Elkhan G. MAMEDGASANOV

TRANSIENT SH WAVES IN ELASTIC LAYER LYING, ON ELASTIC HALF-SPACE

Abstract

Propagation of transient SH (shear horizontal) waves in an homogeneous isotropic elastic layer, lying on elastic half-space, is studied. The waves are created by giving horizontal concentrated tangential load on free surface of a layer. The problem is solved by Laplace and Fourier integral transformations. The inverse transformations are calculated by the Cagniard de Hoop method.

Introduction. Non-one dimensional problems on transient waves propagation in deformable solids with boundaries are complicated problems of continuum mechanics [1,2]. The problems for elastic half-space excited by concentrated loads refer to them. The problem on excitation of elastic half-space surface by normal concentrated load was considered by Lamb. Further, the Lamb problem was generalized for the case of anisotropic, inhomogeneous and linear visco-elastic half-spaces with using simplest models [3-6].

Recently, horizontally polarized (SH) shear harmonic elastic waves [1,2] are intensively studied. Transient SH waves in elastic homogeneous isotropic half-space were investigated in [7].

In the present paper we study two-dimensional transient problem on propagation of horizontally polarized shear wave in elastic layer lying on elastic half-space excited by concentrated shear loads. The problem is solved by the Laplace and Fourier integral transformations. The inverse transformations are determined by the Cagniard-de-Hoop method. Notice that for an elastic half-space this method was developed in the paper [7].

Problem statement. Let permutations field in an elastic layer $0 \leq y \leq h$ and half-space $y \geq h$ be given in the form:

$$\overline{u}^k = \left\{0, 0, w^k(x,y,t)\right\} \quad (k = 1, 2,) \tag{1}$$

where the indices 1 and 2 correspond to a layer and half-space .

Considering that $u^k = 0, \quad v^k = 0$, deformations accept the form:

$$e^k = \frac{\partial u^k}{\partial x} + \frac{\partial v^k}{\partial y} + \frac{\partial w^k}{\partial z} = 0,$$

$$e^k_{xx} = \frac{\partial u^k}{\partial x} = 0, \qquad e^k_{yx} = 0, \qquad e^k_{zx} = \frac{1}{2}\frac{\partial w^k}{\partial x},$$

$$e^k_{xy} = \frac{1}{2}\left(\frac{\partial u^k}{\partial y} + \frac{\partial v^k}{\partial x}\right) = 0, \quad e^k_{yy} = \frac{\partial v^k}{\partial y} = 0, \quad e^k_{zy} = \frac{1}{2}\frac{\partial w^k}{\partial y},$$

$$e^k_{zx} = \frac{1}{2}\left(\frac{\partial u^k}{\partial z} + \frac{\partial w^k}{\partial x}\right) = \frac{1}{2}\frac{\partial w^k}{\partial x}, \; e^k_{yz} = \frac{1}{2}\left(\frac{\partial v^k}{\partial z} + \frac{\partial w^k}{\partial y}\right) = \frac{1}{2}\frac{\partial w^k}{\partial y}, \; e^k_{zz} = \frac{\partial w^k}{\partial z} = 0.$$

Stress tensor components are expressed by the relations

$$\sigma_{11}^k = \sigma_{12}^k = \sigma_{22}^k = \sigma_{33}^k = 0, \quad \sigma_{13}^k = \mu_k \frac{\partial w^k}{\partial x}, \quad \sigma_{23}^k = \mu_k \frac{\partial w^k}{\partial y}, \quad k = 1{,}2, \qquad (2)$$

where μ_k-is shear modulus.

Allowing for (1)-(2) the motion equations

$$\frac{\partial \sigma_{ij}^k}{\partial x_j} = \rho_k \frac{\partial^2 u_i^k}{\partial t^2} \quad u_1^k = u^k, u_2^k = v^k, u_3^k = w^k, x_1 = x, x_2 = y, \ x_3 = z \quad (k = 1{,}2)$$

are represented in the form of hyperbolic type partial differential equations

$$\mu_k \Delta w^k = \rho_k \frac{\partial^2 w^k}{\partial t^2} \qquad (-\infty < x < \infty, \ \ y > 0, \quad t > 0), \quad k = 1{,}2, \qquad (3)$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is a Laplace operator.

