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INTEGRO-DIFFERENTIAL "BOUNDARY" CONDITIONS WITH ORDER EXCEEDING A PARABOLIC SYSTEM ORDER

Abstract

In the paper we consider a mixed problem for parabolic system with disconnected coefficients and with integro-differential "boundary" conditions. Therewith, the order of "boundary" conditions in space variable may exceed the order of the parabolic system. Under correct "boundary" conditions we apply finite integral transformation and obtain analytic representation of the solution of the problem under consideration.

Introduction. At the beginning of the XIX century Fourier suggested a method (method of separation of variables) for integration (of some) of partial linear differential equations under the given boundary and initial conditions (problem 1). Application of the Fourier method to the solution of mixed problems with separated variables reduceS to the problem on expansion of an arbitrary function from some class in eigen functions corresponding to the spectral problem (problem 2). If the operator determined by problem 2 is not self-adjoint, then there is no orthogonality of eigen functions and the problem on the existence of completeness of a system of eigen functions remains open.

In 1827 Cauchy [1] suggested a new method (residue method) for solving problem 1 with constant coefficients. The essence of the method is representation of an arbitrary function $f(x)$ in the form of integral residue from the fraction $\omega(\rho,x)/\mathcal{F}(\rho)$ where the function $\omega(\rho,x)$ for all values of ρ satisfies the considered differential equation, for the values of ρ converting the denominator to "zero", moreover, boundary conditions. In 1917 Y.D.Tamarkin used the results of Birkhoff's paper [2] noticed in the paper [3]:

Consideration of integral residue of the function $\int_a^b \rho^{n-1} G(x,t,\rho)$ (by the Poincare method) only in special cases leads to expantion of an arbitrary function $f(x)$ in fundamental functions on all intervals (a,b) including the ends. In order to get more general results it is necessary to investigate the representation of the function $f(x)$ in the form of the integral

$$\int_a^b G(x,t,\rho)\,L(f)\,dt$$

Here $L(y)$ denotes the left hand side of the differential equation to which fundamental functions, satisfy $G(x,t,\rho)$ is the Green function .

In the paper [4] the formula of Birkhoff [2] and Tamarkin [3] expansions are successfully used and the mixed problems for which boundary condition of the spectral problem are regular, are considered. In the case when boundary conditions of the spectral problem are not regular or when the integrals from the desired function are contained in "boundary" conditions, the question on the existence of the formula of the Birkhoff-Tamarkin expansion remains open.

Investigations of the author [5]-[8] showed that in solving problem 1 the use of Birkhoff-Tamarkin formula is not necessary. Applying the finite integral transformation suggested by the author in the paper [5] problem 1 is solved under more general boundary conditions and weaker restrictions on data of problem 1. In solving problem 2 we introduce the notion of correctness of boundary conditions that are wider than the notion of regularity, i.e. if the conditions of the spectral problem are regular, they are correct by our definition, but the inverse statement is not true.

In the papers [5]-[8] we showed applicability of the finite integral transformation method [5] to the solution of the following mixed problems for conjugation of parabolic systems with discontinuous coefficients.

Find the solution of the system

$$D_t u_i(x,t) - a(t) \sum_{j=0}^{2p_i} A_{ij}(x) D_x^j u_i(x,t) = f_i(x,t),$$

$$x \in (a_i, b_i), \quad t \in (0,T), \quad i = 1, ..., n, \tag{1}$$

with integro-differential "boundary" conditions

$$\sum_{i=1}^{n} \sum_{j=0}^{\chi(i)} \sum_{m=0}^{S(j,i)} a^{1-j}(t) \left\{ \alpha_{jm}^{(i)} D_x^m D_t^j u_i(x,t)|_{x=a_i} + \beta_{jm}^{(i)} D_x^m D_t^j u_i(x,t)|_{x=b_i} + \right.$$

