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VIBRATION AND STABILITY OF A CONIC SHELL FLITTER IN SUPERSONIC GAS FLOW

Abstract

In the proposed paper in development of results of works [5-7], it is considered the problem of the flutter of truncated shell, there has been adduced data of evaluative computations, their comparison with similar ones obtained by the piston theory.

A new feature in he formulation of the problem resides in the fact that in the formula for the excess pressure, the item with the second mixed derivative of the shell flexure with the respect to the time and coordinate is held, the evaluation and qualitative analysis of the approximate solution have shown that taking into account the item can appreciably decline the flutter critical speed.

Introduction. In the papers on supersonic panel flutter of shells (see e.g. [1.2]) the piston theory is used for the pressure of aerodynamic intercoupling between gas flow and shell. Insufficiency of such an approach is discussed in the papers [4-6], and the results of these investigations are used in [6, 7] for new statements of problems on flutter of plates and conical shells. In the suggested paper as development of the results of [5-7] we consider a problem on a truncated conic shell flutter, cite data on evaluative computations and their comparison with similar ones obtained by the piston theory.

A new moment in the problem statement is that the formula for excess pressure contains the item with the second mixed derivative of the shell flexure by time and coordinate; evaluation and qualitative analysis of approximate solution show that taking into account this item may appreciably decrease the flutter critical speed.

1. Aerodynamic intercoupling pressure. Let's consider a circular cone streamlined by supersonic flow without angle of incidence. Origin of a Cartesian coordinate system is in the vertex, the axis x is directed along the velocity vector. In non-deformed state the equation forming $z_1 = kx$, $k = tg\alpha$, α is half-opening of a cone, $\alpha^2 << 1$. A part of the cone $x_1 \le x \le x_2$ is engaged by a shell; by $w(x,t)$ we denote its flexures in axially symmetric case, and have:

$$x_1 \le x \le x_2, \quad z = kx - w(x,t). \tag{1.1}$$

According to plane sections law [8, 9] the state of gas after shock wave is determined from the solution of a plane problem on piston, that moves by the law $\bar{z} = kvt - w(vt,t)$, where v is flow velocity. The solution of this problem by a small parameter method, [9], under additional suggestion $|w(x,t)/kx| << 1$ was obtained in [5], but it was realized for the case of a cone stream-line; below we carry out appropriate analysis.

For the excess pressure on a shell in [7] it was obtained the expression

$$\Delta p = \frac{2\rho^0 D^2}{\chi + 1}\left(1 + \frac{\varepsilon}{4} a(D) - \frac{\chi p^0 (\chi + 1)}{2\rho^0 D^2}\right) +$$

$$+\frac{4\rho^0 D^2}{\chi+1}\left(1+\frac{3\varepsilon}{4}-\varepsilon\frac{11a(D)}{8k}\right)\dot{w}-$$

$$-2\varepsilon\frac{\rho^0 D}{t}\left(1+\frac{5\chi+1}{\chi(\chi+1)}\frac{9a(D)}{8}\right)w-\frac{\rho^0 D}{2}\left(1-\varepsilon\frac{3a(D)}{2\chi(\chi+1)}\right)t\,\ddot{w}-$$

$$-2\varepsilon\frac{3-\chi}{\chi+1}\frac{\rho^0 D}{t^2}\int\limits_0^t w(\xi,t)\,dt-2\varepsilon\frac{1+\chi}{\chi}\frac{\rho^0 D}{t^4}\int\limits_0^t \xi^2 w(\xi,t)\,dt, \tag{1.2}$$

here ρ^0, p^0, α_0 is density, pressure and a sound velocity at the non-perturbed flow; k is a polytropic exponent; $\varepsilon=(\chi-1)/(k+1)$ shock wave velocity is found from the square equation. $\varepsilon Da(D)+2vtg\alpha=2D$, $a(D)=1+2a_0^2/\left((\chi-1)D^2\right)$ that after some denotation $u_0=Mtg\alpha$, $u=D/a_0=Mtg\beta$ is reduced to the following form

