Oktay Sh. SALMANOV

A PROBLEM ON EIGEN VIBRATIONS OF TRANSVERSALLY STIFFENED LIQUID-FILLED CYLINDRICAL SHELL LOADED WITH AXIAL CONTRACTING FORCES

Abstract

In the paper we investigate a problem on free vibrations of liquid-filled cylindrical shells that are regularly reinforced by transversally located ribs and loaded with axial contracting forces. Such kind problem, when there is no liquid, was investigated in the paper [1]. In the paper [2] a problem on vibrations of stiffened liquid-filled cylindrical shells without axial contracting forces was researched.

Using the motion equations of structurally-orthotropic shell, liquid and contact proofness conditions we obtain a frequency equation for finding eigen frequencies of vibrations of a shell reinforced by transverse ribs and loaded with axial contracting forces. The influences of physical and mechanical parameters of ribs and liquid on frequency eigen vibrations of the considered construction are studied. Simplified formulae for calculating eigen frequencies of vibrations of the considered system are obtained on the dase of asymptotic method.

By [1], a system of equations of structurally-orthotropic liquid-filled cylindrical shell loaded with axial contracting forces is of the form:

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{1-v}{2}\frac{\partial^2}{\partial\theta^2}\right)u+\frac{1+v}{2}\frac{\partial^2\vartheta}{\partial\xi\partial\theta}-v\frac{\partial}{\partial\xi}w-\rho_1\frac{\partial^2 u}{\partial t_1^2}=0$$

$$\frac{1+v}{2}\frac{\partial^2\vartheta}{\partial\xi\partial\theta}+\left\{\frac{1-v}{2}\left(1+\eta a^2\right)\frac{\partial^2}{\partial\xi^2}+\left[1+\left(1-\frac{h_s}{r}\right)^2\gamma_s^{(2)}+a^2\right]\frac{\partial^2}{\partial\theta^2}\right\}\vartheta+$$

$$+\left\{-\left[1+\left(1-\frac{h_s}{r}\right)^2\gamma_s^{(2)}\right]\frac{\partial}{\partial\theta}+(2-v)\,a^2\frac{\partial^3}{\partial\xi^2\partial\theta}+\right.$$

$$\left.+\left[a^2-\left(1-\frac{h_s}{r}\right)\delta_s^{(2)}\right]\frac{\partial^3}{\partial\theta^3}\right\}w-\rho_2\frac{\partial^2\vartheta}{\partial t_1^2}=0 \qquad (1)$$

$$-v\frac{\partial u}{\partial\xi}+\left\{-\left[1+\left(1-\frac{h_s}{r}\right)\gamma_s^{(2)}\right]\frac{\partial}{\partial\theta}+(2-v)\,a^2\frac{\partial^3}{\partial\xi^2\partial\theta}\right.$$

$$\left.+\left[a^2-\left(1-\frac{h_s}{r}\right)\delta_s^{(2)}\right]\frac{\partial^3}{\partial\theta^3}\right\}\vartheta+\left[1+\gamma_s^{(2)}+\eta_{s1}^{(2)}+2\left(\delta_s^{(2)}+\eta_{s1}^{(2)}\right)\frac{\partial^2}{\partial\theta^2}+\right.$$

$$\left.+a^2\Delta\Delta+\overline{p}\frac{\partial^2}{\partial\xi^2}+\left(\eta_{s1}^{(2)}+\eta_{s2}^{(2)}\right)\frac{\partial^4}{\partial\theta^4}\right]w+\rho_3\frac{\partial^2 w}{\partial t_1^2}=\frac{R^2\left(1-v^2\right)}{Eh}q_z$$

where $\rho_1=1$, $\rho_2=1+\overline{\rho}_s\overline{\gamma}_s=\rho_3$, $\overline{\gamma}_s^{(2)}=\dfrac{F_s}{L_1 h}(1+k_2)$ (L_1 is shell's length, F_s is the area of cross section of ribs, k_2 is the number of transverse ribs), $\overline{\rho}_s=\dfrac{\rho_s}{\rho}$ (ρ,ρ_s is density of shell and ribs materials, respectively),