$$\left. \int_{a_i}^{b_i} \gamma_{jm}^{(i)}(x) D_x^m D_t^j u_i(x,t)\, dx \right\} = \varphi(t), \quad t \in (0,T), \tag{2}$$

and initial conditions

$$u_i(x,0) = \Phi_i(x), \quad x \in (a_i, b_i), \quad i = 1, ..., n, \tag{3}$$

where A_{ij} are square matrices of order r_i; $a(t)$ is a scalar function; $\alpha_{jm}^{(i)}$, $\beta_{jm}^{(i)}$, $\gamma_{jm}^{(i)}(x)$ are matrices of dimensions $N \times r_i$, $N = 2(d_1 + d_2 + ... + d_n)$, $d_v = p_v r_v$; Φ_i, f_i, u_i are columns of dimension r_i; φ is a column of dimension N; r_i, p_i, n are natural numbers; $\chi(i) - 0$ or 1; T $(0 < T \leq \infty)$, a_i, b_i $(a_i < b_i)$ are finite numbers; u_i $(i = 1, ..., n)$ is a desired solution, and the remaining data contained in (1)-(3) are assumed to be known, and finally $S(j,i)$ are non-negative integers smaller or equal to $2p_i - 1$.

Investigations of the author showed that the finite integral transformation method is applicable to the solution of mixed problems (1)-(3) in the case $S(j,i) \geq 2p_i$; as well, i.e. when order (of differential) in space variable of "boundary" conditions (2) is greater or equal than the order of equation (1).

In the present paper, for simplicity of notation we are restricted by the consideration of the following model problem. Find the solution of the equation

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + f(x,t), \quad x \in (0,1), \quad t > 0 \tag{4}$$

with integro-differential "boundary" conditions

$$\sum_{k=0}^{1} \sum_{j=0}^{S(i,k)} \left\{ \alpha_{ij}^{(k)} \left. \frac{\partial^{k+j} u(x,t)}{\partial t^k \partial x^j} \right|_{x=0} + \beta_{ij}^{(k)} \left. \frac{\partial^{k+j} u(x,t)}{\partial t^k \partial x^j} \right|_{x=1} + \right.$$

$$\left. + \int_0^1 \gamma_{ij}^{(k)}(x) \frac{\partial^{k+j} u(x,t)}{\partial t^k \partial x^j} dx \right\} = \varphi_i(t), \quad t > 0, \quad i = 1, 2, \tag{5}$$

and initial condition

$$u(x,0)=\Phi(x), \quad x\in(0,1), \tag{6}$$

here $u\equiv u(x,t)$ is a desired solution and remaining data contained in (4)-(6) are assumed to be known, and non-negative integers $S(i,k)$ (contained in (5)) in particular may be greater or equal to 2, i.e. order (of differential) in space variable of "boundary" conditions (5) in particular, may be greater or equal than the order of equation (4).

Fulfillment of restrictions 1^0-3^0 is assumed.

1^0. Let $\operatorname{Re}(a^2)>0$ i.e. let $|\arg a|<\pi/4$ where a is some constant

2^0. Let $\gamma_{ij}^{(k)}(x)\in C^l([0,1])$, $l=m+1+2k+j$, $k=0,1$, $j=0,..,S(i.k)$, $i=1,2$, where m is some non-negative integer.

3^0. Let $\Phi(x)\in C^{S1}([0,1])$ where $S1=\max S(i,1)$ for $i=1,2$; the functions $f(x,t)$, $\varphi_i(t)$, $i=1,2$ are continuous for $x\in[0,1]$, $t\geq 0$; for $S>2$ the derivatives $\dfrac{\partial^k f(x,t)}{\partial x^k}$, $k=1,...,S-2$ exists and continuous for $x\in[0,1]$, $t\geq 0$ and $\Phi\left(x\in C^{S-2}([0,1])\right)$, where $S=\max S(i,k)$ for $k=0,1$, $i=1,2$.