$$(3+\chi)u^2-2(\chi+1)u_0u-2=0. \tag{1.3}$$

By passing to the Euler system of coordinates it should be

$$x=vt,\ \dot{w}=\partial w/\partial t+v\partial w/\partial x.$$

Estimate the orders of addends in (1.2)

$$\varepsilon\rho^0 D\frac{w(vt,t)}{t}=\frac{w}{x}\varepsilon\rho^0 Dv=\frac{w}{kx}ke\rho^0 Dv$$

$$\varepsilon\rho^0 D\frac{1}{t^2}\int\limits_0^t w(\xi,t)\,d\xi=\varepsilon\rho^0 D\frac{w\left(\tilde{t},t\right)(t-t_1)}{t^2}=$$

$$=k\varepsilon\rho^0 Dv\,(x-x_1)\frac{w(\tilde{x},t)}{kx^2},\quad x_1\le\tilde{x}\le x;$$

$$\varepsilon\rho^0 D\frac{1}{t^4}\int\limits_0^t \xi w(\xi,t)\,d\xi=\frac{1}{3}\varepsilon\rho^0 D\frac{w\left(\tilde{\tilde{x}},t\right)\left(t^3-t_1^3\right)}{t^4}=$$

$$=\frac{1}{3}k\varepsilon\rho^0 Dv\frac{w\left(\tilde{\tilde{x}},t\right)}{kx}\left(1-\frac{x_1^3}{\tilde{\tilde{x}}^3}\right);\quad x_1\le\tilde{\tilde{x}}\le x;$$

The integrals are computed by the mean-value theorem. Since $[w/kx]<<1$ and $k\sim\varepsilon$ we conclude that the written out addends are of order ε^2 or higher and we can neglect them.

The excess pressure Δp (1.2) consists of a sum of quasistatic q_0 and dynamic q_1 components; assume $w=w_0(x)+w_1(x,t)$, then we get from (1.2)

$$q_0=\frac{2\rho^0 D^2}{\chi+1}H_{01}-\frac{4\rho^0 Dv}{\chi+1}H_1\frac{\partial w_0}{\partial x}-\frac{\rho^0 Dvx}{2}H_2\cdot\frac{\partial^2 w_0}{\partial x^2} \tag{1.4}$$

$$q_1=-\frac{4\rho^0 D}{\chi+1}H_1\left(\frac{\partial w_1}{\partial x}+v\frac{\partial w_1}{\partial x}\right)-$$

$$-\frac{\rho^0 Dx}{2v}H_2\cdot\left(\frac{\partial^2 w_1}{\partial t^2}+2v\frac{\partial^2 w_1}{\partial t\partial x}+v^2\frac{\partial^2 w_1}{\partial x^2}\right) \tag{1.5}$$

$$H_{01}=1+\frac{\varepsilon}{4}a\left(D\right)-\frac{\chi p^0\left(\chi+1\right)}{2\rho^0 D^2};\quad H_1=1+\frac{3\varepsilon}{4}-\frac{11\varepsilon}{8\chi}a\left(D\right);$$

$$H_2=1-\frac{3\varepsilon}{2\chi\left(\chi+1\right)}a\left(D\right) \tag{1.6}$$

Estimate the orders of quantities of addends in the second parenthesis of (1.5).

We compare "density" of apparent additional mass with mass of a shell in a surface unit

$$\frac{\rho^0 Dx}{2v}\frac{1}{\rho h}\sim\frac{\rho^0 kvl}{2\rho vh}=\frac{k\rho^0 l}{2\rho h},\quad l=x_2-x_1; \tag{1.7}$$

