

## APPLIED PROBLEMS OF MATHEMATICS AND MECHANICS

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DISTRIBUTION OF ELECTROMAGNETIC WAVES  
TO CHEMICALLY CHANGING ENVIRONMENTS

## Abstract

*Creation of physical fields has put forward a series of new vital scientific problems. The problem of influence and influence of physical fields concerns to them on physical mechanical and physical and chemical characteristics of materials, and also, on dynamic processes occurring in the nature, and also, on ecology of the nature. In particular, studying of the mechanism of their mutual interaction against each other. For today the part of these questions was found with the scientific sanction owing to specially developed methods of experimental physics.*

*However for today there is no theoretical basis on rational management of processes of mutual interaction of hypothetical, physical fields with real materials and environments, in particular, with changing environments. Absence of an appropriate theoretical basis does not allow to give the answer about the mechanism of interaction of physical fields with chemically, physical chemically or bio-chemically changeable environments. Therefore such applied problems as energy-dynamic intercellular interactions; a problem of punching of micro-electronic schemes influence by the directed electromagnetic fields; a problem of management chemical or biochemical processes by means of influence by electromagnetic fields and many other things, for today, remain unresolved.*

*In clause statement of a problem of distribution of electromagnetic waves in chemically changing environments is given and the generalized equations of electrodynamics of chemically changing environments are offered.*

### 1. The generalized equations of distribution of electromagnetic waves in chemically changing environments

Let's construct the generalized defining equations of electrodynamics of Maxwell's in chemically changing environments. Thus chemically changeability of environment is understood as change of concentration of chemical elements of the environment occurring in chemical reactions.

The theory is under construction in the assumption, that in chemically changing environments for each moment of time two basic curl laws of electrodynamics keep the classical [1,2] kind:

$$\int_{(1)} \mathbf{H}_s ds = \frac{1}{c} \iint_{(S)} r_n dS \quad (1)$$

$$\int_{(1)} \mathbf{E}_s ds = -\frac{1}{c} \frac{d}{dt} \iint_{(S)} B_n dS. \quad (2)$$

Here  $\vec{E}$  and  $\vec{H}$  vectors of electric and magnetic forces;  $\vec{r}$ - a vector of a full electric current;  $\vec{B}$ - a vector of a magnetic induction;  $\tilde{n}$ -speed of light in vacuum.

In the written equations (1) - any way closed contour and (S) - a surface, to it limited. Here  $H_s$  and  $E_s$  are projections of vectors  $\vec{H}$  and  $\vec{E}$  on a vector (1) curve:

$$H_s = H_x \cos(\vec{n}, X) + H_y \cos(\vec{n}, Y) + H_z \cos(\vec{n}, Z) \quad (3)$$

$$E_s = E_x \cos(\vec{n}, X) + E_y \cos(\vec{n}, Y) + E_z \cos(\vec{n}, Z). \quad (4)$$

In the environment in which there are changes of concentration of chemical elements in time, the vector of a full electric current  $\vec{r}$  and a vector of an induction current  $\vec{B}$ , passing through a surface (S) is accepted, that, will be equal:

$$r_n = r_x \cos(\vec{n}, X) + r_y \cos(\vec{n}, Y) + r_z \cos(\vec{n}, Z) \quad (5)$$

$$B_n = B_x \cos(\vec{n}, X) + B_y \cos(\vec{n}, Y) + B_z \cos(\vec{n}, Z). \quad (6)$$

It is necessary to understand changes of chemical properties of the investigated environment as chemical changeability of environment on all considered interval of time  $t_0, t_n$ , passing in chemical reactions. In chemical processes changes can be carried to such changes in time of concentration of elements making this substance, or change of speed of chemical reaction.

In this connection chemical changeability of environment we shall represent as mathematically in the form of function of change of concentration of a chemical element from time in the form of  $f = f(t)$ .

Changes in time of chemical elements of environment will automatically entail to change of electromagnetic properties of this environment. It will mean, that to each small interval of chemical process, i.e. time  $\Delta t$ , will correspond both the concentration maintenance of elements, and inherent to it electromagnetic characteristics of this environment. If to assume as a basis such ideology a vector of the full electric current, passing through such environment, it is possible to present in the form of two composed - a current of conductivity and a current of displacement which factors will be functions from function of chemical changeability of concentration of elements of  $f(t)$  environment, in the form of [3]:

$$\vec{r} = \lambda(f(t), t) \vec{E} + \frac{\partial}{\partial t} [\varepsilon(f(t), t) \vec{E}]. \quad (7)$$

Here  $\lambda(f(t), t)$  and  $\varepsilon(f(t), t)$  there is a function of conductivity of environment and function of the dielectric characteristic the environments depending on function of chemical changeability of concentration of elements of investigated  $f(t)$  environment. Feature of a (7) parity consists that  $\lambda(f(t), t)$  and  $\varepsilon(f(t), t)$  are not constants as in classical electrodynamics, and are functions of chemical process of environment.