For solving problem (4)-(6) first of all we solve the following parametric problem

$$a^2\frac{d^2y}{dx^2}-\lambda^2 y=\psi(x), \qquad x\in(0,1) \tag{7}$$

$$U_i(\lambda,y)=\gamma_i, \quad i=1,2, \tag{8}$$

where

$$U_i(\lambda,y)=\sum_{k=0}^{1}\sum_{j=0}^{S(i,k)}\lambda^{2k}\left\{\alpha_{ij}^{(k)}\frac{\partial^j y}{dx^j}|_{x=0}+\beta_{ij}^{(k)}\frac{\partial^j y}{dx^j}|_{x=0}+\int_0^1\gamma_{ij}^{(k)}(x)\frac{d^j y}{dx^j}dx\right\}, \quad i=1,2.$$

Here and in sequel, we assume that

$$\lambda\in R_\sigma=\left\{\lambda:|\lambda|\geq R, \quad |\arg\lambda|\leq\frac{\pi}{4}+\sigma\right\},$$

where σ is some positive number satisfying the inequality $0<\sigma<\frac{\pi}{4}-|\arg a|$, R is sufficiently great positive number.

We take a system of fundamental particular solutions of homogeneous equation corresponding to (7) in the form [5]

$$y_1\equiv y_1(x,\lambda)=\exp\left(-\frac{\lambda}{a}x\right);$$

$$y_2\equiv y_2(x,\lambda)=\exp\left(-\frac{\lambda}{a}(1-x)\right), \quad x\in[0,1], \ \lambda\in R_\sigma. \tag{9}$$

From (9) we get

$$\begin{array}{ll}|y_1(x,\lambda)|\leq const \quad \exp(-\varepsilon|\lambda|x), & x\in[0,1],\\ |y_2(x,\lambda)|\leq const \quad \exp(-\varepsilon|\lambda|(1-x)), & \lambda\in R_\sigma,\end{array} \tag{10}$$

where ε is some positive number

Assume

$$\Delta(\lambda) = \det \begin{pmatrix} U_1(\lambda, y_1) & U_1(\lambda, y_2) \\ U_2(\lambda, y_1) & U_2(\lambda, y_2) \end{pmatrix} \tag{11}$$

Expanding determinant (11) we have

$$\Delta(\lambda) = \sum_{i=0}^{m} \alpha_i \lambda^{M-i} + \frac{E(\lambda)}{\lambda^{m+1-M}}, \quad \lambda \in R_\sigma, \tag{12}$$

where $M = \max\limits_{k=0,1} \{4k + S(1,k) + S(2,k)\}$, m is from the restriction 2^0, α_i are some numbers, $E(\lambda)$ is some function satisfying the inequality

$$|E(\lambda)| \leq const, \quad \lambda \in R_\sigma.$$

Notice that the number m contained in (12) may be taken sufficiently large (i.e. for $\Delta(\lambda)$ we can obtain more precise asymptotics) if the functions contained in restriction 2^0, are sufficiently smooth.

Definition. *We'll say that "boundary" conditions (8) (or (5)) are correct, even if of one the numbers $\alpha_0, \alpha_1, .., \alpha_m$ (or (12)) differs from zero.*

4^0. Assume that $\psi(x) \in C^{S-2}([0,1])$ for $S \geq 2$ and $\psi(x) \in C[(0,1)]$ for $S \leq 1$ where S is from restriction 3^0.

The following lemma is easily proved in a similar way stated in [5].