Compare the second addend with the third one

$$\left(2v\frac{\partial^2 w_1}{\partial t\partial x}\right):v^2\frac{\partial^2 w_1}{\partial x}\sim\frac{2vw_1}{t_0 l}\frac{l^2}{v^2 w_1}=\frac{2l}{t_0 v}\sim\frac{2lw_0}{v},$$

here $\omega_0\sim\pi c_0\sqrt{h/R}\zeta/l$, $c_0^2=E/\rho$, ζ is a coefficient of order of several units [12], E, ρ is a Young modulus and density of a shell material, R is a mean radius. Finally, $2l\omega_0/v\sim 2\pi\left(c_0/v\right)\zeta\sqrt{h/R}$; for ordinary values of parameters this relation will be a quantity of order of several units (the same result will be obtained if t_0-l/v). Relation (1.7) will be a quantity of order $10^{-1}\tilde{\ }10^{-2}$ and we can neglect them.

With regard to above-introduced denotation, the expressions for q_0, q_1 from (1.4), (1.5) will take the form

$$-q_0=\frac{2\chi p_0}{\chi+1}u^2H_0-\frac{4\chi p^0}{\chi+1}MuH_1\frac{\partial w_0}{\partial x^2}-\frac{\chi p^0}{2}MuxH_2\frac{\partial^2 w_0}{\partial x^2} \tag{1.8}$$

$$q_1=-\frac{4\chi p_0}{\left(\chi+1\right)a_0}uH_1\left(\frac{\partial w}{\partial t}+v\frac{\partial w_1}{\partial x}\right)-\frac{\chi p_0 ux}{a_0}H_2\frac{\partial^2 w_1}{\partial t\partial x}-$$

$$-\frac{\chi p_0}{2}M^2 tg\beta x\cdot H_2\cdot\frac{\partial^2 w_1}{dx^2}, \tag{1.9}$$

here we denote $H_0=1+\varepsilon a^*\left(u\right)/4-1/\left(2u^2\right)$, in expressions for H_1, H_2 we should replace $a\left(D\right)$ by $a^*\left(u\right)=1+2/\left(\left(\chi+1\right)u^2\right)$.

2. Problem statement. The state of the point on a conic surface is determined by the coordinates $s=x/\cos\alpha$, $\theta=\psi\sin\alpha$ where ψ is a polar angle. For small conicity $\alpha^2<<1$, therefore with good exactness $s=x$. Vibrations of the cone is described by the equations [10]

$$D_0^2\Delta w-\Delta_k F-L\left(w,F\right)=\Delta p-\rho h\frac{\partial^2 w}{\partial t^2}$$

$$2\Delta^2 F+2Eh\Delta_k w+L\left(w,w\right)=0. \tag{2.1}$$

Here $D_0=Eh^3/\left(12\left(1-\mu^2\right)\right)$ μ is a Poisson coefficient of a shell material, F is a stress function. In (2.1) the operators are of the form

$$L\left(u,v\right)=\left(\frac{1}{x}\frac{\partial u}{\partial x}+\frac{1}{x^2}\frac{\partial^2 u}{\partial\theta^2}\right)\frac{\partial^2 v}{\partial x^2}+\left(\frac{1}{x}\frac{\partial v}{\partial x}+\frac{1}{s^2}\frac{\partial^2 v}{\partial\theta^2}\right)\frac{\partial^2 u}{\partial x^2}-$$

$$-2\left(\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial u}{\partial \theta}\right)\right)\left(\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial u}{\partial \theta}\right)\right);$$

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{1}{x}\frac{\partial}{\partial x}+\frac{1}{x^2}\frac{\partial^2}{\partial \theta^2};\quad \Delta_k=\frac{1}{xtga}\frac{\partial^2}{\partial x^2}.$$

Similar to representation $w=w_0(x)+w_1(x,t)$ we assume $F=F_0(x)+F_1(x,t)$. Introduce dimensionless quantities keeping the previous denotation for them:

$$x\Longrightarrow x/x_2,\ w_0\Longrightarrow w_0/h,\quad F_0\Longrightarrow F_0/\left(Eh^2x_2\right),$$

$$w_1\Longrightarrow w_1/h_2\quad F_1\Longrightarrow F_1/\left(Eh^2x_2\right).$$

Substituting all in (2.1) we linearize by perturbations w_1, F_1 and make the obvious simplifications. For the principal state functions we get (the substitution $x=x_1+y$ is done).

