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GEOMETRICALLY NONLINEAR AEROELASTIC VIBRATIONS AND STABILITY OF A CYLINDRICAL SHELL

Abstract

In the paper we consider the cylindrical shell flatter problem allowing for geometrical nonlinearuty and influence of the stationary gas surrounding the shell in the statement of [3].

Vibrations and stability of a cylindrical shell with supersonic velocity gas flow were considered in geometrically linear statement and on the basis of piston theory formula in [1,2]. In the paper [3] the refined statement of the problem is given and shown that "piston" formula should be completed by an addend that is equivalent by its meaning to interaction of a shell with Winkler basis. In this paper we consider a problem on cylindrical shell flatter in the statement of [3] allowing for geometrical nonlinearity and influence of stationary gas surrounding the shell.

1. Problem statement

Represent a cylindrical surface $r = R$, $|X| < \infty$; its part $0 \le X \le L$ is occupied by a thin elastic shell, the remaining part of the surface is assumed to be rigid. Inside this construction gas flows in positive direction of the axis X, whose unperturbed parameters are denoted by ρ_0, p_0, a_0, u_0 that is density, pressure, sound velocity, flow velocity, respectively, moreover $\chi_0 p_0 = \rho_0 a_0^2$, where χ_0 is a polytropic index. The flow is supersonic, therefore $M = u_0/a_0 > 1$. Stationary gas with parameters ρ_1, p_1, a_1, χ_1 is in the domain $r \ge R$.

In the paper [3] for internal flow we have obtained formula for aerodynamic interaction pressure

$$\Delta q_0 = -\frac{\chi_0 p_0}{a_0}\left(\frac{\partial w}{\partial t} + u_0\frac{\partial w}{\partial x} + \frac{a_0}{2R}w\right) \tag{1.1}$$

We'll assume it valid for interaction of a shell with external immobile gas as well

$$\Delta q_1 = -\frac{\chi_1 p_1}{a_1}\left(\frac{\partial w}{\partial t} + \frac{a_1}{2R}w\right) \tag{1.2}$$

so that total pressure will equal $\Delta q = \Delta q_0 + \Delta q_1$.

Motion of the shell is described by the known system of equation [4] for deflections w and stress functions Φ:

$$D\Delta^2 w = h\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \Phi}{\partial y^2} + h\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \Phi}{\partial x^2} - 2h\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 \Phi}{\partial x\partial y} + \frac{h}{R}\frac{\partial^2 \Phi}{\partial x^2} + q \tag{1.3}$$

$$\frac{1}{E}\Delta^2\Phi = \left(\frac{\partial^2 w}{\partial w\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2} \tag{1.4}$$

here $D = Eh^3/\left(12\left(1-v^2\right)\right)$ is cylindrical rigidity, h, E and v are shell's thickness, Young modulus and Poisson coefficient of its material. Pressure q is formed of inertial forces $\rho h\partial^2 w/\partial t^2$, aerodynamic interaction Δq and statical pressure drop

$\Delta p = p_1 - p_0$. We represent the function w and Φ in the form of the sum of quasistatic and dynamic constituents: $w = w_0(x) + w_1(x,y,t)$, $\Phi = \Phi_0(x,y) + \Phi_1(x,y,t)$, $y = R\theta$, θ is a polar angle of cylindrical system of coordinates. We substitute this in (1.3), (1.4). For quasistatic state we get

$$\begin{array}{c} Dw_0^{IV} = h\sigma_{xx}^0 w_0^{II} + \frac{h}{R}\sigma_{yy}^0 + \Delta p \\ \frac{1}{E}\Phi_0^{IV} = -\frac{1}{R}w_0^{II} \end{array} \tag{1.5}$$

For dynamic state we'll have (in future we'll omit the lower index)

$$D\Delta^2 w - h\sigma_{xx}^0 \frac{\partial^2 w}{\partial x^2} - h\sigma_{yy}^0 \frac{\partial^2 w}{\partial y^2} - \frac{h}{R}\frac{\partial^2 \Phi}{\partial x^2} - \Delta q = 0 \tag{1.6}$$

$$\Delta^2 \Phi + Ew_0'' \frac{\partial^2 w}{\partial y^2} + \frac{E}{R}\frac{\partial^2 w}{\partial x^2} = 0. \tag{1.7}$$

We'll assume the ends of the shell $x = 0$, $x = l$ to be simply-supported.

