**Abdurrahim F. GULIYEV, Sakina H. HASANOVA**

# THE MIXED BOUNDARY VALUE PROBLEM FOR CORDES TYPE LINEAR NON-DIVERGENT PARABOLIC EQUATIONS OF THE SECOND ORDER

**Abstract**

*The mixed boundary value problem is considered for linear non-divergent parabolic equations of the second order with, generally speaking, discontinuous coefficients satisfying Cordes condition. The one-valued, strongly (almost everywhere) solvability of this problem is proved in the corresponding space Sobolev.*

Let $E_n$ and $R_{n+1}-$ be $n-$ dimensional and $(n+1)$ - dimensional Euclidean space of the points $x=(x_1,x_2,...,x_n)$ and $(t,x)=(t,x_1,x_2,...,x_n)$ respectively, $\mathcal{D}\subset E_n$ be bounded domain with the boundary $\partial\mathcal{D}\in C^2$, $B_R^{x^0}\subset\mathcal{D}$ be $n$ - dimensional open sphere of the radius $R$ with the center at the point $x^0=\left(x_1^0,x_2^0,...,x_n^0\right)$,

$$Q_T=\left\{(t,x)\right|\ 0<t<T<\infty,\ x\in\mathcal{D}\},$$

$$S_T=\left\{(t,x)\right|\ \ 0<t<T,\ x\in\partial\mathcal{D}\},$$

$$Q_R^T=B_R^{x^0}\times(0,T)\,,\mathcal{A}\left(Q_R^T\right)$$

be the set of all functions $u\,(t,x)$ from $C^\infty\left(\overline{Q}_R^T\right)$ with support in $B_\rho^{x^0}\times[0,T]\,,\ \ \rho<R$ for which $u|_{t=0}=0$. Consider in the domain $Q_T$ the mixed boundary value problem for linear parabolic equation of the form

$$\mathcal{L}u=\sum_{i,j=1}^n a_{ij}\,(t,x)\,\frac{\partial^2 u}{\partial x_i\partial x_j}-\frac{\partial u}{\partial t}=f\,(t,x)\,, \tag{1}$$

$$u|_{t=0}=0,\qquad \left.\frac{\partial u}{\partial n}\right|_{S_T}=0 \tag{2}$$

under the assumption that $\|a_{ij}\,(t,x)\|$ is a real symmetrical matrix, moreover for all $(t,x)\in Q_T$ and $\xi\in E_n$ the condition

$$\gamma\,|\xi|^2\leq\sum_{i,j=1}^n a_{ij}\,(t,x)\,\xi_i\xi_j\leq\gamma^{-1}\,|\xi|^2\,,\ \gamma\in(0,1]-const, \tag{3}$$

and also

$$\sigma=\sup_{(t,x)\in Q_T}\frac{\sum\limits_{i,j=1}^n a_{ij}^2\,(t,x)}{\left[\sum\limits_{i=1}^n a_{ii}\,(t,x)\right]^2}<\frac{1}{n-1}. \tag{4}$$

At this we'll suppose that the condition (4) is fulfilled to within non-singular linear transformation, i.e. we can cover the domain $Q_T$ with finite number of the subdomains $Q^1, Q^2, ..., Q^l$ so in every $Q^i$ there exists non-singular linear transformation at which the image of the operator satisfies condition (4) in the image of subdomains $Q^i$ ; $i = 1, 2, ..., l$.

The aim of the present paper is proof identically strong (almost every where) solvability of the mixed boundary value problem $(1)-(2)$ for every $f(t,x) \in L_2(Q_T)$ in the space $\widehat{W}_2^{2,1}(Q_T)$, at condition (3) and Cordes condition on main part of the equation (1).

In the case when the loading coefficients of the operator $\mathcal{L}$ are uniformly continuous in cylindrical domain $Q_T$ and minor coefficients are elements of corresponding Lebesgue spaces then uniform strong (almost everywhere) solvability of the problem $(1)-(2)$ in the space $\widehat{W}_p^{2,1}(Q_T)$, $p \in (1,\infty)$ is proved in $[1]-[2]$. The example indicating the exactness of Cordes condition is in $[3]$. We shall note that problem Dirichle for linear and quasilinear parabolic and elliptic equations of the second order of non-divergent structure with discontinuous coefficients has been studied in $[6]-[10]$. Indicate also papers $[11]-[12]$, in which the strong solvability of Neumann problem for elliptic equations of the second order of non-divergent structure with discontinuous coefficients is studied the indicated fact is transported on the class of non-linear parabolic equations of the second order.

**1. Some auxiliary assertions.** Let agree at first in some notation and definition. Well denote by $u_i$, $u_{ij}$ and $u_t$ the derivatives $\frac{\partial u}{\partial x_i}$, $\frac{\partial^2 u}{\partial x_i \partial x_j}$ and $\frac{\partial u}{\partial t}$ respectively; $i,j = 1,2,...,n$. Let $W_2^{1,0}(Q_T)$ and $W_2^{2,1}(Q_T)$ be Banach spaces of the measurable functions $u(t,x)$ given on $Q_T$ with finite norms

$$\|u\|_{W_2^{1,0}(Q_T)} = \left( \int\limits_{Q_T} \left( u^2 + \sum_{i=1}^n u_i^2 \right) dtdx \right)^{1/2}$$

and

$$\|u\|_{W_2^{2,1}(Q_T)} = \left( \int\limits_{Q_T} \left( u^2 + \sum_{i=1}^n u_i^2 + \sum_{i,j=1}^n u_{ij}^2 + u_t^2 \right) dtdx \right)^{1/2}$$

respectively. Denote by $\widehat{W}_2^{2,1}(Q_T)$ the subspace $W_2^{2,1}(Q_T)$, in which dense set is collection of all functions from $C^\infty\left(\overline{Q}_R\right)$, $u|_{t=0} = 0$, $\left.\frac{\partial u}{\partial n}\right|_{S_T} = 0$.

The function $u(t,x) \in \widehat{W}_2^{2,1}(Q_T)$ is called strong solvability of the mixed boundary value problem $(1)-(2)$, if it satisfies equation (1) almost everywhere in $Q_T$.

Further everywhere the note $C(...)$ means that the positive constant $C$ depends only on the contents of parenthesis.

