injected water. Unlike [1] in the present paper water injection is taken into account in the form of the function $y(\tau)$ at the right hand sides of system (8), and the given system of determining equations are derived from equations (1-7) mentioned above.

The unknown function $\mu(t)$ may be determined with the help of field measurement data by the formula:

$$\mu(\tau) = \frac{2\alpha y(\tau) - \alpha \dot{x}_p - \rho \dot{x}_w}{1+\alpha}\,. \tag{9}$$

We can determined the coefficient σ by rectifying field measurement data

$$Y = X + \sigma, \tag{10}$$

in the coordinates

$$Y = \frac{\rho \dot{x}_w}{\alpha(x_w - x_p)} \quad and \quad X = \frac{\alpha y(\tau) - \mu(\tau)}{\alpha(x_w - x_p)}.$$

In special case, if liquid filtration occurs under constant pressure drop, then $\mu(\tau) = \beta = const$, and system (8) takes the form:

$$\begin{cases} \dot{x}_p = -\beta + \sigma(x_p - x_w) + y(\tau) \\ \rho \dot{x}_w = -\beta + \alpha\sigma(x_w - x_p) + \alpha y(\tau) \end{cases}\,. \tag{11}$$

With the help of the second equation of (11) one can rectify the field measurement data

$$Y = \beta X + \sigma, \tag{12}$$

in the coordinates

$$Y = \frac{\rho \dot{x}_w - \alpha y(\tau)}{\alpha(x_w - x_p)} \quad \text{and} \quad X = -\frac{1}{\alpha(x_w - x_p)}$$

and to define the unknown coefficients σ and β.

The results of rectification for $\alpha = 10$ and $\rho = 1,5$ are given in figure 1.

Fig. 1.

At calculations it was assumed that x_p is oil production rate, x_w is water production rate , $y(\tau)$ is water injection.

It is seen form the figure that $\sigma \approx 0,6$; $\beta \approx 0,9$.

Direct problem.

Under the found values of $\sigma \approx 0,6$; $\beta \approx 0,9$ and $\alpha = 10$, $\rho = 1,5$ from the second equation of (11) by the finite differences method we determine the dynamics of water production rate in time:

$$x_{wn+1} = x_{wn} + \frac{\triangle \tau}{\rho}\left[-\beta + \alpha\sigma(x_{wn} - x_{pn}) + \alpha y_n\right] \tag{13}$$

The results of calculations by formula (13) are given in figure 2. It is seen that the suggested model (13) describes well the dynamics of water production rate in time.

It we assume that the water injection exceeds by intensity of effect on liquid filtration in comparison with pressure force, we can simplify the system of equations (8) and rewrite it in the form:

$$\begin{cases} \dot{z} = \sigma(z - x) + \alpha y(\tau) \\ \dot{x} = \rho(x - z) + \beta y(\tau) \end{cases} \tag{14}$$

Fig. 2.

where $z(\tau)$ is oil production rate, $x(\tau)$ is water production rate, $y(\tau)$ is water injection.

We apply the general scheme of use of system (8) for prediction of oil recovery from formation. To this end by the data on dynamics of oil, water production rate and water injection for some period of time the function $\mu(t)$ and coefficient σ should be estimated. Further, by the given intensity of water injection $y(t)$ one can predict the dynamics of oil production rate x_p and water production rate x_w.

By means of the system (8) we can solve the problem on elaboration of methods for solving a problem on optimal choice of effect technique on oil formation and bottom-hole zone for increasing oil-recovery and maximal oil extraction from the entrails of the earth. This problem is solved with the help of numerical experiment by the way of analytic representation of the function $y(t)$.

References

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