We represent the solution of system (1.6), (1.7) in the form of $w = W(x)\exp(\omega t)\cos n\theta$, $\Phi = F(x)\exp(\omega t)\cos n\theta$; after substitution we get

$$D\Delta_n^2 W - h\sigma_{xx}^0 W'' + h\sigma_{yy}^0 \frac{n^2}{R^2} W - \frac{h}{R}F'' - \Delta q(x) = 0 \tag{1.8}$$

$$\Delta_n^2 F - Ew_0'' \frac{n^2}{R^2} W + \frac{E}{R} W'' = 0 \tag{1.9}$$

here we denote

$$\Delta q(x) = -\omega \frac{\chi_0 p_0}{a_0}\left(1 + \frac{\chi_1 p_1 a_0}{\chi_0 p_0 a_1}\right) W -$$

$$-\frac{\chi_0 p_0}{2R}\left(1 + \frac{\chi_1 p_1}{\chi_0 p_0}\right) W - \chi_0 p_0 M W' - gh\omega^2 W \tag{1.10}$$

$$\Delta_n^2 = \frac{\partial^4}{\partial x^4} - \frac{n^2}{R^2}\frac{\partial^2}{\partial x^2} + \frac{n^4}{R^4}.$$

We introduce dimensionless coordinate $x' = x/l$, remain for it previous denotation, and dimensionless frequency $\Omega = l\omega/a_0$;

Substitute (1.10) into (1.8), (1.9) and get

$$\Delta_n^2 W - B_2 W'' + B_1 M W' + B_0 W - A_0 F'' - \lambda W = 0 \tag{1.11}$$

$$\Delta_0^2 F + D_2 W'' - D_0 w_0'' W = 0; \quad \Delta_n^2 = \frac{\partial^4}{\partial x^4} - 2n^2\alpha^2 \frac{\partial^2}{\partial x^2} + \alpha^4 n^4; \ \alpha = \frac{l}{R}$$

here we introduce the denotation

$$B_0 \frac{\sigma_{yy}^0 n^2 \alpha^2 l^2 h}{D} + \frac{\chi_0 p_0 l^3 \alpha}{2D}\left(1 + \frac{\chi_1 p_1}{\chi_0 p_0}\right); \quad B_1 = \frac{\chi_0 p_0 l^3}{D};$$

$$B_2 = \frac{h\sigma_{xx}^0 l}{D}; \quad A_0 = \frac{h\alpha l}{D}; \ A_1\Omega + A_2\Omega_\lambda^2 + \lambda = 0$$

$$A_1 = \frac{\chi_0 p_0 l^3}{D}\left(1 + \frac{\chi_1 p_1 \alpha_0}{\chi_0 p_0 a_1}\right); \ A_2 = \frac{ghl^2 a_0^2}{D}$$

$$D_0 = En^2\alpha^2; \quad D_2 = E\alpha l.$$

System (1.1) together with boundary conditions compose eigen-values problem. Vibrations are stable, if $\operatorname{Re}\lambda < 0$, unstable if $\operatorname{Re}\lambda > 0$. The boundary of domains $\operatorname{Re}\lambda = 0$ determines the critical values of parameters; in complex plane λ this condition means that λ lies on the stability parabola $A_2(Jm\lambda)^2 = A_1^2 \operatorname{Re}\lambda$.

2. Statical state

From equations of forces equilibrium in median surface we have $\sigma_{xx}^0 = \sigma_1 = const$, $\sigma_{yy}^0 = \sigma_2 = \sigma_2(x)$. We find from the second equation of (1.5) that $\sigma_2 = -(E/R)\,w^0 + b_0$, $b_0 - const$. On the other hand, from Hook's law we get

$$\sigma_1 = \frac{E}{1-v^2}\left(\frac{\partial u_0}{\partial x} + \frac{1}{2}w_0'^2 - \frac{v}{R}w_0\right) \tag{2.1}$$

$$\sigma_2 = \frac{E}{1-v^2}\left(v\frac{\partial u_0}{\partial x} - \frac{w_0}{R} + \frac{v}{2}w_0'^2\right) \tag{2.2}$$

The ends of the shell don't converge therefore $\int\limits_0^l (\partial u_0/\partial x)\,dx = 0$ from (2.1) we get σ_1 and $\partial u_0/\partial x$

$$\sigma_1 = \frac{E}{l\left(1-v^2\right)}\int\limits_0^l \left(\frac{1}{2}w_0'^2 - \frac{v}{R}w_0\right)dx \tag{2.3}$$

$$\frac{\partial u_0}{\partial x} = \frac{1-v^2}{E}\sigma_1 - \frac{1}{2}w_0'^2 + \frac{v}{R}w_0$$

now from (2.2) we have:

$$\sigma_2 = -Ew_0/R + v\sigma_1.$$

Assume $w_0 = hC_0\sin(\pi x/l)$; calculations by formula (2.3), (2.4) give

$$\sigma_1 = \frac{EC_0h}{\left(1-v^2\right)l}\left(\frac{\pi^2C_0h}{4l} - \frac{2v\alpha}{\pi}\right)$$

$$\sigma_2 = \frac{EhC_0\alpha}{l}\sin\frac{\pi x}{l} + \frac{vEC_0h}{\left(1-v^2\right)l}\left(\frac{\pi^2C_0h}{4l} - \frac{2v\alpha}{\pi}\right).$$