The equation (3) is solved under the following initial and boundary conditions:

$$w^k = 0, \ \ \frac{\partial w^k}{\partial t} = 0 \quad for \ \ t = 0 \quad (-\infty < x < \infty, \ \ 0 < y < h), \quad k = 1{,}2, \qquad (4)$$

$$\sigma_{23}^1 = -\delta(x) f(t) \quad for \ \ y = 0, \quad (-\infty < x < \infty, \ t > 0), \qquad (5)$$

$$w^1 = w^2, \quad \sigma_{23}^1 = \sigma_{23}^2 \quad for \quad y = h \quad (-\infty < x < \infty, \ t \geq 0), \qquad (6)$$

$$w^k \to 0 \quad r = \sqrt{x^2 + y^2} \to \infty, \quad k = 1{,}2, \qquad (7)$$

where $f(t)$ is a single-valued piece-wise smooth function vanishing for $t < 0$ and varying no rapidly than exponential function as $t \to \infty$; $\delta(x)$ is a Dirac delta function.

In all sections parallel to the plane xOy the wave pattern is identical (cylindrical waves propagate from the source). According to that has been said we can represent the problem in the form of laminated plane strip subjected to the action of instantaneous concentrated load perpendicular to free surface. On the axis Ox the permutations differ from zero, they are created by the stress σ_{13}^1. At the given statement of the problem the stress $\sigma_{23}^1 = 0$ at all the points of the axis $Ox(y = 0)$, except the origin $x = 0$. As the point $x = 0$ it is given in the form of Diracs δ function.

Problem solution. The problem (3)-(7) is solved with using Fourier integral transformations in coordinate x and Laplace transformations in time t, determined by the relations

$$w^{kF} = \int_{-\infty}^{\infty} w^k(x,y,t) e^{iqx} dx \qquad (\mathrm{Im} \ \ q = 0),$$

$$\overline{w}^k \equiv w^{kL} = \int_{0}^{\infty} e^{-pt} w^k(x,y,t) dt \qquad (\mathrm{Re} \ \ p > 0).$$

Applying these transformations to equations (3) and allowing for the initial conditions (4) we get

$$\mu_k\left(\frac{d^2w^{kLF}}{dy^2}-q^2w^{kLF}\right)=\rho_kp^2w^{kLF}\quad or\quad \frac{d^2w^{kLF}}{dy^2}=\left(q^2+\frac{\rho_kp}{\mu_k}\right)w^{kLF},$$

whose solution will be in the form:

$$w^{kLF}=A_ke^{-n_ky}+B_ke^{n_ky}\qquad (k=1,2). \tag{8}$$

Here $n_k=\sqrt{q^2+\eta_k^2}$, $\eta_k^2=\rho_kp^2/\mu_k$, by the condition (7) $B_2=0$. For isolating one-valued branch of this radical, in the plane q a cut is carried out from the points $\pm i\eta_k$ to the infinity along the rays $\arg q=\arg\eta_k\pm\pi/2$ and it is accepted that for $q=0$ the equality $\sqrt{q^2+\eta_k^2}=\eta_k$ is fulfilled. Then $\mathrm{Re}\sqrt{q^2+\eta_k^2}>0$ for $\mathrm{Im}\, q=0$, $\mathrm{Re}\, p>0$. Substituting the solution (8) to conditions (5) and (6) the problem is reduced to the following system of algebraic equations with respect to integration constants A_1, A_2, B_1:

$$\begin{cases} A_1-B_1=\dfrac{\overline{f}}{\mu_1n_1},\\ A_1e^{-n_1h}+B_1e^{n_1h}-A_2e^{-n_2h}=0,\\ A_1e^{n_1h}-B_1e^{n_1h}-\dfrac{\mu_2n_2}{\mu_1n_1}A_2e^{-n_2h}=0.\end{cases} \tag{9}$$

Hence we get:

$$A_1=\frac{\overline{f}}{\mu_1n_1}\frac{1}{1-\dfrac{\mu_1n_1-\mu_2n_2}{\mu_1n_1+\mu_2n_2}e^{-2n_1h}},$$

$$B_1=A_1\frac{\mu_1n_1-\mu_2n_2}{\mu_1n_1+\mu_2n_2}e^{-2n_1h}, \tag{10}$$

$$A_2=\frac{\overline{f}}{\mu_1n_1}\frac{1}{1-\dfrac{\mu_1n_1-\mu_2n_2}{\mu_1n_1+\mu_2n_2}e^{-2n_1h}}\left(e^{-n_1h}\frac{\mu_1n_1-\mu_2n_2}{\mu_1n_1+\mu_2n_2}e^{-n_1h}\right)e^{-n_2h}.$$