$$\frac{tg\alpha}{12\left(1-\mu^2\right)}H^2\lambda_0^2\Delta^2 w_0-\frac{1}{x_1+y}\frac{\partial^2 F_0}{\partial y^2}+\frac{4\chi p^0}{2E}\cdot\frac{4\chi p^0}{2E}\frac{uu_0}{H\lambda_0}\frac{\partial w_0}{\partial y}+$$

$$+\frac{\chi p_0}{2E}\frac{uu_0\left(x_1+y\right)}{H\lambda_0}H_2\frac{\partial^2 w_0}{\partial y^2}=\frac{2\chi p^0}{(\chi+1)E}\frac{u^2tg\alpha}{H^2\lambda_0^2}H_0=q_0^*; \tag{2.2}$$

here we denote $h/x_2=(h/l)(l/x_2)\equiv H\lambda_0$.

We complete system (2.2) with Navier additional conditions [11]

$$y=0,\quad y=1-x_1: w_0=0,\quad \frac{\partial^2 w_0}{\partial y^2}=0,\quad F_0=0;\quad \frac{\partial^2 F_0}{\partial y^2}=0. \tag{2.3}$$

We assume perturbed state as axially symmetric and put $w_1=W(y)\exp(\omega t)$, $F_1=\Phi(y)\exp(\omega t)$; for the functions W,Φ from (2.1) we get the system:

$$tg\alpha\Delta^2\Phi+\frac{1}{x_1+y}\frac{\partial^2 W}{\partial y^2}=0$$

$$\frac{tg\alpha}{12\left(1-\mu^2\right)}H^2\lambda_0^2\Delta^2 W-\frac{1}{x_1+y}\frac{\partial^2\Phi}{\partial y^2}-\frac{tg\alpha\cdot H\lambda_0}{x_1+y}\frac{\partial F_0}{\partial y}\frac{\partial^2 W}{\partial y^2}-$$

$$\frac{tg\alpha H\lambda_0}{x_1+y}\frac{\partial^2 F_0}{\partial y^2}\frac{\partial W}{\partial y}+\frac{4\chi p_0uu_0}{(\chi+1)EH\lambda_0}H_1\frac{\partial W}{\partial y}+$$

$$+\frac{\chi p^0uu_0}{2EH\lambda_0}H_2\left(x_1+y\right)\frac{\partial^2 W}{\partial y^2}+\frac{tg\alpha\chi p_0}{EH\lambda_0}H_2\left(x_1+y\right)u\Omega\frac{\partial W}{\partial y}+$$

$$+\frac{4\chi p_0utg\alpha}{(\chi+1)EH\lambda_0}H_1\Omega W+\frac{tg\alpha a_0^2}{c_0^2}\Omega^2 W=0. \tag{2.4}$$

Dimensionless frequency is introduced by the relation $\Omega=l\omega/\alpha_0$; $c_0^2=E/\rho_j$, we accept conditions (2.3) as boundary conditions.

The obtained eigen-value problem essentially differs from flutter problems based on the piston theory for Δp by the fact that the second equation contains an item with derivative $\Omega\partial W/\partial y$ and thereby analysis of the problem is appreciably complicated.

