Khankishi B. MAMMADOV, Yelena Y. MUSTAFAEVA

METHODS OF IBM COMPUTATION OF CURVILINEAR COLUMNS ON LARGE DISPLACEMENTS

Abstract

At present usage of IBM opened new opportunities for working out methods of building and construction element deformation research. There appeared new works with more exact solutions of problems which were considered unsolvable for engineers because of their complexity. Problems of elastic flexible columns computation on large displacements are related to them.

In mechanics of columns elastic flexible curvilinear columns take an especially important place. Elastic flexible columns have wide application in different fields of technique – in the field of mechanical engineering, robot constructing, device constructing and other branches of economy. And time devices using columns have got spread not only as clocks but as sensors of stable signals in different automatic devices of land and space technique. Determination of the current time value and measuring time intervals are necessary at solution of mechanical objects control problems in aviation, space research. The accuracy of time device reading largely depends on the elastic element calculation accuracy taking into account real conditions of its work. In this connection the problem of large displacements determination on deformation is very actual when in the process of fine detail deformation its initial configuration changes strongly at that the displacements on deformation become considerable with the length of the construction itself [4].

General theory of slender elastic columns was worked out in the second half of the XIX-th century in works of M.F.Okatov, D.K.Bobilyov, G.Kirchgof [1], A.Klebsha. Its further development is connected with the names of A.Lyav, Ye.L.Nicolay, V.L.Kirpichev, P.F.Popkovich, A.N.Krilov, P.M.Riz, A.I.Lourie, G.U.Janelidze, V.G.Rekach, A.Ya.Driving, Wan Tsui-de [3], Abbasov U.M. and many other investigators. Monographs [2] by Ye.P.Popov should be especially noted where the previous investigations are generalized and there is given a general method of calculation of slender columns plane shape bend in the system of coordinates oriented by force.

In the present paper there are given new differential relationships of the problem allowing to get the integral of the original differential equation in the frames of the theory of slender columns on large displacements.

The paper is dedicated to the calculation of slender elastic three-hinged round arch loaded in the clue by concentrated force $2Fv$ (fig. 2a).

Let's make equilibrium equation of element dS of the deformed column part free of forces and moments (fig.1a). Having chosen the tangent and the normal to the axis at the last point of the element as the coordinates axes and having decomposed all the forces in the direction of these axes we'll obtain the following (fig.1b) [5]

$$\begin{gathered} F_n + (F_q + dF_q)d\varphi - (F_n + dF_n) = 0; \\ F_q - (F_n + dF_n)d\varphi - (F_q + dF_q) = 0 \end{gathered} \tag{1}$$

Neglecting the addends of higher order smallness we'll get from the first equation of (1)

$$dF_n/d\varphi = F_q \tag{2}$$

From the second equation of (1) we have:

$$dF_q/d\varphi = -F_n. \tag{3}$$

Now we'll make momentum equilibrium equation. It has the form

$$M + F_q ds - F_n ds d\varphi - (M + dM) = 0$$

whence we get the known relationship

$$dM/ds = F_q \tag{4}$$

From (2) and (4) it follows that

$$dF_n/dM = d\varphi/ds = \chi \tag{5}$$

Here $\chi = d\varphi/ds$ is the curvature of the column axis.

Thus we have the following relationships for the plane crooked column deformed element computing equations [9]:

$$\chi\frac{dM}{d\varphi} - F_q = 0, \ \frac{dF_n}{d\varphi} - F_q = 0, \ \frac{dF_q}{d\varphi} + F_q = 0 \tag{6}$$

On the other hand for the bent elastic column in the accepted system of coordinates we'll use the known elastic equilibrium exact equation on the curvilinear column plane bend [6]:

$$\chi - \chi_0 = -\frac{M}{B} \tag{7}$$

It is known that when deriving Euler's formula they didn't manage to get numerical value of the column bend as the value of integration constant was undefined [8]. It is connected with that the column bent axis was described by an approximate differential equation. If to use a precise differential equation of the bent axis for the investigation then it is possible to determine both the critical force and the dependence between the compressing force and the column bends. So we'll use the precise expression of the curvature. The precise expression of the curvature as it is known has the form

$$\chi = \pm\frac{y''}{\left[1 + (y')^2\right]^{3/2}} \tag{8}$$

at the same time in the linear theory of column bends in the view of elastic line angular displacement smallness the curvature is usually taken as

$$\chi \approx \pm y''.$$

The curvature χ at the given point of an elastic line is considered positive if the decline angle of the elastic line increases with the increase of the arc length S vice versa the curvature will be negative.

