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INVESTIGATION OF FREE VIBRATIONS ACOUSTIC MEDIUM WITH CYLINDRICAL INCLUSION CONTAINING ELASTICALLY SUSPENDED MASS, BY THE INVERSE METHOD

Abstract

Free vibrations of a cylindric inclusion containing suspended mass situated in elastic medium are investigated by the inverse method. Eigenfrequencies of vibrations are defined at some values of parameters of the system.

Study of joint motion of discrete systems continuum is of great practical value. For example to the readings of data units of measuring devices of wave processes there are introduced noises from their own vibrations, or in interacting with seismic waves, constructions may be considered as discrete systems.

Investigation of behavior of shell constructions with added masses is of great practical interest. Main attention at these problems is given to questions on estimation of damping ability of a construction (under the action of dynamical loads). If rigidity of a shell is great in comparison with rigidity of dampers (springs), in many cases one can ignore shell strains.

Influence of a system of loads interior to a cylindrical shell on its behavior while interacting with spherical pressure wave was considered in [1]. A system of loads (concentrated masses) is fastened to internal surface of a shell by means of elastic strings. The masses are connected with linear characteristics springs among themselves as well. Concentrated masses may only reciprocate. The solution is constructed with using Fourier series (with respect to angular coordinate) and integral transformations. (Laplace-with respect to time, and Fourier-with respect to axial coordinate). The conversion integrals were calculated by means of Gauss-Lagger quadratic formula (for the Fourier transform transformer) and expansion in series by ultraspherical polynomials (for the Laplace transform transformer). Numerical calculations were carried out for a water immersed steel shell under exponential profile spherical wave.

In the paper [2] a rigid inclusion with elastically suspended mass interior to it is considered in acoustic media. Environment forces P and interaction forces with suspended mauss M_2 effect on inclusion of the mass M_1 by means of a rigidity spring L. Spring's effect transmitted to inclusion of the mass M_1 and in the apposite direction with inverse sign on the mass M_2 is proportional to the difference of permutations of the inclusion x_1 and internal mass x_2 , i.e. equals $L\left(x_2 - x_1\right)$. Under the action of these forces the inclusion acquires acceleration defined by Newton's second law:

$$M_1 \frac{d^2 x_1}{dt^2} = P + L\left(x_2 - x_1\right), \qquad (1)$$

here P is the resistance of medium to inclusion motion.

The force $L(x_2 - x_1)$ effects on internal mass, consequently, suspended mass acceleration will be expressed in the following way:

$$M_2 \frac{d^2 x_2}{dt^2} = -L(x_2 - x_1). \tag{2}$$

Thus, motion of a cylindrical shell with oscillator in acoustic medium after passage of wave (non-stationary problem) was investigated in the paper [2].

Investigation of eigen vibrations of the above-described system whose results may be compared with the results obtained in the paper [2] is of great interest alongside with the investigations of non-stationary motion of elastic medium.

So, investigation of eigen vibrations is important from practical point of view, for example, if the inclusion is a construction that has vibrations sources causing resonance. The system may be in resonance state from the influence of external sources of vibrations.

Eigen vibrations of the system may represent interest in working out measuring instruments or on the contrary, negatively tell on the functioning of the existing instruments.

In this connection, in the present paper we study free vibrations of the above-mentioned system in acoustic medium. It is assumed that the mass is elastically suspended, the cylinder continuously moves in the medium.

The considered problem is plane. Vortexless motion of the medium is described by the equation:

$$\Delta\varphi = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}, \tag{3}$$

where a is sound propagation velocity, φ is velocity potential $\overrightarrow{\vartheta} = \operatorname{grad}\varphi$, $\overrightarrow{\vartheta}$ is velocity, Δ is a Laplace operator.

Inclusion moves by the law (1), (2).

For a sylindrical inclusion of radius r_0

$$P = r_0 \int\limits_0^{2\pi} p\cos\theta d\theta, \quad \text{where} \quad p = -\rho\frac{\partial\varphi}{\partial t}, \tag{4}$$

ρ is density.