Allowing for (10), the expressions of double transformations for permutations function will be of the form:

$$w^{1LF}=\frac{\overline{f}}{\mu_1n_1}\frac{1}{1-\psi e^{2n_1h}}\left(e^{-n_1h}+\psi e^{-2n_1h+n_1y}\right), \tag{11}$$

$$w^{2LF}=\frac{\overline{f}}{\mu_1n_1}\frac{1}{1-\psi e^{2n_1h}}\left(e^{-n_1h}+\psi e^{-n_1h}\right)e^{-n_2(y-h)}, \tag{12}$$

where $\psi=\dfrac{\mu_1n_1-\mu_2n_2}{\mu_1n_1+\mu_2n_2}$.

We represent the function w^{2LF} in the form:

$$w^{2LF}=w^{1LF}\Big|_{y=h}e^{-n_2(y-h)}. \tag{13}$$

It is seen from (13) that the function $w^2(x,y,t)$ is a solution for elastic half-space with boundary conditions equal to the inverse transformation of the function $w^{1LF}\,|_{y=h}$, coinciding with the result of the paper [7].

Taking into account that at the right half-plane $\mathrm{Re}\, p > 0$ the condition $\left|e^{-n_1 h}\right| < 1$ is fulfilled, then absolutely convergent series

$$w^{1LF} = \frac{\overline{f}}{\mu_1 n_1} \sum_{k=0}^{\infty} \left(\psi^k e^{-y_1 n_1} + \psi^{k+1} e^{-y_2 n_1} \right), \tag{14}$$

where $y_1 = y + 2kh,\ \ y_2 = 2(k+1)h - y$, exists and will be uniform.

The Cagniard-de Hoop method will be used for calculating the inverse transformations of the function (14).

For the function ψ independent of the parameters p and q, the inverse transformations of the function (14) may be found by direct use of the results of the paper [7]. In the case of harmonic waves ψ is constant.

For calculating the inverse joint transformations of the function (14) we consider the following expression

$$\xi^{LF} = \frac{\overline{f}(p)}{\mu n} e^{-ny}. \tag{15}$$

Applying to (15) the Fourier inverse transformation with respect to q, we get:

$$\overline{\xi}(x, y, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\overline{f}}{\mu\sqrt{q^2+\eta^2}} e^{-iqx - y\sqrt{q^2+\eta^2}} dq. \tag{16}$$

We can write the relation (16) as

$$\overline{\xi}(x, y, p) = \frac{\overline{f}}{2\pi\mu} \int_{-\infty}^{\infty} \frac{1}{\sqrt{q^2+p^2/c^2}} e^{-iqx - y\sqrt{q^2+p^2/c^2}} dq, \tag{17}$$

where $c = \sqrt{\mu/\rho}$ is the velocity of shear wave propagation.

We calculate the Laplace inverse transformation of the function (17) with using the Cagniard-de-Hoop method [8, 9]. The essence of the method is that integral (17) is transformed into the integral of the form of Laplace transformation with respect to t. To this end, at first we make change of integration variable of the form $q = -isp$, and reduce the expression (17) to the form

$$\overline{\xi} = \frac{i\overline{f}c}{2\pi\mu} \int_L \frac{1}{\sqrt{1-c^2s^2}} e^{-p[sx + y\sqrt{c^{-2}-s^2}} ds, \tag{18}$$

Where the straight line L passes through the origin of coordinates in a complex plane s with inclination to positive semi-axis (fig.1). The branches of the radical $\sqrt{c^{-2}-s^2}$ are determined so $\sqrt{c^{-2}-s^2} = c^{-1}$ that for $s = 0$. Moreover, the cuts for isolating this single valued branch pass along real semi-axes $(-\infty, -c^{-1})$ and $(c^{-1}, +\infty)$.

For the integral (18) to look like the Laplace integral it is necessary to deform the contour L to such a path L_1, along which the expression $sx + y\sqrt{c^{-2}-s^2}$ be real. Assume

$$sx + y\sqrt{c^{-2}-s^2} = t \tag{19}$$

where real quantity t should be considered as a parameter changing along the integration path L_1 in a complex plane s.

Fig.1.