3. Basic state. Assume $\omega_0 = K_0 \sin\delta\pi y,\ F_0 = N_0 \sin\delta\pi y,\ \delta(1-x_1) = 1$; substitute this in (2.2) and carry out ordinary projection procedure, and get

$$K_0\left[\frac{tg\alpha H^2\lambda_0^2}{12(1-\mu^2)}\left(\frac{(\delta\pi)^2}{2\delta}-(\delta\pi)J_1+J_2+\frac{1}{2\delta\pi}J_3\right)-A_2J_4\right]+$$

$$+N_0J_5=\frac{2}{(\delta\pi)^3}q_0^* \tag{3.1}$$

$$K_0J_5-N_0tg\alpha\left(\frac{(\delta\pi)^2}{2\delta}-(\delta\pi)J_1+J_2+\frac{1}{2\delta\pi}J_3\right)=0,$$

here we denote $A_2 = \chi p_0 u u_0 H_2/(2EH\lambda_0)$, the numbers J_k are integrals on the interval $[0, 1-x_1]$ from combinations of trigonometric and power functions $(x_1+y)^{\pm m}$, m are integers; on concrete values of integrals we'll speak in an example. From (3.1) we determine parameter N_0 that we'll need in future.

4. Perturbed state. We carry out qualitative analysis of system (2.4) on the basis of two-term approximation: $W = k_1\sin\delta\pi y + k_2\sin 2\delta\pi y$, $\Phi = N_1\sin\delta\pi y + N_2\sin 2\delta\pi y$. After the known procedure, from the first equation of (2.4) we get the system

$$N_1 tg\alpha\cdot S_{11}+N_2 tg\alpha\cdot S_{12}=J_5k_1+4J_6k_2$$

$$N_1 tg\alpha\cdot S_{21}+N_2 tg\alpha\cdot S_{22}=J_{10}k_1+4J_{11}k_2 \tag{4.1}$$

the matrix S_{ij} is expressed by the integrals J_k, about which we spoke at the end of the previous item. Write solution (4.1) in the form $N_i = \alpha_{is}k_s$, the concrete form of the matrix $\{\alpha_{is}\}$ will be given below. Substitute this solution into the second equation of (2.4); after Bubnov-Galerkin procedure we get a system of homogeneous equations with determinant

$$\Delta=\begin{vmatrix} a_{11}+\beta_{11}\Omega+\gamma\Omega^2 & a_{12}+\beta_{12}\Omega \\ a_{21}+\beta_{21}\Omega & a_{22}+\beta_{22}\Omega+\gamma\Omega^2\end{vmatrix}. \tag{4.2}$$

In the sequel, we'll need the values of the parameters B_{ij} and γ; write them out:

$$\beta_{11}=\frac{2A_2}{M\delta\pi}J_{27}+\frac{A_1}{2\delta^3\pi^2};\quad \beta_{22}=\frac{4A_2}{M\delta\pi}J_{26}+\frac{A_1}{2\delta^3\pi^2};$$

$$\beta_{12}=\frac{2A_2}{M\delta\pi}J_{28};\quad \beta_{21}=\frac{4A_2}{M\delta\pi}J_{25};\quad \gamma=\frac{1}{2\delta^3\pi^2}\frac{tg\alpha a_0^2}{c_0^2}; \tag{4.3}$$

$$J_{25}=\int\limits_0^{1-x_1}(x_1+y)\cos\delta\pi y\sin 2\delta\pi y dy;\quad J_{26}=\frac{1}{2}\int\limits_0^{1-x_1}(x_1+y)\sin 4\delta\pi y dy;$$

$$J_{27}=\int\limits_0^{1-x_1}(x_1+y)\sin 2\delta\pi y dy;\quad J_{28}=\int\limits_0^{1-x_1}(x_1+y)\cos 2\delta\pi y\sin\delta\pi y dy;$$

here we denote $A_1 = \dfrac{4kp^0utg\alpha \cdot H_1}{(k+1)\,EH\lambda_0}$.

The following integrals are easily calculated

$$J_{25} = \frac{2}{3\delta\pi}\left(2x_1 + \frac{1}{\delta}\right) = 2J_{28}; \quad J_{27} = -\frac{1}{8\delta^2\pi} = 2J_{26}$$

therefore $\beta_{11} = \beta_{22} = \beta_0$; $\beta_{21} = -\beta_{12} = \beta_1$.

Vibrations are stable, if $\operatorname{Re}\Omega < 0$, otherwise they are unstable, boundaries of stability and unstability domains and respectively, critical combination of parameters are determined by the condition $\operatorname{Re}\Omega = 0$.