Lemma 1. *Let restrictions 1^0, 2^0, 4^0 be fulfilled and "boundary" conditions (8) be correct. Then for $\lambda \in R_\sigma$ problem (7)-(8):*

1) has a unique solution

2) this solution is represented by the formula [5]

$$y(x,\lambda) = \delta[x, \lambda, \gamma_1, \gamma_2] - \delta[x, \lambda, V_1(\lambda, \psi), V_2(\lambda, \psi)] +$$
$$+ \int_0^1 G(x, \xi, \lambda)\psi(\xi)\, d\xi, \quad x \in R_\sigma, \tag{13}$$

where

$$V_i(\lambda, \psi) = \sum_{k=0}^{1} \sum_{2 \leq j \leq S(i,k)} \lambda^{2k} \sum_{0 \leq n \leq [j/2-1]} \frac{\lambda^{2n}}{a^{2(n+1)}} \times$$
$$\times \left\{ \alpha_{ij}^{(k)} \frac{d^l \psi(x)}{dx^l}|_{x=0} + \beta_{ij}^{(k)} \frac{d^l \psi(x)}{dx^l}|_{x=1} + \int_0^1 \gamma_{ij}^{(k)}(x) \frac{d^l \psi(x)}{dx^l} dx \right\},$$
$$l = j - 2 - 2n; \quad i = 1, 2,$$

$[j/2-1]$ is an integer part of $j/2-1$;

$$\delta[x, \lambda, \gamma_1, \gamma_2] = \frac{1}{\Delta(\lambda)} \begin{vmatrix} 0 & y_1(x,\lambda) & y_2(x,\lambda) \\ -\gamma_1 & U_1(\lambda, y_1) & U_1(\lambda, y_2) \\ -\gamma_2 & U_2(\lambda, y_1) & U_2(\lambda, y_2) \end{vmatrix},$$

$$G(x, \xi, \lambda) = g(x, \xi, \lambda) + G_1(x, \xi, \lambda),$$

$$G_1(x, \xi, \lambda) = \frac{1}{\Delta(\lambda)} \Delta_1(x, \xi, \lambda)$$

$$\Delta_1(x, \xi, \lambda) = \begin{vmatrix} 0 & y_1(x,\lambda) & y_2(x,\lambda) \\ U_1(\lambda, g)_x & U_1(\lambda, y_1) & U_1(\lambda, y_2) \\ U_2(\lambda, g)_x & U_2(\lambda, y_1) & U_2(\lambda, y_2) \end{vmatrix}, \tag{14}$$

$g \equiv g(x,\xi,\lambda) = -\frac{1}{2a\lambda}\exp\left(-\frac{\lambda}{a}|x-\xi|\right)$ *is a fundamental solution [5] of equation (7).*

The following theorem is true.

Theorem 1. *Let restrictions* 1^0*-*2^0 *be fulfilled and "boundary" conditions (8) be correct. Then for any numbers* γ_1,γ_2 *and* $\psi(x)\in C([0,1])$ *if* $\psi(x)$ *is Hölder continuous with exponent* $\alpha\,(0<\alpha\leq 1)$ *in* $[\mu_1,\mu_2]$ $(0<\mu_1<\mu_2<1)$ *then it holds*

$$\int\limits_{\mathcal{L}} \lambda^k \delta[x,\lambda,\gamma_1,\gamma_2]\,d\lambda = 0, \qquad 0<x<1, \tag{15}$$

k is any integer:

$$\int\limits_{\mathcal{L}} \lambda^s d\lambda \int\limits_0^1 G(x,\xi,\lambda)\,\psi(\xi)\,d\xi =$$

$$= -s\sqrt{-1}\left(\frac{\pi}{2}+2\sigma\right)\psi(x), \quad s=0,1, \quad 0<\mu_1<x<\mu_2<1, \tag{16}$$

where $\mathcal{L}$ *is an infinite smooth line in* R_σ*, whose sufficiently far part coincides with continuations of the rays* $\arg\lambda = \pm\left(\frac{\pi}{4}+\sigma\right)$ *where integral on the lines* $\mathcal{L}$ *is understood in the sense of the principal value.*