The missing two equations replenish with a choice so-called the equations the environments establishing communication between electromagnetic characteristics of the investigated environment. It is supposed, that the electromagnetic environment in view of function of chemical changeability  $f(t)$  is defined functional by dependences of a vector of electric displacement  $\vec{D}$  from a vector of electric forces  $\vec{E}$  and a

vector of a magnetic induction  $\vec{B}$  from a vector of magnetic  $\vec{H}$  forces, in a following kind:

$$\vec{D} = \varepsilon(f(t), t) \vec{E} \quad (8)$$

$$\vec{B} = \mu(f(t), t) \vec{H}. \quad (9)$$

Here  $\mu(f(t), t)$ - function of magnetic permeability of the environment, depending on function of chemical changeability of concentration of elements of  $f(t)$  environment,  $\vec{D}$ - a vector of electric displacement. The system of the equations (1), (2), (7), (8) and (9) forms the full closed system rather 5 unknown vectors. Thus, knowledge of experimental functions - chemical changeability concentration of elements of  $f(t)$  environment, and also, three functions of electromagnetic characteristics of  $\varepsilon(f(t), t)$ ,  $\mu(f(t), t)$ ,  $\lambda(f(t), t)$  environment, allow to define five unknown vector functions  $\vec{E}$ ,  $\vec{H}$ ,  $\vec{B}$ ,  $\vec{D}$  and  $\vec{r}$ . Let's substitute (8) and (9) in (1) and (2) in view of (3) - (7), and also, we shall apply Stock's formula which says, that circulation of a field along a contour (l) to some surface is equal to integral on the surface (S) from a normal component of a curl, i.e. to a stream of a curl through a surface(S). By virtue of randomness of a surface (S) and  $\vec{n}$  perpendicularity, integrand expressions of equating to zero we shall have [3]:

$$crot\vec{H} = \lambda(f(t), t) \vec{E} + \varepsilon(f(t), t) \frac{\partial \vec{E}}{\partial t} + (\varepsilon'_f f'_t + \varepsilon'_t) \vec{E} \quad (10)$$

$$crot\vec{E} = -\mu(f(t), t) \frac{\partial \vec{H}}{\partial t} - (\mu'_f f'_t + \mu'_t) \vec{H}. \quad (11)$$

Here  $\frac{df(t)}{dt} = f'_t$ -speed of chemical changeability of concentration of elements of environment, i.e. speed of chemical reaction of environment;  $\frac{\partial \varepsilon(f(t), t)}{\partial f} = \varepsilon'_f$ -change of function of dielectric permeability of environment depending on concentration change of elements of environment;  $\frac{\partial \mu(f(t), t)}{\partial f} = \mu'_f$ - change of function of magnetic permeability of environment depending on concentration change of chemical elements of environment. Thus, the equations (10)-(11) are generalized Maxwell's equations on a case of chemical changeability of concentration of elements of environment in time. From (10)-(11) follows, that change of chemical process of the environment of a  $f(t)$  kind entails change of electromagnetic characteristics of environment in time, i.e.  $\varepsilon = \varepsilon(f(t), t)$ ,  $\mu = \mu(f(t), t)$ ,  $\lambda = \lambda(f(t), t)$ . Kinds of these functions should be defined by special experiments.

The first feature of the received equations (10) - (11) consists that factors of these equations are functions of change chemical concentration of elements of  $f(t)$  environment, and also, functions of electromagnetic characteristic  $\varepsilon = \varepsilon(f(t), t)$ ,  $\mu = \mu(f(t), t)$ ,  $\lambda = \lambda(f(t), t)$  environments.

The second feature of the equations (10)-(11) consists available at them last members  $\varepsilon'_f$  and  $\mu'_f$ . Physically it means that vectors of electric force  $\vec{E}$  and magnetic force  $\vec{H}$  environments, show essential dependence on concentration change of elements of environment. If speed of change of chemical concentration of elements

of environment time it will be equal to zero  $f'_t(t) = 0$  i.e.  $f(t) = const$ , in this case the equations (10)-(11) will accept the simplified kind:

$$c \operatorname{rot} \vec{E} = -\mu \left( \tilde{f}(t), t \right) \frac{\partial \vec{H}}{\partial t} - \frac{\partial \mu \left( \tilde{f}(t), t \right)}{\partial t} \vec{H} \quad (12)$$

$$c \operatorname{rot} \vec{H} = \lambda \left( \tilde{f}(t), t \right) \vec{E} + \varepsilon \left( \tilde{f}(t), t \right) \frac{\partial \vec{E}}{\partial t} + \frac{\partial \varepsilon \left( \tilde{f}(t), t \right)}{\partial t} \vec{E}. \quad (13)$$

It means that to each  $\tilde{f}(t)$  value, there will correspond concrete values of electromagnetic characteristics  $\tilde{\lambda}_n, \tilde{\varepsilon}_n, \tilde{\mu}_n$ . Dependence of functions  $\vec{E}$  and  $\vec{H}$  only from time means, that the effect of type of ageing of environment in time in this case takes place. If in the considered environment do not pass any chemical changes the constancy of electromagnetic characteristics in this case takes place, i.e.  $\varepsilon = \varepsilon_0, \mu = \mu_0, \lambda = \lambda_0$ . In this case the equations (10) - (11) coincide with classical Maxwell's equations:

$$c \operatorname{rot} \vec{H} = \lambda_0 \vec{E} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}, \quad c \operatorname{rot} \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}.$$

Thus, the received generalized equations of distribution of electromagnetic waves in chemically changing environments are presented in the form of the equations in private derivatives with variable factors. And factors of the equations are functions of change chemical concentration of elements of environment  $f(t)$  and  $t$  time, and also functions of electromagnetic characteristics  $\varepsilon = \varepsilon(f(t), t), \mu = \mu(f(t), t), \lambda = \lambda(f(t), t)$  environments. In the coordinate form of the equation (19)-(20) are represented by six equations in private derivatives with variable factors concerning nine unknown scalar functions  $E_x, E_y, E_z, H_x, H_y, H_z, \varepsilon = \varepsilon(f(t), t), \mu = \mu(f(t), t), \lambda = \lambda(f(t), t)$ . Three missing equations are determined by the special experimental equations. More precisely, for a finding of  $\varepsilon = \varepsilon(f(t), t), \mu = \mu(f(t), t), \lambda = \lambda(f(t), t)$  functions, for each set class of chemically changing  $f = f(t)$  environment. Carrying out of special experiments is necessary.