Assume $\Omega = i\Omega_0$, $\Omega_0 \neq 0$ and substitute it in (4.2) and as a result we get the system:

$$2\beta_0\gamma\Omega_0^2 = \beta_0\,(a_{22} + a_{11}) + (a_{21} - a_{12})\,\beta_1.$$

$$a_{22}a_{11} - a_{21}a_{12} + \gamma^2\Omega_0^4 - \Omega_0^2\left[\gamma\,(a_{22} + a_{11}) + \beta_0^2 + \beta_1^2\right] = 0$$

From the first equation we define vibration frequency in critical condition

$$\Omega_0^2 = \frac{a_{22} + a_{11}}{2\gamma} + \frac{a_{21} - a_{12}}{2\gamma\beta_0}\beta_1 \qquad (4.4)$$

after substitution (4.3) in the second equation it takes the form

$$-a_{21}a_{12} + \frac{(a_{21} - a_{12})^2}{4\beta_0^2} \qquad \beta_1^2 = \frac{a_{22} + a_{11}}{2\gamma}\left(\beta_0^2 + \beta_1^2\right) +$$

$$+\frac{(a_{22} - a_{11})^2}{4} + \frac{a_{21} - a_{12}}{2\gamma\beta_0}\left(\beta_0^2 + \beta_1^2\right)\beta_1.$$

5. Example and some estimations. For an example we take the values of parameters: $p^0/E \sim 10^{-5}$; $tg\alpha = 0,17$ $\left(\alpha \cong \pi/18 = 10^0\right)$;$H = 3 \cdot 10^{-3}$;$\lambda_0 = 0,5$; $x_1 = 0,5$; $\delta = 2$; by estimating the orders of quantities we'll assume $H_0 \sim H_1 \sim H_2 \sim 1$. From system (3.1) we find $N_0 \cong -2 \cdot 10^{-2}$: the matrix elements a_{ij} are equal (later we'll omit the approximate equality sign): $a_{11} = 0,2$; $a_{12} = -10^{-2}$; $a_{21} = 10^{-2}$; $a_{22} = 8 \cdot 10^{-2}$. In the determinant of (4.2) the elements a_{11}, a_{21}, a_{22} are positive, the element a_{12} is negative. The numbers a_{ij} have the structure: $a_{ij} = a_{ij}^{(1)} + a_{ij}^{(2)}\,(M)\,M^2$ moreover $a_{ij}^{(2)}$, weakly depends on M. In future we'll need the order of relation $\beta_0\beta_1/\gamma$; the calculations give $\beta_0\beta_1/\gamma \sim 10^{-3}M^2$.

A new essential element of the problem statement is the taking into account the item with the second mixed derivative of the flexure in time and coordinate; in expressions (4.4) and (4.5) the parameter β_1 corresponds to it.

The first qualitative result obviously follows from (4.4) increase of vibration frequency in critical condition; this increase may be appreciable, since as estimations show, $\beta_1/\beta_0 \sim 10$.

From the quation (4.5) that for the above-given parameters serves to determine flutter critical velocity implies that $M_{kp}\,(\beta_1 \neq 0) < M_{kp}\,(\beta_1 = 0)$. Indeed, $a_{22}^{(1)} \sim a_{11}^{(1)} \sim 10^{-1}$, $\left|a_{12}^{(1)}\right| \sim a_{21}^{(1)} \sim 10^{-2}$, $\beta_0^2/\gamma \sim 10^{-5}M^2$. Therefore, the left hand side of equality (4.5) appreciable increases for $\beta \neq 0$ in comparison with the case $\beta_1 \neq 0$, and the curve increases in a smaller degree, since the last item is small in comparison with other ones. This conclusion is the most essential, since earlier in the papers [12, 13, 14] it was shown that taking into account the second derivative of the flexure with respect to coordinate in (1.9) leads to decrease of flutter critical velocity, as well.

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