**Lemma1.** If $u(t,x) \in \mathcal{A}\left(Q_R^T\right)$, then

$$\int\limits_{Q_R^T} \left( \sum_{i,j=1}^n u_{ij}^2 + u_t^2 \right) dtdx \le \int\limits_{Q_R^T} (Hu)^2 \, dtdx, \tag{5}$$

where
$$H = \Delta - \frac{\partial}{\partial t}.$$

**Proof.** We have
$$\int\limits_{Q_R^T} (Hu)^2\, dtdx = \int\limits_{Q_R^T} (\Delta u)^2\, dtdx - \int\limits_{Q_R^T} \Delta u u_t dtdx + \int\limits_{Q_R^T} u_t^2 dtdx =$$
$$= \int\limits_{Q_R^T} \sum_{i,j=1}^{n} u_{ii} u_{jj} dtdx - 2 \int\limits_{Q_R^T} \sum_{i=1}^{n} u_{ii} u_t dtdx + \int\limits_{Q_R^T} u_t^2 dtdx =$$
$$= - \int\limits_{Q_R^T} \sum_{i,j=1}^{n} u_{iij} u_j dtdx + 2 \int\limits_{Q_R^T} \sum_{i=1}^{n} u_i u_{ii} dtdx + \int\limits_{Q_R^T} u_t^2 dtdx =$$
$$= \int\limits_{Q_R^T} \sum_{i,j=1}^{n} u_{ij}^2 dtdx + \int\limits_{Q_R^T} \sum_{i=1}^{n} (u_i)_t^2\, dtdx + \int\limits_{Q_R^T} u_t^2 dtdx =$$
$$= \int\limits_{Q_R^T} \left( \sum_{i,j=1}^{n} u_{ij}^2 + u_t^2 \right) dtdx + \int\limits_{B_R^{x^0}} \left( u_i^2\,(T,x) - u_i^2\,(0,x) \right) dtdx.$$
Since $u\,(0,x) = 0$ then hence it follows the required inequality. Lemma is proved.

Let's
$$\delta = \sup_{(t,x)\in Q_T} \left( \sum_{i,j=1}^{n} \left( a_{ij}\,(t,x) - \delta_{ij} \right)^2 \right)^{1/2},$$
where $\delta_{ij}$ is symbol of Kroneker.

**Lemma 2.** *If $\delta < 1$, then for any function $u\,(t,x) \in \mathcal{A}\left(Q_R^T\right)$, it holds the estimation*
$$\int\limits_{Q_R^T} \left( \sum_{i,j=1}^{n} u_{ij}^2 + u_t^2 \right) dtdx \le C_1\,(\delta) \int\limits_{Q_R^T} (\mathcal{L}u)^2\, dtdx \qquad (6)$$

**Proof.** According to lemma 1, we have
$$\left( \int\limits_{Q_R^T} \left( \sum_{i,j=1}^{n} u_{ij}^2 + u_t^2 \right) dtdx \right)^{1/2} \le \|Hu\|_{L_2(Q_R^T)} \le \|\mathcal{L}u\|_{L_2(Q_R^T)} +$$
$$+ \left\| \sum_{i,j=1}^{n} \left( a_{ij}\,(t,x) - \delta_{ij} \right) u_{ij} \right\|_{L_2(Q_R^T)} \le \|\mathcal{L}u\|_{L_2(Q_R^T)} +$$
$$+\delta \left( \int\limits_{Q_R^T} \sum_{i,j=1}^{n} u_{ij}^2 dtdx \right)^{1/2} \le \|\mathcal{L}u\|_{L_2(Q_R^T)} + \delta \left( \int\limits_{Q_R^T} \left( \sum_{i,j=1}^{n} u_{ij}^2 + u_t^2 \right) dtdx \right)^{1/2}.$$

Now it is sufficient to use the inequality $\delta < 1$, and the required inequality (6) proved.

**Lemma 3.** *If* $\delta < 1$ *and* $R \leq 1$, *then for any function* $u(t,x) \in \mathcal{A}\left(Q_R^T\right)$ *the estimation*

$$\|u\|_{W_2^{2,1}\left(Q_R^T\right)} \leq C_2(\delta)\,\|\mathcal{L}u\|_{L_2\left(Q_R^T\right)} \tag{7}$$

*is true.*

**Proof.** Let $\mathcal{K}_R^T = \left\{\left|x_i - x_i^0\right| < R;\ i = 1, 2, ..., n\right\} \times (0,T)$. Continue the function $u(t,x)$ by $\mathcal{K}_R^T \setminus Q_R^T$ and denote by $u$ the obtained in this way function it is clear that $u \in C^\infty\left(\mathcal{K}_R^T\right)$, $u(0,x) = 0$. Let's fix $t' \in (0,T)$ and denote $x' = (x_2, ..., x_n)$. For $x_1 \in \left(x_1^0 - R_1, x_1^0 + R_1\right)$, we have

$$u\left(t', x_1, x'\right) = u\left(t', x_1^0 - R, x_1'\right) + \int\limits_{x_1^0 - R}^{x_1} \frac{\partial}{\partial x_1} u\left(t', \tau, x'\right) d\tau = \int\limits_{x_1^0 - R}^{x_1} \frac{\partial}{\partial x_1} u\left(t', \tau, x'\right) d\tau.$$

Using the Hölder inequality, we obtain

$$u^2\left(t', x_1, x'\right) = \left(\int\limits_{x_1^0 - R}^{x_1} \frac{\partial}{\partial x_1} u\left(t', \tau, x'\right) d\tau\right)^2 \leq$$

$$\leq \int\limits_{x_1^0 - R}^{x_1} d\tau \int\limits_{x_1^0 - R}^{x_1} \left(\frac{\partial}{\partial x_1} u\left(t', \tau, x'\right)\right)^2 d\tau \leq$$

$$\leq \int\limits_{x_1^0 - R}^{x_1^0 + R} d\tau \int\limits_{x_1^0 - R}^{x_1^0 + R} \left(\frac{\partial}{\partial x_1} u\left(t', \tau, x'\right)\right)^2 d\tau = 2R \int\limits_{x_1^0 - R}^{x_1^0 + R} \left(\frac{\partial}{\partial x_1} u\left(t', \tau, x'\right)\right)^2 d\tau.$$

Integrating the both sides of this inequality by $\mathcal{K}_R^T$,we have

$$\int\limits_{\mathcal{K}_R^T} u^2\left(t', x_1, x'\right) dt'dx \leq \int\limits_{\mathcal{K}_R^T} \left(2R \int\limits_{x_1^0 - R}^{x_1^0 + R} \left(\frac{\partial}{\partial x_1} u\left(t', \tau, x'\right)\right)^2 d\tau\right) dt'dx \leq$$

$$\leq 4R^2 \int\limits_{\mathcal{K}_R^T} \left(\frac{\partial}{\partial x_1} u\left(t', \tau, x'\right)\right)^2 dt'dx.$$

Consequently

$$\int\limits_{Q_R^T} u^2(t,x)\, dtdx \leq 4R^2 \int\limits_{Q_R^T} \left(\frac{\partial}{\partial x_1} u\left(t, x_1, x'\right)\right)^2 dtdx,$$

since $u(t,x) \equiv 0$ with out $Q_R^T$ .