$$\delta_s^{(2)}=\frac{h_s}{R}\overline{\gamma}_s^{(2)},\ \eta_{s1}^{(2)}=\frac{E_sJ_{xs}\left(1-v^2\right)(1+k_2)}{EL_1R^2h}\qquad \overline{\eta}_s^{(2)}=\left(\frac{h_s}{R}\right)^2\overline{\gamma}_s^{(2)},$$

$$\eta_{s2}^{(2)}=\frac{E_s\left(1-v^2\right)}{E}\overline{\eta}_s^{(2)},\quad \delta_s^{(2)}=\frac{h_s}{R}\overline{\gamma}_s^{(2)},\quad \gamma_s^{(2)}=\frac{E_s\left(1-v^2\right)}{E}\overline{\gamma}_s^{(2)}$$

$E,\ v$ is elasticity modulus and Poisson coefficient of shell material, respectively, $R,\ h$ is a radius and thickness of the shell, respectively, E_s it is elasticity modulus of ribs material, $a^2=\frac{h_s}{12R^2},\ \Delta=\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\theta^2},\ J_{xs}$ is inertia moment of transverse cross section of the rib with respect to the axis $ox,\ \theta=\frac{y}{R},\ u,\ \vartheta,\ w$ are constituents of permutations of mean surface of a shell, $t_1=\omega_0 t,\ \omega_0=\sqrt{\frac{E}{(1-v^2)\rho_0 R^2}},\ q_z$ is pressure of liquid on a shell.

Linearized wave equation describing small perturtations propagation in ideal compressible liquid is of the form [2] :

$$\Delta\Phi-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}=0, \tag{2}$$

where Φ is potential, c is sound propagation velocity in liquid.

Motion equations of a shell (1) and liquid (2) are complemented by contact conditions. On contact surface "shell-liquid" we observe continuity of reedial velocities and pressures

$$\vartheta_r=\frac{\partial w}{\partial t},\ q_z=-P. \tag{3}$$

Hydrodynamic pressure P and radial velocity ϑ_r in liquid are determined as follows [2] :

$$P=-\rho\frac{\partial\Phi}{\partial t},\quad \vartheta_r=\frac{\partial\Phi}{\partial r}, \tag{4}$$

where ρ is liquid's density.

Complementing the motion (3) equations of shell and liquid by contact conditions we arrive at the problem on eigenvibrations of a liquid-filled stiffened shell. In other words, a problem on ligen vibrations of a liquid-filled stiffened cylindrical shell is reduced to joint integration of shell and liquid theory equations by fulfilling the indicated conditions on their contact surface.

Permutations of a shell will be sought in the form:

$$\begin{aligned} u&=u_0\sin\chi\xi\cos n\theta\cos\omega_1 t_1\\ \vartheta&=\vartheta_0\sin\chi\xi\cos n\theta\cos\omega_1 t_1\\ w&=w_0\sin\chi\xi\cos n\theta\cos\omega_1 t_1\end{aligned} \tag{5}$$

Here, $u_0,\ \vartheta_0,\ w_0$ are the unknown constants; $\chi,\ n$ ware numbers in transverse and peripheral directions, respectively.

We represent the velocity potential Φ in the form:

$$\Phi=A\cos\chi\xi J_n(\gamma r)\cos\theta\sin\omega_1 t_1 \tag{6}$$

where $\gamma^2=-\chi^2+\frac{\omega^2}{c^2},\ J_n$ is n-th order Bessel function of first kind, A is integration constant.

