MECHANICS

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CALCULATIONS OF DILATANTIONAL PLASTIC FAILURES AROUND RUNNING WELL

Abstract

Self-model solution of the problem on sand recovery at bringing open bottom hole well in elastico-plastically failed formation is constructed by numerical method.

Development of elastico-plastic radius, porosity variation, reservoir pressure and effective stresses distribution along the formation are studied. Sand showings in fluid flow is determined by initial adhesion and internal friction, ratio of production rate of fluid and solid masses is determined both by decreasing adhesion of failed part of the plate and alternative dilatant coefficient.

1. Let a plastic zone of a round formation be localized near the bottom hole of central well and matrix flow is caused by a linear fluid drain engaged at the moment $t = 0$ with constant intensities, i.e. we'll give the conditions for intensity of linear drain in a small radius $a(t)$ of conditional well

$$\begin{aligned} Q_f &= -2\pi ahmw = const, \\ Q_s &= -2\pi ah(1-m)\,v = const. \end{aligned} \tag{1.1}$$

Rest conditions are fulfilled at infinity $r/a \to \infty$ and all the parameters of the formation equal initial values independent of the coordinate

$$p = p_0, \quad \sigma^f_{rr} = -(\Gamma - p_0), \quad m = m^0_e. \tag{1.2}$$

At numerical realization we'll give these conditions in sufficiently great radius that corresponds to real radius of power circuit that grows proportional to $\sqrt{t}$.

The well is uncased and therefore radial effective stress on its wall equals zero

$$\sigma^f_{rr} = 0 \Longrightarrow p = p_a \quad at \quad r = a(t). \tag{1.3}$$

We'll determine radius of conditional well from the condition of vanishing effective stress (1.3).

We assume that intensity of fluid withdrawal corresponds to realization of plane deformation of formation matrix and radius of elastico-plastic zone $b(t)$ and well radius $a(t)$ will be transposed to external area of the well. At such statement we have to solve the problem due to time dependence.

Conditions of equality of pressure and radial stresses values and also velocities of solid and liquid particles belonging to internal plastic and external elastic sides

$$p_e = p_n, \quad \sigma_{rr}^{(e)} = \sigma_{ee}^{(n)}, \quad \upsilon_e = \upsilon_n, \quad w_e = w_n. \tag{1.4}$$

should be fulfilled on the boundary $b(t)$.

Notice that radius $b(t)$ is found from Mohr-Coulomb yield condition [1] that is satisfied on its both hand-sides, moreover, here all operating stresses in formation, both initial and generated by bringing of the well should be taken into account.

In external elastic domain $b(t) \le r \le R(t)$ balance of forces operating in liquid and solid phases is of the form [1,2]

$$\frac{\partial p_e}{\partial r} = -\frac{\mu}{k_e} m_e \left(w_e - \upsilon_e \right) \tag{1.5}$$

$$\frac{\partial \sigma_{rr}^{f(e)}}{\partial r} + \frac{\sigma_{rr}^{f(e)} - \sigma_{\theta\theta}^{f(e)}}{r} - \frac{\partial p_e}{\partial r} = 0, \tag{1.6}$$

Mass preservation equation for both phases

$$\frac{\partial m_e \rho_f}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \rho_f w_e \right) = 0, \tag{1.7}$$

$$\frac{\partial \left(1 - m_e\right) \rho_s}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \left(1 - m_e\right) \rho_s \upsilon_e \right) = 0, \tag{1.8}$$

Hook's generalized law for plane deformation

$$\begin{aligned} \sigma_{rr}^{f(e)} &= (K+G) \frac{\partial u}{\partial r} + (K-G) \frac{u}{r} + \varepsilon \rho_e, \\ \sigma_{\theta\theta}^{f(e)} &= (K+G) \frac{\partial u}{\partial r} + (K-G) \frac{u}{r} + \varepsilon \rho_e, \end{aligned} \tag{1.9}$$

where u is radial displacement of solid particles of formation, w_e is real velocity of fluid, μ is its viscosity, p_e is formation pressure, $\varepsilon = \beta_1 K$ is cementation factor of formation matrix, G is shear modulus, K is volume compression factor, m_e, k_e are porosity and permeability factors of formation, respectively, ρ_f, ρ_s are phase densities.