Solving equation (19) with respect to s we find

$$s = \frac{xt \pm y\sqrt{r^2c^{-2} - t^2}}{r_2}, \qquad |t| < rc^{-1}, \;\; r = \sqrt{x^2 + y^2}, \quad x \geq 0, \tag{20}$$

$$s = \frac{xt \pm iy\sqrt{t^2 - r^2c^{-2}}}{r_2}, \qquad |t| > rc^{-1}, \;\; r = \sqrt{x^2 + y^2}, \quad x \geq 0, \tag{21}$$

where the radicals are assumed to be ariphmetical.

For $s = 0$ the expression (19) yields $t = y/c$. In this connection in (20) it is necessary to take the sign "minus" and leave the both signs in (21) in order to get a contour described by the expression (19) provided $t > 0$ and $x \geq 0$. In fig.1 in the area $\mathrm{Re}\, s > 0$ by bold-face curve we depict the contour L_1 for $x \geq 0$ that consists of the segment OM and the curve $N'MN$ described by the expressions:

$$s = \frac{xt - y\sqrt{r^2c^{-2} - t^2}}{r_2}, \qquad \left(\frac{y}{c} < t < \frac{r}{c}, \quad x \geq 0\right), \tag{22}$$

$$s = \frac{xt \pm iy\sqrt{t^2 - r^2c^{-2}}}{r_2}, \qquad \left(t > \frac{r}{c}, \quad x \geq 0\right). \tag{23}$$

The signs "plus" and "minus" in (23) belong to the curves on upper and lower half-planes s, respectively.

As $t \to +\infty$ having accepting in relations (22), (23) $x = r\cos\theta$, $y = r\sin\theta$ $(0 < \theta < \pi/2)$ we have:

$$s(t, r, \theta) = \frac{1}{r}\begin{cases} t\cos\theta - \sqrt{r^2c^{-2} - t^2}\sin\theta & (t < r/c), \\ t\cos\theta \pm i\sqrt{t^2 - r^2c^{-2}}\sin\theta & (t > r/c). \end{cases}$$

We'll assume that r and θ are fixed. In this case $s \to e^{\pm i\theta}t/r$, and the curve L_1, as $t \to +\infty$ will be bounded by the asymptotes outgoing from the origin $s =$

0 at the inclination angle $\pm\theta$ to a real axis (fig.1). As $s\to\infty$ the expression $p\left(sx+y\sqrt{c^{-2}-s^2}\right)$ behaves as follows:

$$p\left(sx+y\sqrt{c^{-2}-s^2}\right)=\begin{cases} \mid p\mid\mid s\mid re^{i(\omega+\varphi+\theta)} & (\operatorname{Re}s>0),\\ \mid p\mid\mid s\mid re^{i(\omega+\varphi-\theta)} & (\operatorname{Re}s<0),\end{cases}$$

where $p=\mid p\mid e^{i\omega}$ $(\mid\omega\mid<\pi/2)$, $s=\mid s\mid e^{i\varphi}$. Since

$$\min\left(\theta,-\frac{\pi}{2}-\omega\right)\le\varphi\le\max\left(\theta,-\frac{\pi}{2}-\omega\right)\quad(\operatorname{Re}s<0),$$

$$\min\left(-\theta,\frac{\pi}{2}-\omega\right)\le\varphi\le\max\left(-\theta,\frac{\pi}{2}-\omega\right)\quad(\operatorname{Re}s>0),$$

we get

$$\begin{aligned} &-\frac{\pi}{2}<\min\left(-\frac{\pi}{2}-\theta,\omega\right)\le(\varphi-\theta+\omega)\le\\ &\le\max\left(-\frac{\pi}{2}-\theta,\omega\right)<\frac{\pi}{2}\qquad(\operatorname{Re}s<0),\\ &-\frac{\pi}{2}<\min\left(\frac{\pi}{2}+\theta,\omega\right)\le(\varphi+\theta+\omega)\le\\ &\le\max\left(\frac{\pi}{2}+\theta,\omega\right)<\frac{\pi}{2}\qquad(\operatorname{Re}s>0).\end{aligned}\tag{24}$$