Proof. According to the conditions of the theorem, from (14), allowing for (12) we get

$$|\delta[x,\lambda,\gamma_1,\gamma_2]| \leq const\,|\lambda|^N (|\gamma_1|+|\gamma_2|)\times$$

$$\times\{\exp(-\varepsilon|\lambda|x)+\exp(-\varepsilon|\lambda|(1-x))\}, \quad x\in[0,1], \quad \lambda\in R_\sigma, \tag{17}$$

$$|G_1(x,\xi,\lambda)| \leq const\,|\lambda|^N \{\exp(-\varepsilon|\lambda|x)+\exp(-\varepsilon|\lambda|(1-x))\};$$

$$x,\xi\,[0,1], \quad \lambda\in R_\sigma \tag{18}$$

where N is some integer.

Using inequalities (17)-(18) and analyticity of the integrand function by $\lambda\in R_\sigma$ we easily understand that formula (15) and equalities

$$\int\limits_{\mathcal{L}} \lambda^k d\lambda \int\limits_0^1 G_1(x,\xi,\lambda)\,\psi(\xi)\,d\xi = 0, \quad 0<x<1, \tag{19}$$

where k is any integer, are fulfilled.

By [5] the following inversion formula

$$\int\limits_{\mathcal{L}} \lambda^s d\lambda \int\limits_0^1 g(x,\xi,\lambda)\,\psi(\xi)\,d\xi = -S\sqrt{-1}\left(\frac{\pi}{2}+2\sigma\right)\psi(x),$$

$$s=0,1, \ \ 0<\mu_1<x<\mu_2<1 \tag{20}$$

holds for fundamental solution $g(x,\xi,\lambda)$.

From (19)-(20) and (14) we get validity of the inversion formula (16). The theorem is proved.

It holds

Theorem 2. *Let restrictions* 1^0*-*3^0 *be fulfilled and integro-differential "boundary" conditions (5) be correct. Then, if problem (4)-(6) has a classic solution, then*

1) it is unique

2) it can be represented by the formula

$$u(x,t)=\frac{1}{\pi\sqrt{-1}}\int\limits_{\mathcal{L}}\lambda\exp\left(\lambda^2 t\right)\mathcal{F}(x,t,\lambda)\,d\lambda,\quad 0<x<1,\quad t>0, \tag{21}$$

where

$$\psi_i(\lambda,t)=\int\limits_0^t\exp\left(-\lambda^2\tau\right)\varphi_i(\tau)\,d\tau+\sum_{j=0}^{S(i,1)}\left\{\alpha_{ij}^{(1)}\frac{d^j\Phi(x)}{dx^j}\bigg|_{x=0}+\right.$$

$$\left.+\beta_{ij}^{(1)}\frac{d^j\Phi(x)}{dx^j}\bigg|_{x=1}+\int\limits_0^1\gamma_{ij}^{(1)}(x)\frac{d^j\Phi(x)}{dx^j}dx\right\},\quad i=1,2;$$

$$\mathcal{F}_0(x,t,\lambda)=-\Phi(x)-\int\limits_0^t\exp\left(-\lambda^2\tau\right)f(x,\tau)\,d\tau;$$

$$\mathcal{F}(x,t,\lambda)=\delta\left[x,\lambda,\psi_1(\lambda,t),\psi_2(\lambda,t)\right]-$$

$$-\delta\left[x,\lambda,V_1(\lambda,\mathcal{F}_0(x,t,\lambda)),V_2(\lambda,F_0(x,t,\lambda))\right]+$$

$$+\int\limits_0^1 G(x,\xi,\lambda)\mathcal{F}_0(\xi,t,\lambda)\,d\xi.$$

Proof . Let problem (4)-(6) have a classic solution.
Then, from (4) we have

$$\int\limits_\alpha^t\exp\left(-\lambda^2\tau\right)\frac{\partial u(x,\tau)}{\partial\tau}d\tau=$$