In internal plastic zone $a(t) \le r \le b(t)$ we write the same equations (1.5)-(1.8), but Hook's law is replaced by kinematical dilatation condition and plastic flow [3-5]

$$\frac{\partial \upsilon_p}{\partial r} + n \frac{\upsilon_p}{r} = 0, \quad N_n \sigma_{\sigma}^{f(n)} - \sigma_{\theta\theta}^{f(n)} = K_n, \tag{1.10}$$

where $n = 1 + \dfrac{2\Lambda \left(3\Lambda + \sqrt{3\left(3 - \Lambda^2\right)} \right)}{3 - 4\Lambda^2}$.

Here Λ is dilatation velocity that for $\Lambda > 0$ characterizes loosening of dense matrix of formation, for $\Lambda < 0$ - compaction of loose matrix, and for $\Lambda = 0$ - incompressibility of rock

$$K_n = \frac{2Y_n \sin\varphi}{\alpha\left(1-\sin\varphi\right)}, \quad N_0 = \frac{1+\sin\varphi}{1-\sin\varphi},$$

α and Y_n are internal friction and adhesion factors, respectively, φ is internal friction angle.

2. Now let's introduce new variables $W = W/t,\ V = v/t,\ U = u/\sqrt{t}$ to the functions of self-model variable $\xi = r/\sqrt{t}$. Then

$$\frac{\partial}{\partial r} = \frac{1}{\sqrt{t}}\frac{d}{d\xi}, \quad \frac{\partial}{\partial t} = -\frac{1}{2t}\xi\frac{d}{d\xi}, \quad V = -\frac{1}{2}\xi\frac{dU}{d\xi} + \frac{1}{2}U. \tag{2.1}$$

If we ignore phase compressibility, system of equations (1.8)-(1.9) for elastic domain $(\xi_b \le \xi) \le \xi_R$ in self-model variables will take the form

$$\frac{dp_e}{d\xi} = -\frac{\mu}{k_e m_e}\left(W_e - V_e\right), \tag{2.2}$$

$$\frac{dV_e}{d\xi} = \frac{\mu\left(1-\varepsilon\right)\xi}{2k_e\left(K+G\right)}m_e\left(W_e - V_e\right) - \frac{V_e}{\xi}, \tag{2.3}$$

$$\frac{dW_e}{d\xi} = \frac{\mu\left(1-\varepsilon\right)\left(m_e-1\right)\xi\left(\xi-2W_e\right)}{2k_e\left(K+G\right)\left(\xi-2W_e\right)}\left(W_e - V_e\right) - \frac{W_e}{\xi}, \tag{2.4}$$

$$\frac{dm_e}{d\xi} = \frac{\mu\left(1-\varepsilon\right)\left(m_e-1\right)m_e}{k_e\left(K+G\right)\left(\xi-2V_e\right)}\xi\left(W_e - V_e\right), \tag{2.5}$$

$$\frac{d\sigma_{rr}^{f(e)}}{d\xi} = 4G\frac{V_e}{\xi^2} - \frac{\mu}{k_e}m_e\left(W_e - V_e\right). \tag{2.6}$$

Interior to plastic zone $(\xi_a \le \xi \le \xi_b)$ the first equation of (1.10) and balance equation (1.6) with the help of relation (1.10) in self-model variable have the forms

$$\frac{dV_n}{d\xi} = -n\frac{V_n}{\xi}, \tag{2.7}$$

$$\frac{d\sigma_{rr}^{(n)}}{d\xi} = -\frac{\sigma_{rr}^{f(n)}\left(1-N_n\right)+K_n}{\xi} - \frac{\mu m_n}{k_n}\left(W_n - V_n\right). \tag{2.8}$$

However, Darcy law (2.2) for moving matrix, phase continuity equations (2.4) and (2.5) do not change their form. Interior to elastic and plastic zones system of equations (2.2)-(2.6) and (2.2), (2.4), (2.5), (2.7), (2.8) with respect to variables p, V, m, W, σ_σ^f are close. Boundary conditions (1.1)-(1.4) will correspond to stationary solution in the space ξ

$$Q_f = -2\pi h\xi_a m_a W_a = const,$$
$$Q_s = -2\pi h\xi_a (1-m_a) V_a = const. \tag{2.9}$$

In this space initial rest conditions (1.2) and conditions on infinity $\xi \to \infty$ coincide. At self-model statement of the problem such a circumstance excludes initial inhomogeneous stress fields.