Thus
$$\int\limits_{Q_R^T} u^2 dtdx \le 4R^2 \int\limits_{Q_R^T} \left(\frac{\partial u}{\partial x_1}\right)^2 dtdx \le 4R^2 \int\limits_{Q_R^T} \sum_{i=1}^n u_i^2 dtdx. \tag{8}$$
Absolutely analogously (8) for $u_i$, we have
$$\sum_{i=1}^n \int\limits_{Q_R^T} u_i^2 dtdx \le 4R^2 \sum_{i,j=1}^n \int\limits_{Q_R^T} u_{ij}^2 dtdx. \tag{9}$$
Now using the condition $R \le 1$, from (8), (9) and lemma 2 it follows the required inequality (7).

**Lemma 4.** *Condition $\delta < 1$ to within non-singular linear transformation coincides with the condition (4).*

**Proof.** Let's make the transformation $\tau = k^2 t,\ y_i = kx_i\ ; i = 1, 2, ..., n$, where
$$k = \left(\frac{\sup\limits_{Q_T} \sum\limits_{i,j=1}^n a_{ij}^2(t,x)}{\inf\limits_{Q_T} \sum\limits_{i=1}^n a_{ii}(t,x)}\right)^{-\frac{1}{2}}.$$
Then if $\|\mathcal{A}_{ij}(\tau, y)\|$ is matrix of leading part of image of the operator $\mathcal{L}$ , then $\mathcal{A}_{ij}(\tau, y) = k^2 a_{ij}(t,x)\, ;\ i,j = 1,2,...,n$. Condition $\delta < 1$ in now variables will take the form
$$\sup_{\widetilde{Q}_T} \sum_{i,j=1}^n \mathcal{A}_{ij}^2(\tau, y) - 2\inf_{\widetilde{Q}_T} \sum_{i,j=1}^n \mathcal{A}_{ii}(\tau, y) + n < 1, \tag{10}$$
where $\widetilde{Q}_T$ is image of the domain $Q_T$ . It is clear, that (10) coincides with the condition (4). Lemma is proved.

**Lemma 5.** *Let relative to the coefficients of the operator $\mathcal{L}$ the conditions (3) and (4) be fulfilled. Then there exists the constants $C_6(\gamma, \sigma, n)$ such that for any function $u(t,x) \in C^\infty\left(\overline{Q}_R^T\right)$, $u|_{t=0} = 0$ at every $R_1 \in (0, R)$ the estimation*
$$\|u\|_{W_2^{2,1}(Q_R^T)} \le C_2 \|\mathcal{L}u\|_{L_2(Q_R^T)} + \frac{C_3}{(R-R_1)^2} \|u\|_{L_2(Q_R^T)} + \frac{C_3}{R-R_1} \|u\|_{W_2^{1,0}(Q_R^T)}$$
*is true.*

**Proof.** Let the function $\eta(x) \in C_0^\infty\left(B_R^{x^0}\right)$ be such that $\eta(x) = 1$ in $B_{R_1}^{x^0}$, $\eta(x) = 0$ without $B_{\frac{R+R_1}{2}}^{x^0}$, $0 \le \eta(x) \le 1$, moreover
$$|\eta_i| \le \frac{C_4}{R-R_1}, \quad |\eta_{ij}| \le \frac{C_4}{(R-R_1)^2}, \quad i,j = 1,2,...,n, \tag{11}$$
where $C_4 = C_4(n)$. Applying to the function $u\eta$ lemma 3, we'll obtain
$$\|u\|_{W_2^{2,1}\left(Q_{R_1}^T\right)} \le C_2 \|\mathcal{L}(u\eta)\|_{L_2(Q_R^T)}\ . \tag{12}$$

But on the other hand

$$\|\mathcal{L}(u\eta)\|_{L_2(Q_R^T)} \le |\mathcal{L}u| + |u| \left| \sum_{i,j=1}^{n} a_{ij}(t,x)\,\eta_{ij} \right| + 2 \left| \sum_{i,j=1}^{n} a_{ij}(t,x)\,u_i \eta_j \right|, \qquad (13)$$

and further allowing for (11)

$$\left| \sum_{i,j=1}^{n} a_{ij}(t,x)\,\eta_{ij} \right| \le \frac{C_5(\gamma,n)}{(R-R_1)^2},$$

$$2 \left| \sum_{i,j=1}^{n} a_{ij}(t,x)\,u_i \eta_j \right| \le 2 \left( \sum_{i,j=1}^{n} a_{ij}(t,x)\,u_i u_j \right)^{1/2} \left( \sum_{i,j=1}^{n} a_{ij}(t,x)\,\eta_i \eta_j \right)^{1/2} \le$$

$$\le 2\gamma^{-1} \left( \sum_{i=1}^{n} u_i^2 \right)^{1/2} \left( \sum_{i=1}^{n} \eta_i^2 \right)^{1/2} \le 2\gamma^{-1} \sum_{i=1}^{n} |u_i| \sum_{i=1}^{n} |\eta_i| \le \frac{2n\gamma^{-1}C_4}{R-R_1} \sum_{i=1}^{n} |u_i|.$$

Thus from (13) we conclude

$$\|\mathcal{L}(u\eta)\|_{L_2(Q_R^T)} \le \|\mathcal{L}u\|_{L_2(Q_R^T)} + \frac{C_5}{(R-R_1)^2} \|u\|_{L_2(Q_R^T)} + \frac{C_6}{R-R_1} \|u\|_{W_2^{2,1}(Q_R^T)}. \qquad (14)$$

Subject to (14) in (12) and denoting by $C_7$ the $max\{C_2C_5,\ C_2C_6\}$ we arrive at the required estimation (11).