From the second and third equations of (6) it follows

$$\frac{d^2 F_n}{d\varphi^2} + F_n = 0$$

the general equation of which is

$$F_n(\varphi) = C_1 \cos\varphi + C_2 \sin\varphi$$

whence

$$F_q(\varphi) = -C_1 \sin\varphi + C_2 \cos\varphi.$$

From the first and second equations of (6) we'll obtain

$$\chi = \frac{dF_n}{dM} \quad \text{or} \quad \frac{dM}{dF_n} = \frac{1}{\chi}. \tag{9}$$

Supposing $B = const$ "$\chi_0 = const$ and differentiating equations (7) by N" we'll have:

$$\frac{d\chi}{dF_n} = -\frac{1}{B}\frac{dM}{dF_n}. \tag{10}$$

Using relationships (9) and (10) we'll get:

$$\frac{1}{\chi} = -B\frac{d\chi}{dF_n}.$$

Separating variables and integrating we find

$$\chi^2 = C_3 - \frac{2F_n}{B}$$

or

$$\chi = \pm\left(C_3 - \frac{2F_n}{B}\right)^{1/2}. \tag{11}$$

Sign plus before expression (11) corresponds to the positive curvature, sign minus - to negative one. Substituting value (11) in equation (7), for the bending element we'll get [11]:

$$M = B\left[\chi_0 \pm \left(C_3 - \frac{2F_n}{B}\right)^{1/2}\right].$$

Having obtained the second derivative of the known expression $y' = \dfrac{dy}{dx} = tg\varphi$ we have:

$$y'' = \frac{d}{dx}\frac{dy}{dx} = \frac{dtg\varphi}{dx} = \frac{1}{\cos^2\varphi}\frac{d\varphi}{dx}.$$

Taking into account $\left[1+(y')^2\right]^{3/2} = \left[1+tg^2\varphi\right]^{3/2} = \dfrac{1}{\cos^3\varphi}$ the exact expression of curvature takes the form:

$$\chi = \pm\frac{y''}{\left[1+(y')^2\right]^{3/2}} = \pm\cos\varphi\frac{d\varphi}{dx}.$$

Thus,

$$\chi = \pm\cos\varphi\frac{d\varphi}{dx}. \tag{12}$$

And taking into consideration $\dfrac{dy}{dx} = tg\varphi$ we'll get

$$\chi = \pm\sin\varphi\frac{d\varphi}{dy}. \tag{13}$$

Integrating separately equations (12) and (13) for the coordinates of the deformed column arbitrary point (sections) we'll obtain outwardly known integrals (the magnitude χ in equations (14) and (15) is curvature after deformation).

$$X + C_4 = \int \chi^{-1}\cos\varphi d\varphi, \tag{14}$$

$$Y + C_5 = \int \chi^{-1}\sin\varphi d\varphi. \tag{15}$$

Arbitrary constants $C_i (i = 1-5)$ are defined from boundary conditions of the concrete constructions exploiting condition.

For convenience of the further calculations we introduce dimensionless parameters: $f_n = F_n : F$, $F_q = F_q : F$, $m = M : Fr$, $\psi = \chi\cdot r$, $X = x : r$, $Y = y : r$, where $\chi_0 = 1/r = const$ initial curvature of axis; χ curvature after deformation, $B = EJ$ rigidity at bend, F_n, F_q, M internal force factors – bending moment, normal and lateral forces; $d\varphi$ central angle of the deformed element, $y' = dy/dx = tg\varphi$; y, x the deformed element section center coordinates, $r = 1/\chi_0$ initial radius of circular column curvature; $f = Fr^2/B$ dimensionless quantity of concentrated force; $C_i (i = 1-5)$ arbitrary constants determined from boundary conditions.

In view of symmetry for one second of arc the boundary conditions are written as follows (fig 2b)

$$f_q(0) = 0,\ f_n(0) = -1,\ X(0) = 0 \text{ for } \varphi = 0$$

$$m(\theta) = 0, Y(\theta) = 0 \text{ for } \varphi = \pm\theta. \tag{16}$$

Satisfying the first and second conditions (16) we'll get $C_1 = -1, C_2 = 0$.