On growing elasto-plastic boundary $\xi_b = b/\sqrt{t}$ elastic effective stresses should satisfy the yield condition (1.10)

$$\sigma_{rr}^{f(e)} = \frac{1}{N_e - 1}\left(4G\frac{V_e}{\xi} + K_e\right), \tag{2.10}$$

and hence we find the boundary ξ. In calculations it was assumed that (1.2) are satisfied for $R/\sqrt{t} = \xi_R = const = 1$ that corresponds to real radius of power circuit growing proportional to $\sqrt{t}$. Notice that yield condition (1.10) was checked at each step of calculations, and velocity of solid particles were chosen so that the condition $\sigma_a^f = 0$ was satisfied on hole well. The considered problem was solved by Runge-Kutt [6] numerical method. After determining p_e from (2.2), V_e, m_e, W_e, $\sigma_{rr}^{f(e)}$ and $\sigma_{\theta\theta}^{f(e)}$ were found sequentially in elastic domain. Then the problem was solved in plastic zone: p_n, V_n, m_n, W_n, $\sigma_{rr}^{f(n)}$ and $\sigma_{\theta\theta}^{f(n)}$.

The solution of corresponding problem has practical sense only for $a(t) \le r_c$ where r_c is a real radius of the well. Fluid and sand production rates were determined according to velocities of filtration and displacement of failed matrix in plastic zone extended as a result of dilatation loosening. For simplicity of calculations we assume that because of occurred failure the value of K_e equals adhesion in a plastic zone.

Carry out calculations for the following data:

$k_0 = 1.0l - 13\ m^2$, $p_0 = 5\ MPa$; $p_R = 16\ MPa$; $G_f = 7.0l - 03\ m^3/sek$; $G_s = 1.8l - 05\ m^3/sek$; $K_e = K_n = 2.4\ MPa$; $\sigma_{rr0}^f = -20\ MPa$; $\beta_1 = 1.0l - 05\ (MPa)^{-1}$; $\mu = 1.0l - 03\ Pa \cdot sek$; $m_0 = 0.3$; $v = 0.25$; $n = 1.1$; $N_e = N_n = 3$.

Dependencies of desired parameters on dimentionless parameter $z = \xi/\xi_n$ in logarithmic coordinate are described in figures 1-5. in this coordinate pressure distribution is subjected almost to linear law (fig.1). By approaching a well the porosity in elastic zone decreases insignificantly, jumps elastico-plastic boundary. In plastic zone the porosity value increased as a result of loosening of formation rock decreases because of compressibility of a matrix at steam pressure drop (fig.2). Distribution of velocities of liquid and solid phases is given in fig.3 and 4. Absolute value of dimensionless effective radial stress $\left(\sigma_{rr}^f = \sigma_{rr}^f / K_e\right)$ continuously increases, however, circular stress $\left(\bar{\sigma}_{\theta\theta}^f = \sigma_{\theta\theta}^f / K_e\right)$ in the boundary jumps to elastic zone (fig.5)

Conclusion. In exact statement a problem on extension of plastic zone at non-associated law of liquid saturated formation around development well is formulated. Step-wise variation of porosity on interface of plastic and elastic zones holds for the permeability factor as well and this is expressed in its turn by porosity. When porosity of both zones are same on this boundary, the similar problem was studied in [7].

Theoretical basis of origin and development of elasticoplastic zones around development well, intensity of solid fragments production at stationary filtration was studied in [4.5]. It is shown that around the well these may arise up to three plastic zones, and the radius of external plastic zone is independent of parameters of external and intermediate plastic zones.

Formation failure around a well reduces to subsidence of free surface of deposit field. Such calculations requires analysis of problems considering vertical shifts [4].

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Received September 04, 2006; Revised October 30, 2006.