**Lemma 6.** *Let relative to the coefficients $\mathcal{L}$ the condition of the previous lemma be fulfilled. Then there exists the constant $C_8(\gamma,\sigma,n)$ such that for any function $u(t,x) \in C^\infty\left(\overline{Q}_R^T\right)$, $u|_{t=0} = 0$ at any $\varepsilon > 0$ the estimation*

$$\|u\|_{W_2^{2,1}\left(Q_{\frac{R}{2}}^T\right)} \le C_2 \|\mathcal{L}u\|_{L_2(Q_R^T)} + \varepsilon \|u\|_{W_2^{2,1}(Q_R^T)} + \frac{C_8}{\varepsilon R^2} \|u\|_{L_2(Q_R^T)}$$

*is true.*

**Proof.** We'll use the following interpolation inequality ([3]) : for any function $u(t,x) \in W_2^{2,1}\left(Q_R^T\right)$ at any $\varepsilon > 0$ the estimation

$$\|u\|_{W_2^{1,0}(Q_R^T)} \le \varepsilon \|u\|_{W_2^{2,1}(Q_R^T)} + \frac{C_9(n)}{\varepsilon} \|u\|_{W_2^{2,1}(Q_R^T)} + \frac{C_8}{\varepsilon R^2} \|u\|_{L_2(Q_R^T)} \qquad (15)$$

is true.

Let's fix an arbitrary $\varepsilon > 0$ and $\varepsilon_1 > 0$ be a number which will be choosen later. According to lemma 5 and the inequality (15)

$$\|u\|_{W_2^{2,1}\left(Q_{\frac{R}{2}}^T\right)} \le C_2 \|\mathcal{L}u\|_{L_2(Q_R^T)} + \frac{4C_3}{R^2} \|u\|_{W_2^{2,1}(Q_R^T)} +$$

$$+\frac{2C_3}{R} \|u\|_{W_2^{1,0}(Q_R^T)} \le C_2 \|\mathcal{L}u\|_{L_2(Q_R^T)} + \frac{4C_3}{R^2} \|u\|_{W_2^{2,1}(Q_R^T)} +$$

$$+\frac{2C_3\varepsilon_1}{R} \|u\|_{W_2^{2,1}(Q_R^T)} + \frac{2C_3C_9}{R\varepsilon_1} \|u\|_{L_2(Q_R^T)}.$$

Now it is enaugh to choose $\varepsilon_1 = \dfrac{\varepsilon R}{2C_3}$, lemma is proved.

**Remark.** If the minor coefficients of the operator $\mathcal{L}$ are bounded, then there exists such $R_0(\gamma, \sigma, n, \mathcal{B}, c)$ that at $R \le R_0$ the assertion of lemma 6 is also true for the operator

$$\mathcal{L}u = \sum_{i,j=1}^{n} a_{ij}(t,x)\, u_{ij} + \sum_{i=1}^{n} b_i(t,x)\, u_i + c(t,x)\, u - u_t = f(t,x)$$

Here $\mathcal{B} = (b_1(t,x), b_2(t,x), ..., b_n(t,x))$.

For $\rho > 0$ the set $\{x : x \in \mathcal{D},\ dist(x, \partial\mathcal{D}) > \rho\}$ denote by $\mathcal{D}_\rho$ .

**Lemma 7**. *Let relative to the coefficients of the operator $\mathcal{L}$ the conditions* (3)$-$(4) *be fulfilled. Then for any function $u(t,x) \in C^\infty\left(\overline{Q}_R^T\right)$, $u|_{t=0} = 0$ , at any $\varepsilon > 0$ and $\rho > 0$ the estimation*

$$\|u\|_{W_2^{2,1}(\mathcal{D}_\rho \times (0,T))} \le C_{10}(\gamma, \sigma, n, \rho, \mathcal{D}) \|\mathcal{L}u\|_{L_2(Q_T)} +$$

$$+\varepsilon \|u\|_{W_2^{2,1}(Q_T)} + \frac{C_{11}(\gamma, \sigma, n, \rho, \mathcal{D})}{\varepsilon} \|u\|_{L_2(Q_T)}$$

*is true.*

**Proof.** Let's fix an arbitrary $\varepsilon > 0$, $\rho > 0$ and $\varepsilon_2 > 0$ be a number which will be choosen later. Let cover $\overline{\mathcal{D}}_\rho$ by the system of spheres $\left\{B_{\frac{\rho}{2}}^{x^i}\right\}$ and choose from this cover the finite subcovering $B^1, B^2, ..., B^N$. It is evident, that the number $N$ depends only on $\rho, n$ and *diam* $\mathcal{D}$. Applying for every $i = 1, 2, ..., N$ lemma 6, we obtain

$$\|u\|^2_{W_2^{2,1}(B^i \times (0,T))} \le 3N \left( C_2^2 \|\mathcal{L}u\|^2_{L_2(Q_T)} + \varepsilon_2^2 \|u\|^2_{W_2^{2,1}(Q_T)} + \frac{C_8^2}{\varepsilon_2^2 \rho^4} \|u\|_{L_2(Q_T)} \right).$$

Summarizing this inequality by $i$ from 1 to $N$ we conclude

$$\|u\|_{W_2^{2,1}(\mathcal{D}_\rho \times (0,T))} \le 3N \left( C_2^2 \|\mathcal{L}u\|^2_{L_2(Q_T)} + \varepsilon_2^2 \|u\|^2_{W_2^{2,1}(Q_T)} + \frac{C_8^2}{\varepsilon_2^2 \rho^4} \|u\|_{L_2(Q_T)} \right).$$

Now it is sufficient to choose $\varepsilon_2 = \dfrac{\varepsilon}{3N}$ and the lemma is proved.

**3. Basic coercive estimation.** The assertion of lemma 7 is true without any demands relative to the domain $\partial\mathcal{D}$. All next assertions of the present paper hold under condition $\partial\mathcal{D} \in \mathcal{C}^2$ which we'll always suppose as fulfilled one.