Direct consequence of generalized Maxwell's equations in case of chemically changeability of environment is solenoid vectors:

$$\lambda(f(t), t) \vec{E} + \frac{\partial}{\partial t} \left[ \varepsilon(f(t), t) \vec{E} \right] \quad \text{and} \quad \frac{\partial}{\partial t} \left[ \mu(f(t), t) \vec{H} \right]. \quad (14)$$

Applying action of the divergence operator ( $\operatorname{div}$ ) to system of the equations (10) - (11), we shall receive:

$$c \operatorname{div} \operatorname{rot} \vec{H} = \operatorname{div} \left\{ \lambda(f(t), t) \vec{E} + \frac{\partial}{\partial t} \left[ \varepsilon(f(t), t) \vec{E} \right] \right\} = 0 \quad (15)$$

$$c \operatorname{div} \operatorname{rot} \vec{E} = -\operatorname{div} \frac{\partial}{\partial t} \left[ \mu(f(t), t) \vec{H} \right] = 0. \quad (16)$$

Equality to zero of formulas (15) - (16) (24) - (25) is connected by that any rotary field has divergence equal to zero, i.e.:

$$\operatorname{div} \operatorname{rot} \vec{E} = 0, \quad \operatorname{div} \operatorname{rot} \vec{H} = 0.$$

By virtue of this position if the left parts of the equations (15) and (16) are equal to zero also the right parts are equal to zero. As, it was required to shown. Now we shall prove solenoid of vectors  $\varepsilon(f(t), t) \vec{E}$  and  $\mu(f(t), t) \vec{H}$ . For the proof of this position we use formulas (15) and (16), i.e.

$$\operatorname{div} \left\{ \lambda(f(t), t) \vec{E} + \frac{\partial}{\partial t} [\varepsilon(f(t), t) \vec{E}] \right\} = 0. \quad (17)$$

From the vector analysis it is known, that;

$$\operatorname{div} [\lambda(f(t), t) \vec{E}] = \lambda(f(t), t) \operatorname{div} \vec{E} + \operatorname{grad} \lambda(f(t), t) \vec{E}. \quad (18)$$

By virtue of that function  $\lambda(f(t), t)$  does not depend on coordinates for this case will be:

$$\operatorname{grad} \lambda(f(t), t) = 0. \quad (19)$$

The formula (18) in view of (19) will become:

$$\operatorname{div} [\lambda(f(t), t) \vec{E}] = \lambda(f(t), t) \operatorname{div} \vec{E}. \quad (20)$$

From (17) and (20), we shall receive:

$$\frac{\lambda(f(t), t)}{\varepsilon(f(t), t)} \operatorname{div} [\varepsilon(f(t), t) \vec{E}] + \frac{\partial}{\partial t} \operatorname{div} [\varepsilon(f(t), t) \vec{E}] = 0. \quad (21)$$

Let's enter size:

$$\rho = \operatorname{div} [\varepsilon(f(t), t) \vec{E}]. \quad (22)$$

Also we shall name its density of an electric charge in chemically changing environment during the moment of time. The equation (21) in view of (22) we shall write down in the form of:

$$\frac{\partial \rho}{\rho} = - \frac{\lambda(f(t), t)}{\varepsilon(f(t), t)} dt. \quad (23)$$

The integral of the equation (23) will look like:

$$\rho = \rho_0 e^{- \int_0^t \frac{\lambda(f(t), t)}{\varepsilon(f(t), t)} dt}. \quad (24)$$

According to (22) at  $t = 0$  value,  $\rho_0$  it will be equal:

$$\rho_0 = \operatorname{div} [\varepsilon(f(t), t) \vec{E}] |_{t=0} = \operatorname{div} [\varepsilon(f(0)), 0 \vec{E}_0]. \quad (25)$$

From (24) follows that vectors  $\varepsilon(f(t), t) \vec{E}_0$  in chemically changing environments for each value  $t$  it will solenoid necessary, that value of density of an electric charge in chemically changing  $\rho = [\varepsilon(f(t), t) \vec{E}]$  environment at  $t=0$  will equal to zero, i.e.

$$\operatorname{div} [\varepsilon(f(0), 0) \vec{E}_0] = 0. \quad (26)$$

The proof solenoid of a vector  $[\mu(f(t), t)\vec{H}]$  is similarly spent. For this case the necessary condition solenoid of a vector  $[\mu(f(t), t)\vec{H}]$  will be in the form of:

$$\operatorname{div} [\mu(f(0), 0)\vec{H}_0] = 0.$$

Now we shall find a necessary condition for a while which would provide solenoid of a vectors  $\vec{E}$  and  $\vec{H}$ .