Substitute the necessary expressions into the first of (1.5) and carry out Bubnov-Galerkin procedure; we get an equation to determine parameter C_0:

$$C_0\left(1 + \frac{12\left(1-v^2\right)\alpha^2l^2}{\pi^4h^2}\right) + \sigma_1\frac{12\left(1-v^2\right)l^2}{E\pi^4h^2}\left(C_0 - \frac{4v\alpha l}{\pi h}\right) -$$

$$-\Delta p\frac{4l^4}{D\pi^5h} = 0.$$

3. Flatter problem.

Represent the solution by two-term approximations:

$$W_1 = C_1\sin\pi x + C_2\sin 2\pi x; \quad F = H_1^*\sin\pi x + H_2^*\sin 2\pi\alpha.$$

Substitute into the second of (1.11), after Bubnov-Galerkin procedure we find the coefficients H_1^*, H_2^*:

$$H_1^* = \frac{\pi^2C_1}{\left(n^2\alpha^2+\pi^2\right)^2}\left(D_2 - \frac{8}{3\pi}D_0C_0h\right) = H_1C_1;$$

$$H_2^* = \frac{4\pi^2 C_2}{(n^2\alpha^2 + 4\pi^2)^2}\left(D_2 - \frac{2}{15\pi}D_0C_0h\right) = H_2C_2.$$

From the first equation of (1.11) we get similarly a homogeneous system with respect to C_1, C_2:

$$(A_{11} - \lambda)\, C_1 + \frac{8}{3}B_1MC_2 = 0; \quad -\frac{8}{3}B_1MC_1 + (A_{22} - \lambda)\, C_2 = 0 \tag{3.1}$$

here we denote

$$\begin{aligned} A_{11}\left(\alpha^2n^2 + \pi^2\right)^2 + \pi^2B_2 + B_0 + \pi^2A_0H_1 \\ A_{22}\left(\alpha^2n^2 + 4\pi^2\right)^2 + 4\pi^2B_2 + B_0 + 4\pi^2A_0H_1. \end{aligned} \tag{3.2}$$

Characteristic equation of system (3.1) is of the form and solution

$$\lambda^2 - \left((A_{22} + A_{11})\,\lambda + A_{22}A_{11} + \left(\frac{8}{3}B_1M\right)\right)^2 = 0 \tag{3.3}$$

Carry out qualitative analysis of the result. For
$M \leq M_0^3 = 3\,(A_{22} - A_{11})\,/\,(16B_1)$ eigen-values (3.3) are real and positive, vibrations are stable. For $M > M_0$ from stability parabola equation we find:

$$\left(\frac{16}{3}M_{4p}\right) = \frac{(A_{22} + A_{11})\,2A_1}{A_2} + (A_{22} - A_{11})^2$$

We form the difference
$A_{22} - A_{11} = 3\pi^2\left(5\pi^2 + 2\pi^2\alpha^2\right) + 3\pi^2B_2 + \pi^2A_0\,(4H_2 - H_1)$.
Simple estimates show that $|4H_2 - H << |B||$ and the sign is defined by the stress sign σ_1. Hence it follows that at excessive external pressure the velocity M_0 will be less than in the case $\Delta p = 0$ (linear statement). The same estimate relates to the sum $A_{22} + A_{11}$, therefore we conclude: excessive pressure $\Delta p > 0$ leads to decreasing critical velocity of flatter.

References

[1]. Bolotin V.V. *Nonconservative problems of elastic stability theory.* // M., Fizmatgiz, 1961, 340 p. (Russian)

[2]. Novichkov Yu.N. *Flatter of plates and shells* // Mekh. deform. Tverdogo tela (Itogi nauki i tekhniki, VINITI), v.1.1, M., 1978, pp.67-122. (Russian)

[3]. Najafov M.A. *Statement of a conical shell flutter problem* // Proceedings of IMM of National Academy of Azerbaijan, v.XXIV (XXXII), pp.211-216, Baku, 2006.

[4]. Grigolyuk E.I., Kabanov V.V. *Stability of shells.* // M., Nauka, 1978, 369 p. (Russian)

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