**Lemma 8.** *Let relative to the coefficient of the operator $\mathcal{L}$ the conditions (3)-(4). Then there exist positive constants $\rho_1(\gamma, \sigma, n, \partial\mathcal{D})$ such that for any function $u(t,x) \in \widehat{W}_2^{2,1}(Q_T)$ at every $\varepsilon > 0$ the estimation*

$$\|u\|_{W_2^{2,1}\left(\mathcal{D}/\mathcal{D}_{\rho_1} \times (0,T)\right)} \le C_{12}(\gamma, \sigma, n, \mathcal{D}) \|\mathcal{L}u\|_{L_2(Q_T)} +$$

$$+\varepsilon \|u\|_{W_2^{2,1}(Q_T)} + \frac{C_{13}(\gamma, \sigma, n, \mathcal{D})}{\varepsilon} \|u\|_{L_2(Q_T)}$$

*is true.*

**Proof.** It is sufficient to prove the lemma for the functions $u(t,x) \in C^{\infty}\left(\overline{Q}_T\right)$, $u|_{t=0} = 0$, $\left.\dfrac{\partial u}{\partial n}\right|_{S_T} = 0$. Besides not losing generally, we'll suppose that the coefficients of the operator $\mathcal{L}$ are infinite differentiable $\overline{Q}_T$ . Let's fix an arbitrary $\varepsilon > 0$ and the point $x^0 \in \partial D$. Make orthogonal transformation of the coordinate $x \to y$ such that the tangent hyperplane to $\partial\widetilde{\mathcal{D}}$ at the point $y^0$ will be perpendicular to the axis $Oy_n$. Here $\widetilde{\mathcal{D}}$ and $y^0$ are images of the domain $\mathcal{D}$ and the point $x^0$ respectively at, such transformation. Denote by $\widetilde{u}(t,y)$ the image of the function $u(t,x)$. Well suppose for simplicity that the domain $\partial\widetilde{\mathcal{D}}$ at intersection $\partial\widetilde{\mathcal{D}}$ with some neighbourhood $O_h$ of the point $y^0$ is given by the equation $y_n = \varphi(y_1, y_2, ..., y_{n-1})$ with twice continuously differentiable function $\varphi$ and the part $\widetilde{\mathcal{D}}$ adjacent to $\partial\widetilde{\mathcal{D}} \cap O_h$ belongs to the set $\{y|\, y_n > \varphi(y_1, y_2, ..., y_{n-1})\}$. Let $\mathcal{A}(t,x) = \|a_{ij}(t,x)\|$ be a matrix of leading coefficients of the operator $\mathcal{L}$, $\widetilde{\mathcal{A}}(t,y) = \|\widetilde{a}_{ij}(t,y)\|$, where $\widetilde{a}_{ij}(t,y)$ are leading coefficients of the image $\widetilde{\mathcal{L}}$ of the operator $\widetilde{\mathcal{L}}$ at our transformation; $i,j = 1,2,...,n$. Show now that the eigen numbers of the matrices $\mathcal{A}$ and $\widetilde{\mathcal{A}}$ coincide. Really, fix an arbitrary point $(t,x) \in Q_T$ and let $\lambda$ be an arbitrary eigen number of the matrix $\mathcal{A}$ and $x^{\lambda}$ be corresponding to it eigen vector. By virtue of orthogonality of our transformation there exists a non-degenerated matrix $T$ such that $\widetilde{\mathcal{A}} = T^{-1}\mathcal{A}T$. Denote by $T^{-1}x^{\lambda}$ the $y^{\lambda}$. We have

$$\widetilde{\mathcal{A}}y^{\lambda} = T^{-1}\mathcal{A}x^{\lambda} = \lambda T^{-1}\mathcal{A}x^{\lambda} = \lambda y^{\lambda}.$$

On the other hand we can write condition (4) in the following form

$$\sigma = \sup_{Q_T} \frac{\sum\limits_{i=1}^{n} \lambda_i^2(t,x)}{\left[\sum\limits_{i=1}^{n} \lambda_i(t,x)\right]^2} < \frac{1}{n-1},$$

where $\lambda_i(t,x)$ are eigen numbers of the matrix $\mathcal{A}(t,x)$; $i = 1,2,...,n$. Thus the condition (4) is fulfilled also for the operator $\widetilde{\mathcal{L}}$, moreover with the same constant $\sigma$. Analogously it is shown that for the operator $\widetilde{\mathcal{L}}$ the conditions (3) are fulfilled (with the same constant $\gamma$). Let's make one more transformation $z_i = y_i$ ; $i = 1,2,...,n$, $z_n = y_n - \varphi(y_1, y_2, ..., y_{n-1})$. Let $\mathcal{L}', \mathcal{D}'$ and $z^0$ be images of the operator $\widetilde{\mathcal{L}}$, of the domain $\widetilde{\mathcal{D}}$ and the point $y^0$ respectively at our transformation, and $a'_{ij}(t,z)$ be leading coefficients of the operator $\mathcal{L}'$; $i,j = 1,2,...,n$. It is easy to see that

$$a'_{ij}(t,z) = \sum_{k,l=1}^{n} \widetilde{a}_{kl}(t,y)\frac{dz_i}{dy_k}\frac{dz_j}{dy_l};\ i,j = 1,2,...,n.$$

Therefore

$$a'_{ij}(t,z) = \widetilde{a}_{kl}(t,y);\ \text{if}\ \ 1 \le i,j \le n-1,$$

$$a'_{nj}(t,z) = -\sum_{k=1}^{n-1} \widetilde{a}_{kj}(t,y)\frac{d\varphi}{dy_k} + \widetilde{a}_{nj}(t,y);\ \text{if}\ \ 1 \le j \le n-1,$$

$$a'_{nn}(t,z) = \sum_{k,l=1}^{n} \widetilde{a}_{kl}(t,y)\frac{d\varphi}{dy_k}\frac{d\varphi}{dy_l} - 2\sum_{k=1}^{n-1} \widetilde{a}_{nk}(t,y)\frac{d\varphi}{dy_k} + \widetilde{a}_{nn}(t,y).$$