$$=a^2\int\limits_\alpha^t\exp\left(-\lambda^2\tau\right)\frac{\partial^2 u(x,\tau)}{\partial x^2}d\tau+\int\limits_\alpha^t\exp\left(-\lambda^2\tau\right)f(x,\tau)\,d\tau\,,$$

where α is some positive number.
Consequently,

$$\left(a^2\frac{\partial^2}{\partial x^2}-\lambda^2\right)\int\limits_\alpha^t\exp\left(-\lambda^2\tau\right)u(x,\tau)\,d\tau=\exp\left(-\lambda^2 t\right)u(x,t)-$$

$$-\exp\left(-\lambda^2\alpha\right)u(x,\alpha)-\int\limits_\alpha^t\exp\left(-\lambda^2\tau\right)f(x,\tau)\,d\tau\,,\quad x\in(0,1)\,,\quad (t>0) \tag{22}$$

From (5) we get

$$U_i\left(\lambda,\int\limits_\alpha^t\exp\left(-\lambda^2\tau\right)u(x,\tau)\,d\tau\right)=\psi_{i\alpha}(\lambda,t)-\exp\left(-\lambda^2 t\right)v_i(t)\,,\ \ i=1,2; \tag{23}$$

where

$$v_i(t)=\sum_{j=0}^{S(i,1)}\left\{\alpha_{ij}^{(1)}\frac{\partial^j u(x,t)}{\partial x^j}\bigg|_{x=0}+\beta_{ij}^{(1)}\frac{\partial^j u(x,t)}{\partial x^j}\bigg|_{x=1}+\int\limits_0^1\gamma_{ij}^{(1)}\frac{\partial^j u(x,t)}{\partial x^j}dx\right\},$$

$$\psi_{i\alpha}(\lambda,t)=\int\limits_\alpha^t\exp\left(-\lambda^2\tau\right)\varphi_i(\tau)\,d\tau+\exp\left(-\lambda^2\alpha\right)v_i(\alpha)\,,\quad i=1,2.$$

For $\lambda \in R_\sigma$ by the uniqueness of the solution of problem (7)-(8) according to formula (13), from (22)-(23) we have

$$\int\limits_\alpha^t \exp\left(-\lambda^2\tau\right) u\left(x,\tau\right) d\tau = \delta[x,\lambda,\psi_{1\alpha}\left(\lambda,t\right) -$$

$$-\exp\left(-\lambda^2\tau\right) v_1\left(t\right), \psi_{2\alpha}\left(\lambda,t\right) - \exp\left(-\lambda^2 t\right) v_2\left(t\right)] -$$

$$-\delta[x,\lambda,V_1\left(\lambda,\exp\left(-\lambda^2 t\right) u\left(x,t\right) + F_\alpha\left(x,t,\lambda\right)\right),$$

$$V_2\left(\lambda,\exp\left(-\lambda^2 t\right) u\left(x,t\right) + F_\alpha\left(x,t,\lambda\right)\right)] +$$

$$+\int\limits_0^1 G\left(x,\xi,\lambda\right)\left\{\exp\left(-\lambda^2 t\right) u\left(\xi,t\right) + F_\alpha\left(\xi,t,\lambda\right)\right\} d\xi,$$

$$x \in [0,1], \quad t \geq \alpha > 0, \quad \lambda \in R_\sigma \tag{24}$$

where

$$F_\alpha\left(x,t,\lambda\right) = -\exp\left(-\lambda^2\alpha\right) u\left(x,\alpha\right) - \int\limits_\alpha^t \exp\left(-\lambda^2\tau\right) f\left(x,\tau\right) d\tau.$$