Consequently, the index of a exponent equal to $-p\left(sx+y\sqrt{c^{-2}-s^2}\right)$ has a negative real part as $\mid s\mid\to\infty$ in the area between the curves L and L_1 (fig.1). Therefore, using the Jordan lemma [10] we can deform integration contour L in L_1 passing in such order: $N'MOMN$. Moreover, a segment of a real axis OM will pass twice in opposite directions along the lower and upper banks. Then we deform the contour L in L_1, make change of variable s for t by formula (19), and get $(x\ge0)$:

$$\bar{\xi}=\frac{-i\overline{fc}}{2\pi\mu}\int\limits_{L_1}Fds=\frac{-i\overline{fc}}{2\pi\mu}\left[\int\limits_{N'MO}Fds+\int\limits_{OMN}Fds\right]=$$

$$=\frac{-i\overline{fc}}{2\pi\mu}\left[\int\limits_{+\infty}^{y/c}F\frac{ds}{d\tau}d\tau+\int\limits_{y/c}^{+\infty}F\frac{ds}{d\tau}d\tau\right]=\frac{\overline{f}}{\pi\mu}\int\limits_{y/c}^{+\infty}\operatorname{Im}\left[\frac{y}{(sx-t)}\frac{ds}{dt}\right]e^{-pt}dt.\tag{25}$$

Here in the integrand expression s is expressed by the relations (22), (23). And in the relation (23) the sign "plus" is taken. When obtaining relation (25) it was taken into account that the curves $N'MO$ and OMN are symmetric with respect to a real axis, and the integrand function is complexly-conjugated in complexly conjugated points s and $\bar{s}$, respectively on $OMN$and $N'MO$. Moreover, at these points the function $\left(sx+y\sqrt{c^{-2}-s^2}\right)$ takes real values equal to t.

We can write relation (25) for $x\ge0$, $y\ge0$ in the form:

$$\bar{\xi}(x,y,p)=\frac{\overline{f}}{\pi\mu}\int\limits_{0}^{\infty}\operatorname{Im}\left[\frac{y}{(sx-t)}\frac{ds}{dt}\right]H(t-r/c)e^{-pt}dt,\tag{26}$$

where $H(t-r/c)$ is a Heavyside unit function. Since the integral in the relation (26) is a Laplace integral, we have

$$\bar{\xi}(x,y,p)=\overline{f}\left\{\frac{1}{\pi\mu}\operatorname{Im}\left[\frac{y}{(sx-t)}\frac{ds}{dt}\right]H(t-r/c)\right\}^L.$$

Then the original $\xi(x,y,t)$ is found in the form

$$\xi(x,y,t)=\frac{1}{\pi\mu}f(\tau)*\operatorname{Im}\left[\frac{y}{(sx-t)}\frac{ds}{dt}\right]H(t-r/c), \tag{27}$$

where the asterisks between the functions mean their convolution

$$f(t)*g(t)=\int_0^t f(t-\tau)g(\tau)d\tau.$$

Taking the expression of the function $s(t)$ into account in (27) we finally get:

$$\xi(x,y,t)=\frac{1}{\pi\mu}\int_{r/c}^{t}\frac{f(t-\tau)d\tau}{\sqrt{\tau^2-r^2/c^2}}\ . \tag{28}$$

Using (27) and denoting the original of the function $\overline{\psi}^k$ by $\psi_k(t)$ we determine the original of the function (14) in the form:

$$w^1(x,y,t)=\frac{1}{\mu_1\pi}\sum_{k=0}^{n}\left(\psi_k(t)*\int_{r_1/c_1}^{t}\frac{f(t-\tau)d\tau}{\sqrt{\tau^2-r_1^2/c_1^2}}+\right.$$

$$\left.+\psi_{k+1}(t)*\int_{r_2/c_1}^{t}\frac{f(t-\tau)d\tau}{\sqrt{\tau^2-r_2^2/c_1^2}}\right),\qquad c_1=\sqrt{\frac{\mu_1}{\rho_1}}. \tag{29}$$

Thus, for final determination of $w^1(x,y,t)$ it is necessary to know the expression of the original of the function:

$$\overline{\psi}^k\equiv\left(\frac{\mu_1n_1-\mu_2n_2}{\mu_1n_1+\mu_2n_2}\right)^k=\left(\frac{\mu_1\sqrt{q^2+p^2/c_1^2}-\mu_2\sqrt{q^2+p^2/c_2^2}}{\mu_1\sqrt{q^2+p^2/c_1^2}+\mu_2\sqrt{q^2+p^2/c_2^2}}\right)^k,$$

$$c_2=\sqrt{\frac{\mu_2}{\rho_2}},\quad (k=0,1,2,...).$$

The Fourier inverse transformation $\overline{\psi}$ is written in the form

$$\overline{\psi}=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-iqx}\frac{\mu_1\sqrt{q^2+p^2/c_1^2}-\mu_2\sqrt{q^2+p^2/c_2^2}}{\mu_1\sqrt{q^2+p^2/c_1^2}+\mu_2\sqrt{q^2+p^2/c_2^2}}dq.$$