Since $\frac{d\varphi}{dy_i}\left(y^0\right)=0$ for $i=1,2,...,n-1$ then there exists $h_1\left(y^0,\varphi\right)$ such that at $h\le h_1$ at intersection $\mathcal{D}'\cap\left(B_h^{z^0}\times(0,T)\right)$ the condition (4) (with the constant $\sigma'=\frac{\sigma+\frac{1}{n-1}}{2}$) is fulfilled. Besides for the operator $\mathcal{L}'$ in indicated intersection the (3) are fulfilled (with constant $\frac{\gamma}{2}$). Assume that $r=r\left(z^0\right)=h_1\left(y^0,\varphi\right)$ and let $u'(t,z)$ be image of the function $\widetilde{u}(t,y)$ at our transformation. It is clear that in variables $z$ the intersection $\mathcal{D}'\cap B_r^{z^0}$ represent hemisphere $B_r^+=\left\{z:\left|z-z^0\right|<r\ ;\ z_n>0\right\}$. Continue the function $u'(t,z)$ and $\mathcal{L}'$ coefficients of the operator $\mathcal{L}'$ by the even form by the hyperlane $z_n=0$ in $B_r^{z^0}\backslash B_r^+$ and denote by $u'(t,z)$ and $\mathcal{L}'$ the obtained in this way function and the operator respectively. Since $u'(t,z)\in W_2^{2,1}\left(B_r^{z^0}\times(0,T)\right)$, then according to lemma 6

$$\left\|u'\right\|_{W_2^{2,1}\left(B_{\frac{r}{2}}^{z^0}\times(0,T)\right)}\le C_2\left\|\mathcal{L}'u'\right\|_{L_2\left(B_r^{z^0}\times(0,T)\right)}+$$

$$+\varepsilon_3\left\|u'\right\|_{W_2^{2,1}\left(B_r^{z^0}\times(0,T)\right)}+\frac{C_8}{\varepsilon_3 r^2}\left\|u'\right\|_{L_2\left(B_r^{z^0}\times(0,T)\right)},\qquad(16)$$

where $\varepsilon_3>0$ will be choosen later. But on the other hand each of norms at the right-hand side (16) represent the corresponding norm taken by semi-cylinder $Q_r^+=B_r^+\times(0,T)$ and multiplied by $\sqrt{2}$. Therefore from (16) we conclude

$$\left\|u'\right\|_{W_2^{2,1}\left(Q_{\frac{r}{2}}^+\times(0,T)\right)}\le C_2\left\|\mathcal{L}'u'\right\|_{L_2\left(Q_r^+\right)}+$$

$$+\varepsilon_3\left\|u'\right\|_{W_2^{2,1}\left(Q_r^+\right)}+\frac{C_8}{\varepsilon_3 r^2}\left\|u'\right\|_{L_2\left(Q_r^+\right)}.\qquad(17)$$

Cover $\partial\mathcal{D}'$ by the system of spheres $\left\{B_{\frac{r}{2}}^{z^i}\right\}$ and choose from this cover finite subcoverining $B^1,B^2,...,B^M$. At this the number $M$ is determined only by the quantities $\gamma,\sigma,n$ and the domain $\mathcal{D}$. Writing out the inequality of the form (17) for every semi-cylinder $B_r^+\left(z^i\right)\times(0,T)\,;\ i=1,2,..,M$ raising both sides of obtained inequalities to power 2 and summarizing by $i$ from 1 to $M$ we obtain

$$\left\|u'\right\|^2_{W_2^{2,1}(\mathcal{B}\times(0,T))}\le 3M\left(C_2\left\|\mathcal{L}'u'\right\|_{L_2(\mathcal{D}'\times(0,T))}+\right.$$

$$\left.+\varepsilon_2^3\left\|u'\right\|_{W_2^{2,1}(\mathcal{D}'\times(0,T))}+\frac{C_8^2}{\varepsilon_3^2 r_0^4}\left\|u'\right\|_{L_2'(\mathcal{D}'\times(0,T))}\right),$$

where $\mathcal{B}=\bigcup\limits_{i=1}^{M}B_{\frac{r}{2}}^+\left(z^i\right)$ and $r_0=\min\left\{r\left(z^1\right),r\left(z^2\right),...,r\left(z^M\right)\right\}$. Returning to the variables $x$ and noting that pre-image $\mathcal{B}$ contains the set $\mathcal{D}/\mathcal{D}_{\rho_1}$ with some $\rho_1(\gamma,\sigma,n,\mathcal{D})$ we conclude

$$\|u\|_{W_2^{2,1}\left(\mathcal{D}/\mathcal{D}_{\rho_1}\times(0,T)\right)}\le$$

$$\le C_{14}\|\mathcal{L}u\|_{L_2(Q_T)}+C_{15}\varepsilon_3\|u\|_{W_2^{2,1}(Q_T)}+\frac{C_{16}}{\varepsilon_3}\|u\|_{L_2(Q_T)},$$

where the constants $C_{14}, C_{15}$ and $C_{16}$ depend only on $\gamma, \sigma, n$ and the domain $\mathcal{D}$. Now it is sufficient to choose $\varepsilon_3 = \dfrac{\varepsilon}{C_{16}}$ and the lemma is proved.

It follows the following lemma form lemmas 7 and 8.

**Lemma 9.** *Let relative to the coefficients of the operator $\mathcal{L}$ the conditions (3)-(4) are fulfilled. Then for any function $u(t,x) \in \widehat{W}_2^{2,1}(Q_T)$ the estimation*

$$\|u\|_{W_2^{2,1}(Q_T)} \le C_{17}(\gamma, \sigma, n, \mathcal{D}) \left( \|\mathcal{L}u\|_{L_2(Q_T)} + \|u\|_{L_2(Q_T)} \right) \tag{18}$$

*is true.*

**Theorem 1.** *Let relative to the coefficients of the operator $\mathcal{L}$ the conditions (3)-(4) be fulfilled. Then there exist the constants $C_{18}(\mathcal{L}, n, \mathcal{D})$, $T_0(\mathcal{L}, n)$ such that if $T \le T_0$ then for any function $u(t,x) \in \widehat{W}_2^{2,1}(Q_T)$ the estimation*

$$\|u\|_{W_2^{2,1}(Q_T)} \le C_{18} \|\mathcal{L}u\|_{L_2(Q_T)} .$$

*is true.*

**Proof.** Let $t \in (0,T)$. We have

$$u(t,x) = \int_0^t u_t(t,x)\, d\tau.$$

Using the Hölder inequality we obtain

$$u(t,x) \le T^{1/2} \left( \int_0^t (u_t(t,x))^2 d\tau \right)^{1/2},$$

and consequently

$$u^2(t,x) \le T \int_0^T (u_t(\tau,x))^2 d\tau.$$

Integrating the both sides of this inequality by $Q_T$ and raising to power $\frac{1}{2}$ we have

$$\|u\|_{L_2(Q_T)} \le T \|u_t\|_{L_2(Q_T)} . \tag{19}$$

Subject to (19) in (18), we come to the estimation

$$\|u\|_{W_2^{2,1}(Q_T)} \le C_{17} \|\mathcal{L}u\|_{L_2(Q_T)} + C_{17} T \|u\|_{W_2^{2,1}(Q_T)} .$$