In (24) as $\alpha \to +0$ passing to limit and using (6) we get

$$\int\limits_0^t \exp\left(-\lambda^2\tau\right) u\left(x,\tau\right) d\tau =$$

$$= \delta\left[x,\lambda,\psi_1\left(\lambda,t\right) - \exp\left(-\lambda^2\tau\right) v_1\left(t\right), \psi_2\left(\lambda,t\right) - \exp\left(-\lambda^2 t\right) v_2\left(t\right)\right] -$$

$$-\delta[x,\lambda,V_1\left(\lambda,\exp\left(-\lambda^2 t\right) u\left(x,t\right) + \mathcal{F}_0\left(x,t,\lambda\right)\right),$$

$$V_2\left(\lambda,\exp\left(-\lambda^2 t\right) u\left(x,t\right) + \mathcal{F}_0\left(x,t,\lambda\right)\right)] +$$

$$+\int\limits_0^1 G\left(x,\xi,\lambda\right)\left\{\exp\left(-\lambda^2 t\right) u\left(\xi,t\right) + \mathcal{F}_\alpha\left(\xi,t,\lambda\right)\right\} d\xi, \quad x \in [0,1], \quad t \geq 0, \quad \lambda \in R_\sigma.$$

Consequently,

$$V\left(x,t,\lambda\right) = \mathcal{F}\left(x,t,\lambda\right), \quad x \in [0,1], \quad t \geq 0, \quad \lambda \in R_\sigma, \tag{25}$$

where

$$V\left(x,t,\lambda\right) = \int\limits_0^t \exp\left(-\lambda^2\tau\right) u\left(x,\tau\right) d\tau + \exp\left(-\lambda^2 t\right) \delta\left[x,\lambda,v_1\left(t\right),v_2\left(t\right)\right] +$$

$$+\exp\left(-\lambda^2 t\right) \delta\left[x,\lambda,V_1\left(\lambda,u\left(x,t\right)\right),V_2\left(\lambda,u\left(x,t\right)\right)\right] -$$

$$-\exp\left(-\lambda^2 t\right) \int\limits_0^1 G\left(x,\xi,\lambda\right) u\left(\xi,t\right) d\xi. \tag{26}$$

By [5] it holds the inversion formula

$$\int\limits_{\mathcal{L}} \lambda d\lambda \int\limits_0^t \exp\left(\lambda^2\left(t-\tau\right)\right) \varphi\left(\tau\right) d\tau = \left(\frac{\pi}{2} - 2\sigma\right)\sqrt{-1}\varphi\left(t\right), \quad t > 0, \tag{27}$$

where $\varphi\left(t\right)$ is a continuous and piecewise differentiable function .

Using formulae (15)-(16) and (27) we have

$$\int\limits_{\mathcal{L}} \lambda d\lambda \int\limits_0^t \exp\left(\lambda^2 (t-\tau)\right) u(x,\tau)\, d\tau = \left(\frac{\pi}{2} - 2\sigma\right)\sqrt{-1}u(x,t),$$

$$\int\limits_{\mathcal{L}} \lambda\delta\left[x,\lambda,v_1(t),v_2(t)\right] d\lambda = 0,$$

$$\int\limits_{\mathcal{L}} \lambda\delta\left[x,\lambda,V_1(\lambda,u(x,t)),V_2(\lambda,u(x,t))\right] d\lambda = 0,$$

$$\int\limits_{\mathcal{L}} \lambda d\lambda \int\limits_0^1 G(x,\xi,\lambda)\, u(\xi,t)\, d\xi =$$

$$\left(\frac{\pi}{2} + 2\sigma\right)\sqrt{-1}u(x,t), \quad 0<x<1, \quad t>0. \tag{28}$$

Multiplying the both hand sides of (25) by $\lambda \exp\left(\lambda^2 t\right)$ and integrating over $\mathcal{L}$, using formulae (26)-(28) we get validity of formula (21). The theorem is proved.

As the function $\exp\left(\lambda^2 t\right)$ for $t>0$ and $|\lambda| \to \infty$ along the lines $\mathcal{L}$ decreases by exponential growth, then imposing definite conditions of the data of problem (4)-(6) we can easily see by direct verification that the function $u(x,t)$ definable by formula (21) is a classic solution of problem (4)-(6).

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