Satisfying the fourth condition (16) with account of values C_1 and C_2 we have:

$$C_3 = 1 - 2f cos\theta.$$

Thus taking into account the values C_1, C_2 and C_3 in the expressions of curvature and bending moment we have

$$\Psi(\varphi) = \pm[1 + 2f(cos\varphi - cos\theta)]^{1/2},$$

$$m(\varphi) = \frac{1}{f}\{1 \mp [1 + 2f(cos\varphi - cos\theta)]^{1/2}\}.$$

Substituting the curvature value into $Y(\varphi)$ expression we'll get

$$Y(\varphi) + C_5 = \int [1 + 2f(cos\varphi - cos\theta)]^{-1/2} \sin\varphi d\varphi$$

or

$$Y(\varphi) + C_5 = \pm\ \frac{1}{f}\{1 + 2f(cos\varphi - cos\theta)\}^{1/2}.$$

At $\varphi = \pm\theta$ we have that $y(\theta) = 0$. Then

$$C_5 = \pm\frac{1}{f}$$

or

$$Y(\varphi) = \pm\ \frac{1}{f}\{[1 + 2f(cos\varphi - cos\theta)]^{1/2} - 1\}.$$

On the other hand at $\varphi = 0,\ X(\varphi) = 0$

$$\text{or} \quad 0 + C_4 = \int [1 + 2f(cos\varphi - cos\theta)]^{-1/2} cos\varphi d\varphi\Big|_{\varphi=0}. \tag{17}$$

At $\varphi = \theta,\ X(\varphi) = X_B$

$$X_B + C_4 = \int [1 + 2f(cos\varphi - cos\theta)]^{-1/2} cos\varphi d\varphi\Big|_{\varphi=\theta}. \tag{18}$$

From equations (17) and (18) it follows that

$$X_B = \int\limits_{\theta}^{\theta} [1 + 2f(cos\varphi - cos\theta)]^{-1/2} cos\varphi d\varphi. \tag{19}$$

Further using values C_1, C_2 and C_3 we'll get [10]

$$\int\limits_{-\theta}^{+\theta} [1 + 2f(cos\varphi - cos\theta)]^{-1/2} d\varphi = \alpha$$

or

$$\int\limits_{0}^{0} [1 + 2f(cos\varphi - cos\theta)]^{-1/2} d\varphi = \frac{\alpha}{2}. \tag{20}$$

Solving equation (20) by numerical methods at the given f and α, we define angular deformation θ in the joint.

There exist different approaches to the solution of such problems on IBM. One of the approaches is making corresponding programs for every concrete case.

Let's describe an algorithm of angular deformation θ calculation. It consists of two stages: the problem discretization, i.e. equation (20), by means of replacement of the integral in the left hand side of (20) with an integral sum, and solution of the obtained nonlinear equation with respect to θ.

Let's for convenience write equation (20) in the form

$$\int_0^\theta F(\varphi,\theta)\,d\varphi = \frac{\alpha}{2}, \tag{21}$$

where $F(\varphi,\theta) = \dfrac{1}{\sqrt{1+2f(\cos\varphi-\cos\theta)}}$.

The principle difficulty of equation (21) is in the presence of the searched parameter θ both in the integrand and in the limits of integration. As the result of discretization we'll get the equation [7]:

$$\int_0^\theta F(\varphi,\theta)\,d\varphi \approx \sum_{i=1}^N F(\varphi,\theta)\frac{\theta}{N}, \quad \varphi_i = \frac{\theta}{N}(i-1), \quad i=1,2,...,N,$$

where N is the number of intervals into which the interval $[0,\theta]$ of integration is divided, $h=\dfrac{\theta}{N}$ a uniform step of partition (discretization), $\varphi_0 = 0 < \varphi_1 < \varphi_2 < ... < \varphi_N = \theta$ uniform grid of interval $[0,\theta]$ with step h. Then integral equation (21) is replaced by the equation nonlinear with respect to θ

$$\Phi(\theta) = \sum_{i=1}^N F\left(\frac{i-1}{N}\theta,\theta\right)\frac{\theta}{N} - \frac{\alpha}{2} = 0, \tag{22}$$

where function $\Phi(\theta)$ is defined by the formula:

$$\Phi(\theta) = \sum_{i=1}^N \frac{1}{\sqrt{1+2f\left(\cos\dfrac{(i-1)}{N}\theta - \cos\theta\right)}}\frac{\theta}{N} - \frac{\alpha}{2}.$$