Considering $q=-isp$ where p is a real number, and assuming $sx=t$ we get

$$\overline{\psi}(x,y,p)=\frac{-ip}{2\pi}\int_0^{\infty}\frac{c_1^{-1}\mu_1\sqrt{1-s^2c_1^2}-c_2^{-1}\mu_2\sqrt{1-s^2c_2^2}}{c_1^{-1}\mu_1\sqrt{1-s^2c_1^2}+c_2^{-1}\mu_2\sqrt{1-s^2c_2^2}}e^{-psx}ds=$$

$$=-\frac{ip}{\pi x}\int_0^{\infty}\frac{\mu_1\sqrt{\dfrac{x^2}{c_1^2}-t^2}-\mu_2\sqrt{\dfrac{x^2}{c_2^2}-t^2}}{\mu_1\sqrt{\dfrac{x^2}{c_1^2}-t^2}+\mu_2\sqrt{\dfrac{x^2}{c_2^2}-t^2}}e^{-pt}dt.$$

Thus, $\psi(x,y,t)$ will be expressed by the relation

$$\psi(x,y,t)=\frac{1}{\pi x}\operatorname{Im}\frac{\partial}{\partial t}\left[\frac{\mu_1\sqrt{t^2-x^2c_1^{-2}}-\mu_2\sqrt{t^2-x^2c_2^{-2}}}{\mu_1\sqrt{t^2-x^2c_1^{-2}}+\mu_2\sqrt{t^2-x^2c_2^{-2}}}\right]$$

$$\frac{1}{\pi x}\frac{\partial}{\partial t}\left[\frac{2\mu_1\mu_2\sqrt{t^2-x^2c_1^{-2}}}{t^2\left(\mu_1^2-\mu_2^2\right)}-\frac{\sqrt{x^2c_2^{-2}-t^2}}{x^2(\mu_1\rho_1-\mu_2\rho_2)}\right]. \tag{30}$$

Notice that the functions ψ_k, $k=2,3,...$ are obtained from $\psi(x,y,t)$ with using the convolution of functions given by the recurrent relations

$$\psi_k(x,y,t)\int_0^t\psi_{k-1}(x,y,t-\tau)\psi_1(x,y,t)d\tau,\quad k=2,3,... \tag{31}$$

where $\psi_1=\psi$.

Now, on the basis of relation (13) we determine the function $w^2(x,y,t)$. To conduct this operation, by (29) we write the function $w^1(x,y,t)$ in the form:

$$w^1(x,h,t)=\frac{1}{\mu_1\pi}\sum_{k=0}^{\infty}\left[\psi_k(t)+\psi_{k+1}(t)\right]*\int_{r_1/c_1}^{t}\frac{f(t-\tau)d\tau}{\sqrt{\tau^2-r^2c_1^{-2}}}. \tag{32}$$

Now, calculate the inverse transformation of the function $e^{-n_2\alpha}$, where $\alpha=y-h$. We denote the original of this function by $\beta(x,y,t)$. Obviously,

$$\beta^{LF}=\mu\frac{\partial}{\partial y}\xi^{LF}\quad for\quad \overline{f}(p)=1.$$

By (28) we have:

$$-\mu\xi^{LF}\Big|_{\overline{f}(p)=1}=-\frac{1}{\pi}\frac{1}{\sqrt{t^2-r^2c^{-2}}},$$

whence

$$\beta(x,y,t)=\frac{1}{\pi}\frac{\partial}{\partial y}\frac{1}{\sqrt{t^2-r^2c^{-2}}}=-\frac{1}{\pi c^2}\frac{y}{\sqrt{\left(t^2-r^2c^{-2}\right)^3}}. \tag{33}$$

Using (33) we define the form of the function w^2 in the form

$$w^2(x,y,t)=\int_{-\infty}^{\infty}\int_0^t w^1(\xi,h,\tau)\beta(x-\xi,y-h,t-\tau)d\tau d\xi.$$

When obtaining this relation, it was assumed that all the met singular integrals converge uniformly with respect to parameters and we can differentiate them in parameter under the sign of integral.