**Proof.** Opening the equation (15) in view of a condition (19) and formulas (20), we shall receive:

$$\lambda(f(t), t) \operatorname{div} \vec{E} + \frac{\partial \varepsilon(f(t), t)}{\partial t} \operatorname{div} \vec{E} + \varepsilon(f(t), t) \frac{\partial}{\partial t} \operatorname{div} \vec{E} = 0. \quad (27)$$

Here  $\frac{d\varepsilon(f(t), t)}{dt}$  there is a full differential:

$$\frac{d\varepsilon(f(t), t)}{dt} = \frac{\partial \varepsilon}{\partial f} \frac{df}{dt} + \frac{\partial \varepsilon}{\partial t} dt. \quad (28)$$

The equation (27) in view of (28) is led to a kind:

$$\frac{d(\operatorname{div} \vec{E})}{\operatorname{div} \vec{E}} + \frac{\lambda + \frac{d\varepsilon(f(t), t)}{dt}}{\varepsilon(f(t), t)} dt = 0. \quad (29)$$

The decision (29) it will be equal:

$$\operatorname{div} \vec{E} = \operatorname{div} \vec{E}_0 e^{-\int_0^t \frac{\lambda(f(t), t) + \frac{d\varepsilon(t), t}{dt}}{\varepsilon(f(t), t)} dt}. \quad (30)$$

From the formula (30) it is simple to receive a necessary condition solenoid of a  $\vec{E}$  vector, in the form of:

$$\left( \operatorname{div} \vec{E} \right) \Big|_{t=0} = \operatorname{div} \vec{E}_0 = 0.$$

The similar condition is simple for receiving for maintenance solenoid of a magnetic  $\vec{H}$  vector, in the form of:

$$\left( \operatorname{div} \vec{H} \right) \Big|_{t=0} = \operatorname{div} \vec{H}_0 = 0.$$

From the constructed generalized equations of distribution of electromagnetic waves in chemically changeable environments (10) - (11), i.e. in case of chemical changeability elements concentration of environment in time, it is possible to deduce the equations, in which each of vectors  $\vec{E}$  and  $\vec{H}$  would enter separately. For this purpose we shall write down the equations (10) - (11) in the form of:

$$\operatorname{rot} \vec{H} = \tilde{\lambda}(f(t), t) \vec{E} + \varepsilon(f(t), t) \frac{\partial \vec{E}}{\partial t} \quad (31)$$

$$\operatorname{rot} \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} - \tilde{\mu} \vec{H}. \quad (32)$$

Where:

$$\tilde{\lambda}((f(t), t)) = \tilde{\lambda} = \lambda((f(t), t)) + \frac{\partial \varepsilon((f(t), t))}{\partial f} \frac{df(t)}{dt} + \frac{\partial \varepsilon((f(t), t))}{\partial t} \quad (33)$$

$$\tilde{\mu}((f(t), t)) = \tilde{\mu} = \frac{\partial \mu((f(t), t))}{\partial t} \frac{df(t)}{dt} + \frac{\partial \mu((f(t), t))}{\partial t}. \quad (34)$$

Functions (33) and (34) we shall name the generalized electromagnetic characteristic functions of the investigated environment on a case of change of its chemical concentration of elements. Feature of these generalized electromagnetic characteristic functions consists in their obvious dependence on function of chemical changeability of  $f(t)$  environment, i.e. changes of chemical concentration of elements of environment.

To the left and right parts of the equation (32) we shall apply operation of a rotor ( $rot$ ), we shall receive:

$$rot \ rot \vec{E} = -rot \left[ \mu(f(t), t) \frac{\partial \vec{H}}{\partial t} + \tilde{\mu}(f(t), t) \vec{H} \right]. \quad (35)$$

By virtue of that functions  $\mu((f(t), t))$  and  $\tilde{\mu}((f(t), t))$  on coordinates do not depend, these functions can be born for the operator, secondly, we shall consider, that

$$rot \frac{\partial}{\partial t} = \frac{\partial}{\partial t} rot.$$

We shall have In this case:

$$rot \left[ \mu(f(t), t) \frac{\partial \vec{H}}{\partial t} \right] = \mu(f(t), t) \frac{\partial}{\partial t} rot \vec{H} \quad (36)$$

$$rot \left[ \tilde{\mu}(f(t), t) \vec{H} \right] = \tilde{\mu}(f(t), t) rot \vec{H}. \quad (37)$$

Substituting (36) - (37) in (29), we shall receive:

$$rot \ rot \vec{E} = -\mu(f(t), t) \frac{\partial}{\partial t} rot \vec{H} - \tilde{\mu}(f(t), t) rot \vec{H}. \quad (38)$$

From the vector analysis it is known, that

$$rot \ rot \vec{E} = grad \ div \vec{E} - \Delta \vec{E}. \quad (39)$$

Substituting (39) in (38) and resolving it together with the equation (31) we shall receive one equation concerning a vector-function  $\vec{E}$  in the form of:

$$\frac{\partial^2 \vec{E}}{\partial t^2} + A(f(t), t) \frac{\partial \vec{E}}{\partial t} + B(f(t), t) \vec{E} = a^2(f(t), t) \left( \Delta \vec{E} - grad \ div \vec{E} \right). \quad (40)$$

Where

$$a^2(f(t), t) = \frac{c^2}{\mu(f(t), t) \varepsilon(f(t), t)}$$

$$A(f(t), t) = \frac{1}{\varepsilon(f(t), t)} \frac{\partial \varepsilon(f(t), t)}{\partial f} \frac{df}{dt} + \frac{\tilde{\lambda}(f(t), t)}{\varepsilon(f(t), t)}$$

$$\begin{aligned}
& + \frac{\tilde{\mu}(f(t), t)}{\mu(f(t), t)} + \frac{1}{\mu(f(t), t) \varepsilon(f(t), t)} \frac{\partial \varepsilon}{\partial t} \\
B(f(t), t) &= \frac{1}{\varepsilon(f(t), t)} \frac{\partial \tilde{\lambda}(f(t), t)}{\partial f} \frac{df}{dt} + \\
& + \frac{\tilde{\mu}(f(t), t) \tilde{\lambda}(f(t), t)}{\mu(f(t), t) \varepsilon(f(t), t)} + \frac{1}{\mu(f(t), t) \varepsilon(f(t), t)} \frac{\partial \lambda}{\partial t}. \quad (41)
\end{aligned}$$