Equation (22) is solved by Newton method:

$$\theta_n = \theta_{n-1} - \frac{\Phi(\theta_{n-1})}{\Phi'(\theta_{n-1})},$$

where the initial value θ_0 is determined from the condition:

$$\Phi(\theta_0)\cdot\Phi''(\theta_0) > 0.$$

Discretization is made by the method of rectangles, the optimal fragmentation N is specified by the formula

$$\max_{[0,\theta]} \frac{|f''(z)|}{12n^2}\theta^3 < \varepsilon.$$

$\alpha = \pi$ Table 1

f (Fr^2/EI)	θ (rad)	$M:Fr$	$y:r$	$Fn:F$	$Fq:F$	$x:r$	$\delta:r$
		$\varphi=0$		$\varphi=\theta$			
0.01	1.5808	1.0050	1.0050	0.0100	0.9999	0.9921	0.0158
0.02	1.5910	1.0100	1.0100	0.0202	0.9998	0.9841	0.0318
0.03	1.6011	1.0149	1.0149	0.0303	0.9995	0.9760	0.0480
0.04	1.6114	1.0198	1.0198	0.0406	0.9992	0.9678	0.0644
0.05	1.6217	1.0247	1.0247	0.0509	0.9987	0.9595	0.0810
0.10	1.6745	1.0485	1.0485	0.1035	0.9946	0.9166	0.1667
0.20	1.7841	1.0924	1.0924	0.2117	0.9773	0.8246	0.3509
0.30	1.8969	1.1291	1.1291	0.3204	0.9473	0.7263	0.5474
0.40	2.0097	1.1572	1.1572	0.4250	0.9052	0.6252	0.7497
0.50	2.1194	1.1759	1.1759	0.5215	0.8532	0.5247	0.9506
0.60	2.2236	1.1857	1.1857	0.6074	0.7944	0.4280	1.1440
0.70	2.3206	1.1877	1.1877	0.6815	0.7318	0.3371	1.3258
0.80	2.4098	1.1836	1.1836	0.7440	0.6682	0.2531	1.4937
0.90	2.4912	1.1748	1.1748	0.7958	0.6055	0.1765	1.6470
1.0	2.5650	1.1626	1.1626	0.8383	0.5451	0.1069	1.7862
1.5	2.8424	1.0803	1.0803	0.9556	0.2948	-0.1535	1.6931
2.0	3.0162	0.9974	0.9974	0.9921	0.1251	-0.3182	1.3636
2.5	3.1306	0.9266	0.9266	0.9999	0.0110	-0.4314	1.1371
3.0	3.2091	0.8679	0.8679	0.9977	-0.0674	-0.5152	0.9696
3.5	3.2645	0.8189	0.8189	0.9925	-0.1226	-0.5807	0.8386
4.0	3.3044	0.7776	0.7776	0.9868	-0.1621	-0.6341	0.7318
4.5	3.3336	0.7422	0.7422	0.9816	-0.1908	-0.6790	0.6420
5.0	3.3549	0.7116	0.7116	0.9773	-0.2117	-0.7176	0.5647
5.5	3.3706	0.6847	0.6847	0.9739	-0.2270	-0.7514	0.4971
6.0	3.3820	0.6609	0.6609	0.9712	-0.2381	-0.7815	0.4371
6.5	3.3903	0.6396	0.6396	0.9692	-0.2461	-0.8084	0.3832