Then there exists the constant $T_0(\mathcal{L}, n)$ such that at $T \le T_0$

$$T < \frac{1}{2C_{17}}$$

The theorem is proved

**4. Solvability of the mixed boundary value problem.** Now consider the mixed boundary value problem (1)-(2).

**Theorem 2.** Let in domain $Q_T$ be given the coefficients of the operator $\mathcal{L}$ satisfying the conditions (3)-(4). Then if $T \le T_0$ and $\partial\mathcal{D} \in C^2$ then the mixed boundary value problem (1)-(2) is identically strongly solvable in the space $\widehat{W}_2^{2,1}(Q_T)$ at every $f(t,x) \in L_2(Q_T)$. At this for solution $u(t,x)$ the estimation

$$\|u\|_{W_2^{2,1}(Q_T)} \le C_{18} \|f\|_{L_2(Q_T)} \tag{20}$$

is true.

**Proof.** Let's prove the theorem by the method of continuation by parameter. Introduce for $\tau \in [0,1]$ the family of the operator

$$\mathcal{L}^\tau = \tau\mathcal{L} + (1-\tau) H$$

It is easy to see that the conditions (3) and (4) are fulfilled for the operator $\mathcal{L}^\tau$ with the constant $\gamma$ and $\sigma$ respectively. Show this on the the example of condition (4). According to lemma 4 the last to within non-singular linear transformation coincides with the condition $\delta < 1$. Let $a_{ij}^\tau(t,x)$ be leading coefficients of the operator $\mathcal{L}^\tau$; $i,j = 1,2,...,n$ and

$$\delta^\tau = \sup_{Q_T} \left( \sum_{i,j=1}^n \left(a_{ij}^\tau(t,x) - \delta_{ij}\right)^2 \right)^{1/2}.$$

We have

$$\delta^\tau = \sup_{Q_T} \left( \sum_{i,j=1}^n \left(\tau a_{ij}(t,x) + (1-\tau)\delta_{ij} - \delta_{ij}\right)^2 \right)^{1/2} =$$

$$= \tau \sup_{Q_T} \left( \sum_{i,j=1}^n \left(a_{ij}(t,x) - \delta_{ij}\right)^2 \right)^{1/2} = \tau\delta \le \delta.$$

Hence it follows that the assertion of theorem 1 is true for the operator $\mathcal{L}^\tau$ with the constant $C'_{18}$ not depending on $\tau$. Denote by $E$ the set of point of the section $[0,1]$ for which the problem

$$\mathcal{L}^\tau u = f(t,x); \quad (t,x) \in Q_T, \quad u \in \widehat{W}_2^{2,1}(Q_T), \tag{21}$$

has solution. Note that by virtue of theorem 1 this solution is unique. Now show that the set $E$ is nonempty and it is open and closed simultaneously relative to $[0,1]$.

Then $E$ coincides with the segment $[0,1]$ and in particular the problem (21) is identically solvable at $\tau = 1$ when $\mathcal{L}^1 = \mathcal{L}$ . At this the estimation (20) follows from theorem 1. Nonemptiness of the set $E$ follows from that the problem (21) is solvable at $\tau = 0$ (see [3]). Show that the set $E$ is open relative to $[0,1]$. Let $\tau^0 \in E$, $\tau \in [0,1]$ be such that $\left|\tau^0 - \tau\right| < \alpha$ where $\alpha > 0$ will be choosen later. Represent the problem (21) in the form

$$\mathcal{L}^{\tau^0} u = f(t,x) + \left(\mathcal{L}^{\tau^0} - \mathcal{L}^\tau\right) u; \quad (t,x) \in Q_T, \quad u \in \widehat{W}_2^{2,1}(Q_T). \tag{22}$$

It is easy to see that $\mathcal{L}^{\tau^0}-\mathcal{L}^{\tau}=\left(\tau^0-\tau\right)(\mathcal{L}-H)$. Consider auxiliary problem

$$\mathcal{L}^{\tau^0}u=f(t,x)+\left(\tau^0-\tau\right)(\mathcal{L}-H)\,v;\quad (t,x)\in Q_T,\quad u\in \widehat{W}_2^{2,1}(Q_T)\,, \tag{23}$$

where $v(t,x)\,\widehat{W}_2^{2,1}(Q_T)$. Acting as in theorem 1 we can show that

$$\|(\mathcal{L}-H)\,u\|_{L_2(Q_T)}\le C_{19}(\mathcal{L},n)\,\|v\|_{\widehat{W}_2^{2,1}(Q_T)}\,.$$

Thus the operator $\mathcal{M}$ associating to every function $\vartheta(t,x)\in\widehat{W}_2^{2,1}(Q_T)$ the solution $u(t,x)$ of the problem (23) is determined, i. e. $u=\mathcal{M}v$. Show that at corresponding way choosen by $\alpha$ the operator $\mathcal{M}$ is contractive. Let $u^1=\mathcal{M}v^1$, $u^2=\mathcal{M}v^2$. We have

$$\mathcal{L}^{\tau^0}\left(u^1-u^2\right)=\left(\tau^0-\tau\right)(\mathcal{L}-H)\left(v^1-v^2\right);\quad u^1-u^2\in\widehat{W}_2^{2,1}(Q_T)$$