Conditions of equality of normal constituents of velocity to the surface of holder is of the form:

$$\frac{\partial\varphi}{\partial r} = \frac{dx_1}{dt}\cos\theta. \tag{5}$$

In the paper [3] it is shown that for calculation of eigen frequencies and amplitudes of elastic element vibrations in liquid, for example, of cylindrical heatreleasing element in atomic nuclear reactor or heatexchange device tubes, it is necessary to know the quantity of adjoint mass and damping force. Besides, these characteristics depend on the location of fixed boundaries, surrounding the cylinder [4,5]. Therefore, assuming that from the external side a medium is bounded with fixed surface $r = r_1$ or in the case of unbounded medium, a node of standing wave settles down on the surface $r = r_1$, we'll have the condition

$$\vartheta|_{r=r_1} = \left.\frac{\partial\varphi}{\partial r}\right|_{r=r_1} = 0. \tag{6}$$

Equation (1) in cylindrical coordinates has the following form:

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial\varphi}{\partial r}+\frac{1}{r^2}\cdot\frac{\partial^2\varphi}{\partial\theta^2}=\frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}. \quad (7)$$

The method of separation of variables is used for finding eigen frequencies. Thus, we can represent the solution of equation (7) in the form:

$$\begin{array}{l}\varphi\left(r,\theta,t\right)=R\left(r\right)e^{iwt}\cos\theta\\ x_1=Be^{iwt};\quad x_2=Ce^{iwt}\end{array} \quad (8)$$

where B and C are the unknowns to be defined.

Considering (8) equation (7) takes the form:

$$R''+\frac{1}{r_1}R'+\left(1-\frac{1}{r_1^2}\right)R=0, \quad (9)$$

where $r=\frac{a}{w}r_1$.

The solution of equation (9) has the form [6]:

$$R=EJ_1\left(\frac{wr}{a}\right)+DN_1\left(\frac{wr}{a}\right), \quad (10)$$

where E and D are the unknowns to be defined.

Here $J_1\left(\frac{wr}{a}\right), N_1\left(\frac{wr}{a}\right)$ and are Bessel and Neumann cylindrical functions.

Substituting (8) into (1), (2) and (5) allowing for (4) we get a system of algebraic homogeneous equations with respect to constants E, B, C and D:

$$\begin{cases}\left(M_1w^2-L\right)B+\rho r_0\pi Riw+LC=0\\ C\left(M_2w^2-L\right)+LB=0\\ R'-iwB=0\\ R'=0\end{cases} \quad (11)$$

For the existence of non-trivial solution of system (11) we equate the principal determinant of the mentioned system to zero.

$$\begin{vmatrix}\rho r_0\pi J_1 iw & M_1w^2-L & L & \rho r_0\pi N_1 iw\\ 0 & L & M_2w^2-L & 0\\ J_1'\left(\frac{r_0w}{a}\right) & -iw & 0 & N_1'\left(\frac{r_0w}{a}\right)\\ J_1'\left(\frac{rw}{a}\right) & 0 & 0 & N_1'\left(\frac{r_0w}{a}\right)\end{vmatrix}=0\,. \quad (12)$$

As a result, we get the frequency equation:

$$J_1'\left(\frac{wr_0}{a}\right)N_1'\left(\frac{wr_1}{a}\right)\left(M_1M_2w^4-Lw^2\left(M_1+M_2\right)\right)-$$

$$-J_1'\left(\frac{wr_1}{a}\right)N_1'\left(\frac{wr_0}{a}\right)\left(M_1M_2w^4-L\left(M_1+M_2\right)w^2\right)-$$

$$-J_1'\left(\frac{wr_1}{a}\right)N_1'\left(\frac{wr_0}{a}\right)\rho\pi r_0\left(M_2w^2-L\right)w^2=0. \quad (13)$$

Let's introduce the following denotation:

$$\frac{L}{M_1}=k_1^2 \quad \frac{L}{M_2}=k_2^2 \quad \frac{L}{\rho r_0^2\pi}=k_0^2 \quad m=\frac{\rho\pi r_0}{M_1}.$$

Then equation (13) takes the following form:

$$J_1'\left(\frac{wr_0}{a}\right)N_1'\left(\frac{wr_1}{a}\right)\left(w^2-k_1^2-k_2^2\right)-J_1'\left(\frac{wr_1}{a}\right)N_1'\left(\frac{wr_0}{a}\right)\left(w^2-k_1^2-k_2^2\right)-$$

$$J_1'\left(\frac{wr_1}{a}\right)N_1\left(\frac{wr_0}{a}\right)m\frac{1}{r_0}\left(w^2-k_2^2\right)=0. \tag{14}$$

Now pass to dimensionless quantities. To this end we introduce the following denotation:

$$\frac{wr_0}{a}=\overline{w}, \quad \overline{k_1}=\frac{k_1r_0}{a}, \quad \overline{k_2}=\frac{k_2r_0}{a}, \quad \overline{k_0}=\frac{k_0r_0}{a}, \quad \frac{wr_1}{a}=\overline{w}^1$$