$\alpha = \pi/2$ Table 2

f (Fr^2/EI)	θ (rad)	$M : Fr$	$y : r$	$F_n : F$	$F_q : F$	$x : r$	$\delta : r$
		$\varphi = 0$		$\varphi = \theta$			
0.01	0.7869	0.2935	0.2935	0.7060	0.7082	0.7068	0.0010
0.02	0.7884	0.2942	0.2942	0.7049	0.7093	0.7064	0.0020
0.03	0.7900	0.2948	0.2948	0.7039	0.7103	0.7060	0.0030
0.04	0.7915	0.2955	0.2955	0.7028	0.7114	0.7057	0.0040
0.05	0.7931	0.2961	0.2961	0.7017	0.7125	0.7153	0.0051
0.10	0.8009	0.2994	0.2994	0.6961	0.7180	0.7035	0.0104
0.20	0.8170	0.3062	0.3062	0.6844	0.7291	0.6996	0.0215
0.30	0.8339	0.3133	0.3133	0.6720	0.7405	0.6951	0.0334
0.40	0.8514	0.3205	0.3205	0.6589	0.7522	0.6910	0.0461
0.50	0.8697	0.3281	0.3281	0.6450	0.7642	0.6863	0.0599
0.60	0.8888	0.3358	0.3358	0.6303	0.7763	0.6812	0.0747
0.70	0.9087	0.3439	0.3439	0.6147	0.7887	0.6758	0.0906
0.80	0.9295	0.3522	0.3522	0.5982	0.8013	0.6701	0.1078
0.90	0.9512	0.3607	0.3607	0.5807	0.8141	0.6640	0.1263
1.00	0.9737	0.3695	0.3695	0.5622	0.8270	0.6754	0.1463
1.50	1.0008	0.4168	0.4168	0.4528	0.8911	0.6177	0.2745
2.00	1.2514	0.4675	0.4675	0.3140	0.9494	0.5649	0.4741

$\alpha = \pi/3$ Table 3

f (Fr^2/EI)	θ (rad)	$M : Fr$	$y : r$	$-F_n : F$	$Fq : F$	$x : r$	$\delta : r$
		$\varphi = 0$		$\varphi = \theta$			
0.01	0.5241	0.1341	0.1341	0.8658	0.5004	0.5000	0.0002
0.02	0.5245	0.1343	0.1343	0.8656	0.5008	0.4999	0.0004
0.03	0.5250	0.1344	0.1344	0.8653	0.5012	0.4999	0.0006
0.04	0.5255	0.1345	0.1345	0.8651	0.5016	0.4998	0.0008
0.05	0.5259	0.1347	0.1347	0.8649	0.5020	0.4997	0.0010
0.10	0.5283	0.1354	0.1354	0.8637	0.5041	0.4995	0.0020
0.20	0.5331	0.1369	0.1369	0.8612	0.5082	0.4990	0.0041
0.30	0.5380	0.1384	0.1384	0.8587	0.5124	0.4984	0.0063
0.40	0.5430	0.1399	0.1399	0.8562	0.5167	0.4979	0.0085
0.50	0.5481	0.1415	0.1415	0.8535	0.5211	0.4973	0.0109
0.60	0.5533	0.1431	0.1431	0.8508	0.5255	0.4967	0.0133
0.70	0.5587	0.1447	0.1447	0.8480	0.5300	0.4961	0.0158
0.80	0.5641	0.1464	0.1464	0.8451	0.5347	0.4955	0.0184
0.90	0.5697	0.1481	0.1481	0.8421	0.5394	0.4948	0.0211
1.00	0.5754	0.1498	0.1498	0.8390	0.5441	0.4941	0.0239
1.50	0.6057	0.1590	0.1590	0.8221	0.5694	0.4904	0.0396
2.00	0.6398	0.1692	0.1692	0.8022	0.5970	0.4859	0.0589
2.50	0.6781	0.1805	0.1805	0.7788	0.6273	0.4806	0.0829
3.00	0.7212	0.1931	0.1931	0.7510	0.6603	0.4742	0.1134
3.50	0.7698	0.2070	0.2070	0.7180	0.6960	0.4665	0.1531
4.00	0.8247	0.2233	0.2233	0.6788	0.7343	0.4571	0.2071
4.50	0.8862	0.2391	0.2391	0.6323	0.7747	0.4458	0.2877

Reliability of the investigation is confirmed by satisfying coincidence of some obtained results with the known in literature partial solutions of the problems [2].

The obtained results for the cases $\alpha = \pi$, $\alpha = \pi/2$ and $\alpha = \pi/6$ are given in tables $1, 2, 3$. Note that for the case $\alpha = \pi$ the given system turns into a double-hinged ring when the forces are applied to the hinges as the results for that (fig.3) and this (fig.2) coincide [9].

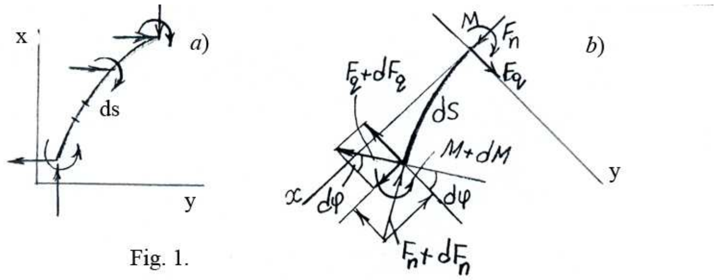

Fig. 1.