Then according to theorem 1

$$\left\|u^1-u^2\right\|_{W_2^{2,1}(Q_T)}\le C_{18}\alpha C_{19}\left\|v^1-v^2\right\|_{W_2^{2,1}(Q_T)}$$

and it is sufficient to choose $\alpha=\dfrac{1}{2C_{18}C_{19}}$. Then the operator $\mathcal{M}$ has a fixed point $u=\mathcal{M}u$. But at $v=u$ the problem (23) coincides with the problem (22), i.e. with (21). The openness of the set $E$ is proved. Now prove its closure. Let $\tau^m\in E$; $m=1,2,...,$ $\tau^0=\lim\limits_{m\to\infty}\tau^m$ snow that $\tau^0\in E$. Denote by $u^m(t,x)$ the solution of the boundary value problem

$$\mathcal{L}^{\tau^m}u^m=f(t,x)\,;\quad (t,x)\in Q_T,\quad u^m\in\widehat{W}_2^{2,1}(Q_T)\;.$$

According to theorem 1

$$\|u^m\|_{W_2^{2,1}(Q_T)}\le C_{18}\,\|f\|_{L_2(Q_T)}\;.$$

Thus the sequence $\{u^m(t,x)\}$ is bounded by the norm $W_2^{2,1}(Q_T)$. Hence it follows that it is weakly compact, i.e. exist subsequence $m_k\to\infty$ at $k\to\infty$ and the function $u(t,x)\in\widehat{W}_2^{2,1}(Q_T)$ such that $u(t,x)$ is weak limit in $\widehat{W}_2^{2,1}(Q_T)$ of the subsequence $\{u^{m_k}(t,x)\}$ at $k\to\infty$. Hence in particular it follows that for any function $\varphi(t,x)\in C^\infty\left(\overline{Q}_T\right)$

$$\left\langle\mathcal{L}^{\tau^0}u^{m_k},\varphi\right\rangle\to\langle\mathcal{L}^{\tau},\varphi\rangle\,;\ k\to\infty,$$

where $\langle u,\vartheta\rangle=\displaystyle\int_{Q_T}u\vartheta dtdx$. But

$$\left\langle\mathcal{L}^{\tau^0}u^{m_k},\varphi\right\rangle=\left\langle\left(\mathcal{L}^{\tau^0}-\mathcal{L}^{\tau_{m_k}}\right)u^{m_k},\varphi\right\rangle+\left\langle\mathcal{L}^{\tau_{m_k}}u^{m_k},\varphi\right\rangle=i_1+i_2$$

We have

$$|i_1|\le\left|\tau^0-\tau^{m_k}\right|\left|\langle(\mathcal{L}-H)\,u^{m_k},\varphi\rangle\right|\le$$

$$\leq \left|\tau^0 - \tau^{m_k}\right| C_{20}(\varphi) C_{19} \|u^{m_k}\|_{W_2^{2,1}(Q_T)} \leq$$
$$\leq C_{18} C_{19} C_{20} \left|\tau^0 - \tau^{m_k}\right| \|f\|_{L_2(Q_T)}$$

Thus $i_1 \to 0$ at $k \to \infty$. On the other hand $i_2 = \langle f, \varphi\rangle$. So for any function $\varphi(t,x) \in C^\infty\left(\overline{Q}_T\right)$

$$\left\langle \mathcal{L}^{\tau^0} u, \varphi \right\rangle = \langle f, \varphi\rangle$$

It means that $\mathcal{L}^{\tau^0} u = f(t,x)$ almost every where in $Q_T$ , i.e. $\tau^0 \in E$. The theorem is proved.

**Theorem 3.** The problem (1)-(2) is identically solvable in space $\widehat{W}_2^{2,1}(Q_T)$ under conditions of the previous theorem without the assumption $T \leq T_0$.

**Proof.** Let $T > 0$, $Q^{(1)} = Q_{T_0}$. We shall consider in $Q^{(1)}$ the solution $u^{(1)}$ of the mixed problem $\mathcal{L}u^{(1)} = f$ in $Q^{(1)}$, $u^{(1)}\big|_{t=0} = 0$, $\left.\dfrac{\partial u}{\partial n}\right|_{S\left(Q^{(1)}\right)} = 0$. Under the theorem 2 this solution is unique and belongs to a class $\widehat{W}_2^{2,1}\left(Q^{(1)}\right)$. Not losing in a generality it is possible to consider, that $u^{(1)}(T_0/2,\ x) \in W_2^2(\mathcal{D})$. We shall consider, further, in cylindrical domain $Q^{(2)} = \mathcal{D} \times (T_0/2,\ 3T_0/2)$ a boundary problem

$$\mathcal{L}v = F \quad \text{in} \quad Q^{(2)},\ v|_{t=0} = 0,\ \left.\frac{\partial v}{\partial n}\right|_{S\left(Q^{(2)}\right)} = 0, \tag{24}$$

where $F(t,x) = f(t,x) - f(T_0/2,\ x) - \dfrac{\partial u^{(1)}}{\partial t}(T_0/2,\ x)$, $S\left(Q^{(2)}\right) = \partial\mathcal{D} \times [T_0/2,\ 3T_0/2]$. Clearly, that $F \in L_2\left(Q^{(2)}\right)$, therefore the problem (24) has the unique solution $v \in W_2^{2,1}\left(Q^{(2)}\right)$. From here follows, that function $u^{(2)}(t,x) = v(t,x) + u^{(1)}(T_0/2,\ x)$ almost everywhere in $Q^{(2)}$ satisfies to the equation $\mathcal{L}u^{(2)} = f$ . Let

$$u(t,x) = \begin{cases} u^{(1)}(t,x) & \text{for} \quad (t,x) \in Q_{T_0/2} \\ u^{(2)}(t,x) & \text{for} \quad (t,x) \in Q^{(2)} \end{cases}$$

According to the theorem 2 function $u$ coincides in $Q^{(2)}$ with function $u^{(1)}$, i. e. $u \in W_2^{2,1}\left(Q_{3T_0/2}\right)$ and $\mathcal{L}u = f$ almost everywhere in $Q_{3T_0/2}$. If $3T_0/2 < T$ continue further described above process. For finite number of steps we shall construct the solution in all cylinder $Q_T$ so on each step the height of the cylinder in which the solution is construction, increases for a constant equal $T_0/2$. The theorem is proved.

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**Abdurrahim F. Guliyev, Sakina H. Hasanova**
Institute of Mathematics and Mechanics of NAS of Azerbaijan.
9, F. Agayev str., AZ1141, Baku, Azerbaijan.
Tel.: (99412) 439-47-20 (off.)
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