Using formulae (4) and contact conditions (3) for q_z we can get:

$$q_z=\frac{\rho E\omega_1 J_n(\gamma R)}{(1-v^2)\rho_0\gamma J_n'(\gamma R)}w_0\cos\chi\xi\cos\theta\cos\omega_1 t_1 \tag{7}$$

After substitution of (5) in (1), allowing for (7), the problem is reduced to homogeneous system of linear algebraic equations of third order

$$a_{i1}u_0 + a_{i2}v_0 + a_{i3}w_0 = 0 \qquad (i = 1, 2, 3)\,, \tag{8}$$

where

$$a_{11} = -\chi^2 - \frac{1-v}{2}n^2 + \omega_1^2 = \widetilde{a}_{11} + \omega_1^2;$$

$$a_{12} = \chi n\frac{1+v}{2};\ a_{13} = -v\chi, \quad a_{21} = \frac{1+v}{2}\chi n;$$

$$a_{22} = -\frac{1-v}{2}\left(1+4a^2\right)\chi^2 - \left[1 + \left(1-\frac{h_s}{R}\right)^2\gamma_s^{(2)} + a^2\right]n^2 +$$

$$\rho_2\omega_1^2 = \widetilde{a}_{22} + \rho_2\omega_1^2;$$

$$a_{23} = \left[1 + \left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]n + (2-v)\,a^2\chi^2 n -$$

$$-\left[a^2 - \left(1-\frac{h_s}{R}\right)\delta_s^{(2)}\right]n^3;\ a_{31} = v\chi;$$

$$a_{32} = -\left[1 + \left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]n - (2-v)\,a^2\chi^2 n - \left[a^2 - \left(1-\frac{h_s}{R}\right)\delta_s^{(2)}\right]n^3$$

$$a_{33} = 1 + \gamma_s^{(2)} + \eta_{s1}^{(2)} - 2\left(\delta_s^{(2)} + \eta_{s1}^{(2)}\right)n^2 + a^2\left(n^2+\chi^2\right)^2\left(\eta_{s1}^{(2)} + \eta_{s1}^{(2)}\right)n^4$$

$$+\eta_c^{(1)}\chi^4 - \rho_2\omega_1^2 - \chi^2\overline{p} - \frac{\omega_1^2\rho J_n(\gamma R)}{\rho_0 h J_n'(\gamma R)} =$$

$$= \widetilde{a}_{33} - \omega_1^2\varphi_1 - \chi^2\overline{p};\ \varphi_1 = \rho_2 + \frac{\rho J_n(\gamma R)}{\rho_0 h J_n'(\gamma R)}.$$

Non-trivial solution of the system of linear algebraic equations (8) is possible only if ω_1 is the root of its determinant. Determinant ω_1 is reduced to transcendental equation, since ω_1 is contained in the arguments of Bessel unction J_n :

$$\begin{vmatrix} \widetilde{a}_{11} + \omega_1^2 & a_{12} & a_{13} \\ a_{21} & \widetilde{a}_{22} + \rho_2\omega_1^2 & a_{23} \\ a_{31} & a_{32} & \widetilde{a}_{33} - \varphi_1\omega_1^2 - \chi^2\overline{p} \end{vmatrix} = 0 \tag{9}$$

We can write it in the form $\left(\omega_1^2 = \lambda\right)$:

$$a_1(\lambda)\,\lambda^3 + a_2(\lambda)\,\lambda^2 + a_3(\lambda)\,\lambda + a_4(\lambda) = 0 \tag{10}$$

The coefficients $a_i(\lambda)$ $(i = 1, 2, ..., 4)$ are of the form :

$$a_1(\lambda) = \rho_2\varphi_1;\quad a_2(\lambda) = \varphi_1\left(\rho_2\widetilde{a}_{11} + \widetilde{a}_{22}\right) - \left(\rho_2\widetilde{a}_{33} - \chi^2\overline{p}\right);$$

$$a_3(\lambda) = -\left(\widetilde{a}_{33} - \chi^2\overline{p}\right)\left(\rho_2\widetilde{a}_{11} + \widetilde{a}_{22}\right) +$$

$$+\varphi_1\widetilde{a}_{11}\widetilde{a}_{22} + \rho_2 a_{13}a_{31} + a_{32}a_{23} - \varphi_1 a_{21}a_{12};$$

$$a_4(\lambda) = \left(\widetilde{a}_{33} - \chi^2\overline{p}\right)\left(a_{21}a_{12} - \widetilde{a}_{11}\widetilde{a}_{22}\right) -$$

$$-a_{21}a_{32}a_{13} - a_{12}a_{23}a_{31} + a_{31}a_{13}\widetilde{a}_{22} + a_{32}a_{23}\widetilde{a}_{11}.$$