Well-known, that character of course of any chemical reaction in environments, as a rule, happen ion-exchange, electronic-exchange or auto-exchange type to allocation of heat. By virtue of told follows that the chemically changeable environment becomes some source of additional energy. In this connection at the description of similar processes in the equation (40) member  $(\text{grad div } \vec{E}) \neq 0$  it should be distinct from zero. However at weak changeability of concentration of elements of environment or at absence in the environment of electric charges or its infinitesimal value, as the first approach it is possible to accept equality to zero. In this special case the equation (40) will enter the name in the form of:

$$\frac{\partial^2 \vec{E}}{\partial t^2} + A(f(t), t) \frac{\partial \vec{E}}{\partial t} + B(f(t), t) \vec{E} = a^2(f(t), t) \Delta \vec{E}. \quad (42)$$

It is the generalized cable equation as it represents distribution of an electric current on chemically changing cable, i.e. on a cable in which there are weak changes of chemical concentration of elements of environment without occurrence in it any additional energy. Similar by it is possible to receive differential the equation for a vector of magnetic force. For this purpose to the equation (31) we shall apply operation, we shall receive:

$$\text{rot rot } \vec{H} = \tilde{\lambda}(f(t), t) \text{rot } \vec{E} + \varepsilon(f(t), t) \frac{\partial}{\partial t} \text{rot } \vec{E}. \quad (43)$$

Resolving in common the equations (32) and (42) and considering thus (39), we shall receive:

$$\frac{\partial^2 \vec{H}}{\partial t^2} + C(f(t), t) \frac{\partial \vec{H}}{\partial t} + D(f(t), t) \vec{H} = a^2(f(t), t) [\Delta \vec{H} - \text{grad div } \vec{H}]. \quad (44)$$

Here:

$$\begin{aligned}
C(f(t), t) &= \frac{1}{\mu(f(t), t)} \left[ \frac{\partial \mu(f(t), t)}{\partial f} \frac{df}{dt} + \frac{\partial \mu(f(t), t)}{\partial t} + \tilde{\mu}(f(t), t) + \right. \\
& \left. + \frac{\tilde{\lambda}(f(t), t)}{\varepsilon(f(t), t)} \mu(f(t), t) \right] \\
D(f(t), t) &= \frac{1}{\mu(f(t), t)} \left[ \frac{\partial \mu(f(t), t)}{\partial f} \frac{df}{dt} + \frac{\partial \mu(f(t), t)}{\partial t} + \right. \\
& \left. + \frac{\tilde{\lambda}(f(t), t)}{\varepsilon(f(t), t)} \tilde{\mu} f(t), t) \right]
\end{aligned}$$



$$a^2(f(t), t) = \frac{c^2}{\mu(f(t), t) \varepsilon(f(t), t)}. \quad (45)$$

Thus, the defining equations (40) and (45), distributions of electric and magnetic waves describing process to chemically changing environments are constructed, i.e. in view of change of chemical concentration of elements of environment  $f(t)$ . These equations represent the equations in private derivatives of the second order with variable factors. And factors of the equations depend only on  $t$  time.

It is necessary to note following basic features of the received generalized equations (40) and (44). In - the first, by virtue of that expressions  $\text{grad div } \vec{E} \neq 0$  and  $\text{grad div } \vec{H} \neq 0$  are distinct from zero. That operators  $(\Delta \vec{E} - \text{grad div } \vec{E})$  and  $(\Delta \vec{H} - \text{grad div } \vec{H})$  will be the sum of members of private derivatives of the second order of the mixed type. It means that the received decisions of such generalized equations will be not harmonious character of distribution of waves in chemically changing environments. The second feature of the received generalized equations (40) and (44) consists in the following. By virtue of that the right parts of these equations depend only on coordinates, and the left parts of these equations depend only on  $t$  time, such class of the vector equations supposes application to them of a mathematical method of division of variables. Such by it is possible the vector equations in private derivatives with variables for factors on time (40) and (44) to reduce to system of the ordinary differential equations with variable factors on  $t$  time.

## 2. Atechnique of experimental definition of electromagnetic characteristics in chemically changing environments.

One of features of electromagnetic characteristics in  $(\varepsilon, \mu, \lambda)$  environments is their raised sensitivity differently sort of changeability of investigated environments (infringement of uniformity of a material, change of a modular condition of environment, influence from outside of any sort of electromagnetic influences, chemical or biochemical changes in investigated environments and i.e.). Therefore below one of experimental ways of their definition will be resulted.

Kinds of function  $\varepsilon = \varepsilon(f(t), t)$ ,  $\mu = \mu(f(t), t)$ ,  $\lambda = \lambda(f(t), t)$  should be defined from a double sort of experiments: the first, chemical characteristics should be defined by methods of chemical kinetics. The second, electromagnetic characteristics should be defined by methods of experimental physics.

By method of chemical kinetics is being constructed the equation of chemical reaction which result defines concentration change of chemical elements in the liquid environment in the form of  $f(t) = f_0 e^{-at}$ . And chemical reaction passes for final time  $0 \leq t \leq t^*$  for this time interval value of concentration of a chemical element  $f(t)$  will be is in an  $f_0 \geq f(t) \geq f_0 e^{-at^*}$  interval. On the other hand, also known methods of experimental physics define electromagnetic characteristics of environment  $\varepsilon, \mu, \lambda$  during the initial moment of  $t = 0$  time, after end of chemical reaction, i.e. at  $t = t^*$ , and also, in some discrete values of time. Processing these experimental data it is simple to receive dependences  $\varepsilon = \varepsilon(f(t), t)$ ,  $\mu = \mu(f(t), t)$ ,  $\lambda = \lambda(f(t), t)$ .