As a result, equation (14) takes the form:

$$\overline{k}=\sqrt{\frac{F_1}{F_2}}, \tag{15}$$

where

$$F_1=c\overline{w}^4\left(J_0\left(\overline{w}\right)N_0\left(c\overline{w}\right)-J_0\left(c\overline{w}\right)N_0\left(\overline{w}\right)\right)+\overline{w}^3\left(J_1\left(c\overline{w}\right)N_0\left(\overline{w}\right)-\right.$$

$$\left.-J_0\left(\overline{w}\right)N_0\left(c\overline{w}\right)\right)+c\overline{w}^3\left(J_0\left(c\overline{w}\right)N_1\left(\overline{w}\right)-J_1\left(\overline{w}\right)N_0\left(c\overline{w}\right)\right)+$$

$$+\overline{w}^2\left(J_1\left(\overline{w}\right)N_1\left(c\overline{w}\right)-J_1\left(c\overline{w}\right)N_1\left(\overline{w}\right)\right)+$$

$$+c\overline{w}^3mJ_0\left(c\overline{w}\right)N_1\left(\overline{w}\right)-\overline{w}^2J_1\left(c\overline{w}\right)N_0\left(\overline{w}\right)$$

$$F_2=-c\overline{w}\left(J_0\left(c\overline{w}\right)N_1\left(\overline{w}\right)-J_1\left(c\overline{w}\right)N_1\left(\overline{w}\right)\right)m-$$

$$-\left(1+b^2\right)\left(\overline{w}^3c\left(J_0\left(\overline{w}\right)N_0\left(c\overline{w}\right)-J_0\left(c\overline{w}\right)N_0\left(\overline{w}\right)\right)\right)+$$

$$+\overline{w}\left(J_1\left(c\overline{w}\right)N_0\left(\overline{w}\right)-J_0\left(\overline{w}\right)N_1\left(c\overline{w}\right)\right)+c\overline{w}\left(J_0\left(c\overline{w}\right)N_1\left(\overline{w}\right)-\right.$$

$$\left.-J_1\left(\overline{w}\right)N_0\left(\overline{w}\right)\right)+J_1\left(\overline{w}\right)N_1\left(c\overline{w}\right)-J_1\left(c\overline{w}\right)N_1\left(\overline{w}\right)$$

$$\overline{k}_1=b\overline{k}_2 \quad \overline{k}_2=\overline{k} \quad \overline{k}_1=b\overline{k} \quad \overline{w}^1=c\overline{w}.$$

For the given values of the parameters the graphs $\overline{k}-\overline{w}$ (fig. 1) where $\overline{w}=A$ were constructed on the interval $0-250$ for $\overline{k}$ and $0-0,6$ for $\overline{w}$

$$\begin{array}{l}
\overline{k}=X\left(A\right), \quad m=1, \quad b=2, \quad C=10,\\
\overline{k}=Y\left(A\right), \quad m=1, \quad b=2, \quad C=20,\\
\overline{k}=Z\left(A\right), \quad m=1, \quad b=2, \quad C=30,\\
\overline{k}=N\left(A\right), \quad m=1, \quad b=2, \quad C=40.
\end{array}$$

It is seen from the figure that in the first mode $\overline{k}$ increases according to increase of $\overline{w}$. In the next modes $\overline{k}$ decreases according to increase of $\overline{w}$. This may be explained by the opposing motion of the cylinder and medium. The number of curves preceding the considered curve corresponds to the number of nodes of standing wave in the medium.

As is sen from figure 1 the graphs of different vibrations modes have vertical asymptotes, whose abscissas respond to nodal surfaces of standing wave.

In order to define frequencies of the system $\overline{w}$ for the given frequency of vibrations of the oscillator $\overline{k}$, on the graph $\overline{k}-\overline{w}$ (fig.1) we draw a horizontal straight line with ordinate $\overline{k}$ and having measured abscissa of intersection points of this horizontal line with the branches of the graph $\overline{k}-\overline{w}$, we get the quantities $\overline{w}_k$ by means of which we can calculate the values $\overline{w}_k = \dfrac{\overline{w}_k a}{r_0}$ of the spectrum of frequencies of eigen vibrations of the system.

We can carry out investigation not only graphically, but also in analytical way, for example, differentiating the expression (15) with respect to $\overline{w}$ and determine the character of change of frequencies of the system.

The graphs $\overline{k}-\overline{w}$ of allow to analyze the character of dependence of frequency on the rigidity of the oscillator spring.

Fig. 1. A graph of dependence of frequency of the oscillator $\overline{k}$ on the frequency of the system $\overline{w}$.

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Received September 5, 2007; Revised December 25, 2007