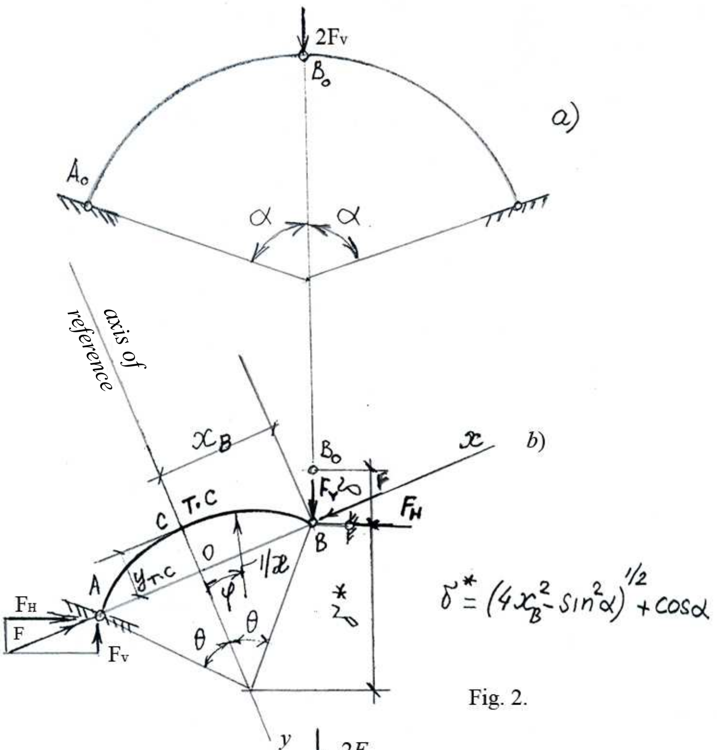

Fig. 2.

Fig. 3.

References

[1]. Kirchoff G. *Ueber das Gleichgewicht und die Bewegung eines unendlich dūnnen elastischen Stabes.*-Journal fūr die reine und angewandte Mathematik (Berlin), 1859, Bd 56 No 4, SS. 285-313; see also: Kirchoff G. Vorlesungen ūber mathematische Physik. Mechanik.-Leipzig: B.G. Teubner, 1876, 466 S.

[2]. Popov Ye.P. *Theory and computation of flexible elastic columns*, M.: Science, 1986, 296 p.

[3]. Wan Tsui-De. *Applied theory of elasticity.* –M.: Fizmatgiz 1959, translation from English I.N.Zemlyanskii.

[4]. Svetlitsky V.A. *Mechanics of columns.* part I. Static, Moscow, Higher school, 1987, 320 p.

[5]. Driving A.Ya. *To nonlinear theory of elastic columns. – In jour. Constructive mechanics and comp.of constr.*, M.; No 1, 1973, pp. 11-14.

[6]. Timoshenko S. P. *History of science and material resistance.* Transl. from English, M., 2006, 536 p.

[7]. Bakhvalov N.S. *Numerical methods.* – M.: Science, 1973, 631 p.

[8]. Alexandrov A.V. *Material resistance.* 13-th edition , M., 2004 , 591 p.

[9]. Abbasov U..M., Mammadov Khan.B., *Computation of double-hinged ring of symmetrical loading in the domain of large displacements, Investigations on constructive mechanics of spatial systems*, Moscow, UDN, 1990, pp. 18-23.

[10]. Mammadov Khan.B. *Bend of elastic ring with internal hinges on large displacements,* News of BSU, series of phys.math. sci., 2002., No 3, pp.132-141.

[11]. Mammadov Khan.B. *On bend of elastic column by concentrated force at large displacements*, News of Bak. St. Univ. ser. of phys.math. sci., 2002., No 4, pp.146-152.

Khankishi B. Mammadov
Azerbaijan State Pedagojic University
34, U. Hajibayov, AZ 1000, Baku, Azerbaijan
Tel.: (99412) 496 41 82

Yelena Y. Mustafaeva
Baku State University
23, Z.I. Khalilov str., AZ1148, Baku, Azerbaijan.
Tel.: (99412) 439 04 06

Received September 6, 2007; Revised December 20, 2007