Example: We shall give an example about influence of chemical changeability of the liquid environment on electromagnetic characteristics of this environment

[4]. It is known, that at change of a modular condition of water its dielectric permeability  $\varepsilon$  changes as follows: at temperature  $T = 20^{\circ}C$  a dielectric constant of  $\varepsilon_{t=0} = 83$  water, dielectric constant steam  $\varepsilon_{t=t_{steam}} = 1.006$ , a dielectric constant of air  $\varepsilon_{t=t_{air}} = 1.00057$ , the dielectric constant of vacuum  $\varepsilon_{t=t_{vacuum}} = 1$ . From here follows, that a dielectric constant of water for the period of time  $(0, t_{steam})$ , being in a liquid condition, changes from 83 up to 1.006 i.e. for this period  $\dot{Y}.I$ . The characteristic of water  $\varepsilon$  changes almost on two orders. The reason of such phenomenon is occurring crystallization of chemical elements being in water under action of temperature, i.e. in water there is a certain chemical reaction. In other words, salts (or so-called chemical micro additives) being water at action of temperature crystallize and besieged on a bottom of a vessel, that water turns to the distilled water, i.e. in a liquid without salt additives. If now to lead corresponding measurements  $\varepsilon$  to the discrete moments of time, inside of an interval  $(0 \leq t_n \leq t_{steam})$  received values  $\varepsilon_n(t_n)$  will be between numbers. On the basis of the received table simply to construct dependence  $\varepsilon(t) = \varepsilon_0 e^{-at}$ .

### 3. The defining equations of distribution of electromagnetic waves on exponential law of a changing chemical environment

Let's consider a special case when concentration change of chemical elements of environment, and also, electromagnetic characteristics of environment change by exponential law of a [3] kind. At the following entry condition;

$$f(t) = f_0 e^{-a_f t}$$

$$f(t)|_{t=0} = f_0 \quad (46)$$

$$a_f = -\frac{1}{t^*} \ln \frac{f(t^*)}{f_0} \quad (47)$$

$$\varepsilon(f(t), t) = \varepsilon_0 e^{-a_\varepsilon t} \quad (48)$$

$$\mu(f(t), t) = \mu_0 e^{-a_\mu t} \quad (49)$$

$$\lambda(f(t), t) = \lambda_0 e^{-a_\lambda t} \quad (50)$$

Here  $f(t^*)$ - value of chemical concentration of the element received from experiment at  $t = t^*$ ;  $f_0$ -value chemical concentration of an element at  $t = 0$ . Taking the logarithm parities (46), (48) - (50), for each value of time  $t$ , we shall receive following system:

$$t = \frac{1}{a_f} \ln \frac{f_0}{f(t)} = \frac{1}{a_\varepsilon} \ln \frac{\varepsilon_0}{\varepsilon(t)} = \frac{1}{a_\mu} \ln \frac{\mu_0}{\mu(t)} = \frac{1}{a_\lambda} \ln \frac{\lambda_0}{\lambda(t)} \quad (51)$$

From here we shall define laws of change of electromagnetic characteristics of chemically changing environment, i.e. dependence of function of conductivity of  $\lambda(f(t), t)$  environment, function of the dielectric characteristic of  $\varepsilon(f(t), t)$  environment, function of magnetic permeability of  $\mu(f(t), t)$  environment depending on function of change of concentration of chemical elements of environment  $f(t)$  in the form of:

$$\varepsilon(f(t), t) = \varepsilon_0 \left[ \frac{f(t)}{f_0} \right]^{\frac{a_\varepsilon}{a_f}} \quad (52)$$

$$\mu(f(t), t) = \mu_0 \left[ \frac{f(t)}{f_0} \right]^{\frac{a_\mu}{a_f}} = \mu_0 \left( \tilde{f}(t) \right)^{\frac{a_\mu}{a_f}} \quad (53)$$

$$\lambda(f(t), t) = \lambda_0 \left[ \frac{f(t)}{f_0} \right]^{\frac{a_\lambda}{a_f}} = \lambda_0 \left( \tilde{f}(t) \right)^{\frac{a_\lambda}{a_f}} \quad (54)$$

$$\frac{df(f)}{dt} = -a_f f_0 e^{-a_f t} = -a_f f_0 \tilde{f}(t) \quad (55)$$

Here  $0 \leq \tilde{f}(t) = \frac{f(t)}{f_0} \leq 1$ . Thus, functions of electromagnetic characteristics of chemically changing,  $\lambda(f(t), t)$ ,  $\varepsilon(f(t), t)$  and  $\mu(f(t), t)$  environment will be expressed through one  $\tilde{f}(t)$  function, which represents itself relative value of chemical change of concentration of elements of environment. By virtue of that function of chemical change of concentration of elements of environment  $f(t)$  at  $t = 0$  will have constant value equal

$$f(t)|_{t=0} = f_0$$

then values of derivative electromagnetic characteristics in this point, according to (52) - (54), will be equal to zero, i.e.

$$\frac{\partial \varepsilon(0)}{\partial t} = 0,$$

$$\frac{\partial \mu(0)}{\partial t} = 0, \quad \frac{\partial \lambda(0)}{\partial t} = 0$$