In special case, when $k=0$ and $\overline{f}=1$ we have:

$$w^{2LF}=2\frac{1}{\mu_1n_1+\mu_2n_2}e^{-n_1h}e^{-n_2(y-h)}.$$

Hence for $\overline{w}^2$ we get

$$\overline{w}^2\frac{1}{\pi}\int\limits_{-\infty}^{\infty}\frac{1}{\mu_1\sqrt{q^2+p^2/c_1^2}+\mu_2\sqrt{q^2+p^2/c_2^2}}e^{-h\sqrt{q^2+p^2/c_1^2}-(y-h)\sqrt{q^2+p^2/c_2^2}}dq. \quad (34)$$

Following the Cagniard-de-Hoop method, allowing for $q=-isp$, and also introducing the denotation

$$h\sqrt{c_1^{-2}-s^2}+(y-h)\sqrt{c_2^{-2}-s^2}=t. \quad (35)$$

we get

$$\overline{w}^2=-\frac{\overline{f}}{\pi}\int\limits_{L}\frac{1}{\mu_1\sqrt{c_1^{-2}-s^2}+\mu_2\sqrt{c_2^{-2}-s^2}}\frac{\partial s}{\partial t}e^{-st}dt,$$

Whence the desired solution

$$w^2(x,y,t)=-\frac{1}{\pi}\,\mathrm{Re}\,\frac{1}{\mu_1\sqrt{c_1^{-2}-s^2}+\mu_2\sqrt{c_2^{-2}-s^2}}\frac{\partial s}{\partial t}.$$

is defined.

Moreover, this function expresses leading front of permutations area in a half-space. Here $s(t,y)$ $\dfrac{\partial s}{\partial t}$ are determined from (35), moreover

$$\frac{\partial s}{\partial t}=-\frac{\sqrt{c_1^{-2}-s^2}\sqrt{c_2^{-2}-s^2}}{s\left[h\sqrt{c_2^{-2}-s^2}+(y-h)\sqrt{c_1^{-2}-s^2}\right]}.$$

Conclusion

1) Analytic solution of a two-dimensional problem on propagation and reflection of transient cylindrical waves in elastic layer with plane parallel surfaces lying on an elastic half-space, is found;

2) The fields of interferential cylindrical waves in a layer and transient waves in a half-space are determined;

3) The method of application of joint Laplace-Fourier integral transformations by the Caniard-de-Hoop method to the problems of mathematical physics, is developed.

References

[1]. Louzar M.,Labrouni, Chevalier. *Propagation of SH waves in laminated elastic composite materials, effect of frequency and number of layers.* Mech., 10, 2006. pp.65-171.

[2]. Kuznetsov S.V. *SH waves in laminated plates.* Quart. Appl. Math., 64, 2006, pp.153-165.

[3]. Achenbach J.D. *Wave propagation in elastic solids.* North-Holland, Amsterdam, 1973.

[4]. Poruchikov V.B. *Metholds of dynamical elasticity theory.* M.: Nauka, 1986. (Russian)

[5]. Petrashen G.I., Marchuk G.I., Ogurtsov K.I. *On a Lamb problem in the case of half-space.* Ucheniye zapiski LGU, issue 21, No 135, ser. mat. nauk, 1956, pp.71-118. (Russian)

[6]. Shemiakin E.I. *The Lamb problem for elastic after effect medium.* DAN SSSR, 104, No2, 1955, pp.193-196. (Russian)

[7]. Mamedgasanov E.G. *Propagation of transient SH waves in elastic half-space.* Doklady NAN Azerbaijan, No2, 2007.

[8]. Cagniard L. *Reflection and refraction of progressive wave.* Translated by E.A.Flinn and C.H.Dix, New York, McGraw-Hill Book Company, 1962.

[9]. De Hoop A.T. *A modification of Cagniard's method for solving seismic pulse problems.* Appl. Sci. Res. Sect., B., v.8, No4, pp.349-356.

[10]. Lavrent'ev M.A., Shabat B.V. *Methods of the theory of complex variable functions* M.: Nauka, 1973.

Elkhan G.Mamedgasanov
Institute of Mathematics and Mechanics of NAS of Azerbaijan.
9, F.Agayev str., AZ1141, Baku, Azerbaijan.
Tel.: (99412) 439 47 20 (off.)

Received July 17, 2007; Revised October 24, 2007.