For the following analysis we should select parameters is equation (10) that essentially influence on the coefficients $a_i(\lambda)$ $(i = 1, 2, ..., 4)$. Dimensionless flexural stiffness of transversally stiffened ribs, $\eta_s^{(2)}$ ($\eta_s^{(2)}$ the greatest from the quantities $\eta_{s1}^{(2)}$, $\eta_{s2}^{(2)}$), a^2, n, χ, ω_1^2 and also dimensionless eccentricities of transversal ribs $\delta_s^{(2)}$ are such parameters for the considered structurally-orthotropic shell. Since $\delta_s^{(2)} \le \sqrt{\eta_s^{(2)}}$, by analyzing the order of coefficients $a_i(\lambda)$ we accept $\delta_s^{(2)} \approx \sqrt{\eta_s^{(2)}}$. Moreover, in order to simplify the frequency equation (10) we'll consider the areas of low frequencies of the considered system. Then, for great n $(x << n)$ we use formulae for logarithmic derivatives of Bessel function J_n [3]

$$\frac{J_n'(x)}{J_n(x)} \approx \frac{n}{x} - \frac{x}{2n} \tag{11}$$

Within the scheme of incompressibility of the liquid we retain only the first term in the expansion (11), and thus, relative error of computation in the vicinity of a shell will equal $O\left((x/n)^2\right)$. In order to carry out asymptotic analysis of frequency equation (10) we consider the area of low frequencies the considered system. Since in futire we'll be interested only in low frequencies of flexural vilerations, we can simplify this equation rejecting the addends with λ^2 and λ^3. As a result, we get:

$$\lambda = \frac{\left(\widetilde{a}_{33} - \chi^2\overline{p}\right)\left(a_{21}a_{12} - \widetilde{a}_{11}\widetilde{a}_{22}\right) -}{\left(\widetilde{a}_{33} - \chi^2\overline{p}\right)\left(\rho_2\widetilde{a}_{11} + \widetilde{a}_{22}\right) -}$$

$$\frac{-a_{21}a_{32}a_{13} - a_{12}a_{23}a_{31} + a_{31}a_{13}\widetilde{a}_{22} + a_{32}a_{23}\widetilde{a}_{11}}{-\varphi_1\widetilde{a}_{11}\widetilde{a}_{22} - \rho_2 a_{13}a_{31} - a_{32}a_{23} - \varphi_1 a_{21}a_{12}} \tag{12}$$

Formula (12) allows to calculate low frequencies of flexural vibrations of a transversally stiffened liquid-filled cylindrical shell loaded with axiall contracting forces. The analysis of this formula shows that frequencies of free vibrations of the considered system increases according to increase of contracting forces.

Reference

[1]. Amiro I.Ya., Zarutskii V.A. *Theory of ridge shell. Methods of shell analysis.* "Naukova Dumka", 1980, p. 367 (Russian).

[2]. Volmir A.S. *Shells in liquid and gas flow. Hydroelasticity problems.* Moscow, Nauka, 320 p. (Russian).

[3]. Latifov F. S. *Vibrations of a shell with elastic and liquid medium.* Baku, Elm, 1999, 164 p. (Russian).

Oktay Sh. Salmanov
Institute of Mathematics and Mechanics of NAS of Azerbaijan.
9, F. Agayev str., AZ1141, Baku, Azerbaijan.
Tel.: (99412) 439-47-20 (off.)

Received January 19, 2007; Revised April 10, 2007