Now we shall write down expressions  $\frac{\partial \varepsilon(0)}{\partial t} = 0$ ,  $\frac{\partial \mu(0)}{\partial t} = 0$ ,  $\frac{\partial \lambda(0)}{\partial t} = 0$  and  $\tilde{\mu}$  through  $\tilde{f}(t)$  function. According to (52) - (55) we shall have:

$$\begin{aligned} \frac{\partial \varepsilon(t)}{\partial f} &= \varepsilon_0 \frac{a_\varepsilon}{a_f} f_0^{-\frac{a_\varepsilon}{a_f}} f^{\frac{a_\varepsilon}{a_f}-1} = \frac{a_\varepsilon \varepsilon_0}{a_f f_0} \left[ \tilde{f}(t) \right]^{\frac{a_\varepsilon}{a_f}-1} \\ \frac{\partial \mu(t)}{\partial f} &= \mu_0 \frac{a_\mu}{a_f} \frac{1}{f_0} \left[ \tilde{f}(t) \right]^{\frac{a_\mu}{a_f}-1} = \frac{a_\mu \mu_0}{a_f f_0} \left[ \tilde{f}(t) \right]^{\frac{a_\mu}{a_f}-1} \\ \tilde{\lambda} &= \lambda_0 \left[ \tilde{f}(t) \right]^{\frac{a_\lambda}{a_f}} - a_\varepsilon \varepsilon_0 \left[ \tilde{f}(t) \right]^{\frac{a_\varepsilon}{a_f}} = \lambda + \frac{\partial \varepsilon}{\partial f} \frac{df(t)}{dt} \\ \tilde{\mu} &= -\mu_0 a_\mu \left[ \tilde{f}(t) \right]^{a_f} = -a_\mu \mu(f(t), t) \\ \frac{\partial \tilde{\lambda}}{\partial f} &= \frac{1}{f_0} \left[ \lambda_0 \frac{a_\lambda}{a_f} \tilde{f}^{-\frac{a_\lambda}{a_f}-1}(t) - \varepsilon_0 \frac{a_\varepsilon^2}{a_f} \tilde{f}(t) \right]^{\frac{a_\varepsilon}{a_f}-1}. \end{aligned} \quad (56)$$

In this case factors of the equations of electrodynamics of changing environments  $A(f(t), t)$ ,  $B(f(t), t)$ ,  $C(f(t), t)$ ,  $D(f(t), t)$ ,  $a^2(f(t), t)$  (41) and (45) in view of (46), (52) - (55) and (56) expressed through function, will become:

$$\begin{aligned} A(f(t), t) &= -2a_\varepsilon \left[ 1 - \frac{\lambda_0}{2\varepsilon_0 a_\varepsilon} \tilde{f}^{\frac{a_\lambda - a_\varepsilon}{a_f}} \right] - a_\mu \\ B(f(t), t) &= -\frac{a_\varepsilon^2}{f_0 a_f} \tilde{f}^{-1}(t) \left[ 1 - \frac{\lambda_0 a_\lambda}{\varepsilon_0 a_\varepsilon^2} \tilde{f}^{\frac{a_\lambda - a_\varepsilon}{a_f}} \right] \end{aligned}$$

$$\begin{aligned}
C(f(t), t) &= \frac{\lambda_0}{\varepsilon_0} \tilde{f}^{\frac{a_\lambda - a_\varepsilon}{a_f}}(t) - 2a_\mu - a_\varepsilon \\
D(f(t), t) &= -a_\mu \left[ 1 - a_\varepsilon + \frac{\lambda_0}{\varepsilon_0} \tilde{f}^{\frac{a_\lambda - a_\varepsilon}{a_f}}(t) \right] \\
a^2(f(t), t) &= \frac{c^2}{\mu_0 \varepsilon_0} \tilde{f}^{-\frac{a_\mu - a_\varepsilon}{a_f}}(t). \tag{57}
\end{aligned}$$

Thus, being set exponential kind of law changeability of chemical concentration elements of the environment in a kind  $\tilde{f} = \tilde{f}(t)$  and considering (57), the generalized equations of distribution of electromagnetic waves by exponential law changing in the chemical environment will have a following appearance:

$$\begin{aligned}
\frac{\partial^2 \vec{E}}{\partial t^2} - \left\{ 2a_\varepsilon \left[ 1 - \frac{\lambda_0}{2\varepsilon_0 a_\varepsilon} \tilde{f}^{\frac{a_\lambda - a_\varepsilon}{a_f}}(t) \right] + a_\mu \right\} \frac{\partial \vec{E}}{\partial t} - \\
- \frac{a_\varepsilon^2}{f_0 a_f} \tilde{f}^{-1}(t) \left[ 1 - \frac{\lambda_0 a_\lambda}{\varepsilon_0 a_\varepsilon^2} \tilde{f}^{\frac{a_\lambda - a_\varepsilon}{a_f}}(t) \right] \vec{E} = \\
= \frac{c^2}{\mu_0 \varepsilon_0} \tilde{f}^{-\frac{a_\mu + a_\varepsilon}{a_f}}(t) \left[ \Delta \vec{E} - \text{grad div } \vec{E} \right] \tag{58}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \vec{H}}{\partial t^2} + \left[ \frac{\lambda_0}{\varepsilon_0} \tilde{f}^{\frac{a_\lambda - a_\varepsilon}{a_f}}(t) - 2a_\mu - a_\varepsilon \right] \frac{\partial \vec{H}}{\partial t} - \\
- a_\mu \left[ 1 - a_\varepsilon - \frac{\lambda_0}{\varepsilon_0} \tilde{f}^{\frac{a_\lambda - a_\varepsilon}{a_f}}(t) \right] \vec{H} = \\
= \frac{c^2}{\mu_0 \varepsilon_0} \tilde{f}^{-\frac{a_\mu + a_\varepsilon}{a_f}}(t) \left[ \Delta \vec{H} - \text{grad div } \vec{H} \right]. \tag{59}
\end{aligned}$$

Here  $\varepsilon_0, L, \mu_0, a_\varepsilon, a_\mu, a_\lambda$ - the constants defined from special experiments,  $c^2 = \frac{1}{\varepsilon_0 \mu_0}$ - expression of speed of light in emptiness. The equation (58) is simple for writing down in more compact kind:

$$\begin{aligned}
a_1 \tilde{f}^{b_1}(t) \frac{\partial^2 \vec{E}}{\partial t^2} - a_2 \tilde{f}^{b_1}(t) \left[ 1 - a_3 \tilde{f}^{b_2}(t) \right] \frac{\partial \vec{E}}{\partial t} - \\
- a_4 \tilde{f}^{b_1 - 1}(t) \left[ 1 - a_5 \tilde{f}^{b_3}(t) \right] \vec{E} = \Delta \vec{E} - \text{grad div } \vec{E} \tag{60}
\end{aligned}$$

Here

$$\begin{aligned}
a_1 &= \frac{\varepsilon_0 \mu_0}{c^2}, \\
a_2 &= \frac{\varepsilon_0 \mu_0}{c^2} (2\lambda_\varepsilon + a_\mu) \\
a_3 &= \frac{\lambda_0}{2\varepsilon_0 a_\varepsilon (2\lambda_\varepsilon + a_\mu)}, \\
a_4 &= \frac{\varepsilon_0 \mu_0}{c^2} \frac{a_\varepsilon^2}{f_0 a_f}, \\
a_5 &= \frac{\lambda_0 a_\lambda}{\varepsilon_0 a_\varepsilon^2}. \tag{61}
\end{aligned}$$

It is similarly possible to write down in a compact kind and the formula (59). Let the found experimental function of changeability of chemical concentration of elements of environment looks like:

$$\tilde{f}(t) = \frac{f(t)}{f_0} = e^{-\theta t}. \quad (62)$$

Here  $\theta$  - experimental constant. Considering (62) equation (60) will enter the name in a following compact kind:

$$\begin{aligned} & a_1 e^{-\theta b_1 t} \frac{\partial^2 \vec{E}}{\partial t^2} - a_2 e^{-\theta b_1 t} \left[ 1 - a_3 e^{-\theta b_2 t} \right] \frac{\partial \vec{E}}{\partial t} - \\ & - a_4 e^{-\theta(b_1-1)t} \left[ 1 - a_5 e^{-\theta b_3 t} \right] \vec{E} = \Delta \vec{E} - \text{grad div } \vec{E}. \end{aligned} \quad (63)$$

#### 4. About one one-dimensional problem of distribution of electromagnetic waves by exponential law of a changing chemical environment

Let's consider one one-dimensional private a case, i.e. a case solenoid of a vector of an electric  $\text{div} \vec{E} = 0$  field. This case is possible under condition of weak changeability of concentration of elements of environment or at absence in the environment of electric charges or its infinitesimal value. And a vector of electric  $\vec{E} = \vec{E}(x, t)$  force will be function of coordinate and time. In this case a regional problem of distribution of an electromagnetic wave (63), signs a following kind:

$$\begin{aligned} & a_1 e^{-\theta b_1 t} \frac{\partial^2 E}{\partial t^2} - a_2 e^{-\theta b_1 t} \left[ 1 - a_3 e^{-\theta b_2 t} \right] \frac{\partial E}{\partial t} - \\ & - a_4 e^{-\theta(b_1-1)t} \left[ 1 - a_5 e^{-\theta b_3 t} \right] E = \frac{\partial^2 E}{\partial x^2} \end{aligned} \quad (64)$$

$$E(x) |_{x=0} = 0$$

$$E(x) |_{x=1} = 0 \quad (65)$$

$$E(x, t) |_{t=0} = \varphi(x)$$

$$\frac{\partial E(x, t)}{\partial t} |_{t=0} = \psi(x). \quad (66)$$

Here  $\varphi(x)$  and  $\psi(x)$  the set functions. By virtue of that the right part of this equation depends only on coordinate  $x$ , and the left part depends only on time  $t$  such vector equation supposes application to it of a method of division of variables. Therefore (64) - (66) it is possible to search for the decision of a regional problem in the form of division of variables:

$$E(x, t) = X(x) \cdot T(t). \quad (67)$$

In this case the system (64) - (66) will be reduced to system of the ordinary differential equations following a kind:

$$X''(x) + \lambda X(x) = 0 \quad (68)$$

$$T''(t) - \frac{a_2}{a_1} [1 - a_3 e^{-\theta b_2 t}] T'(t) - \frac{a_4 e}{a_1} \left[ 1 - a_5 e^{-\theta b_3 t} - \frac{1}{a_4 e} \lambda e^{\theta b_1 t} \right] T(t) = 0. \quad (69)$$

By virtue of that connected with changeability of the chemical environment of function  $e^{-\theta b_1 t}$ ,  $e^{-\theta b_2 t}$ ,  $e^{\theta b_3 t}$  at a final interval of time final values accept, secondly, factors of the equation (69) are analytical, therefore the decision of system (68)-69 under regional conditions (65) - (66) does not represent mathematical complexity. Therefore the decision of a task in view can be written down in the form of converging of some:

$$E(x, t) = \sum_{k=0}^{\infty} X_k(x) \cdot T_k(t). \quad (70)$$

Here function  $T_k(t)$  is the decision of the equation (69) which looks like converging of some.

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