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MATHEMATICAL MODELS OF TRANSPORT SYSTEMS WITHOUT OVERTAKING

Abstract

Mathematical models of moving particles describing behaviour of transport systems are constructed. It is shown that when the number of particles is more than three, fact on random binomial walk of separately considered particle doesn't hold.

1. Introduction

Recently investigation of mathematical models of moving particles is of great interest. It is connected with the fact that on the one hand such models are widely used in different applications, and on the other hand behaviour of complex models may be studied by means of simulation in computers. Applications of such models are transport problems that are described by behaviour of moving particles both on straight and close contours, quening systems and others. Unexpected effects arising in transport system such as traffic jam, road capacity and etc. may be revealed in such models.

In [1] it is constructed a simplified mathematical model of moving particles on a straight line without overtaking where unexpected effect-random binomial walk of separately considered particle is revealed, that in its turn admits to explain origin of traffic jam in transport systems depending on intensity of traffic flow. In [2] this result was generalized for the case of movement of many particles on a straight line and boundary values of intensity of moving particles at which traffic jams arise, are found.

Mathematical models of moving particles constructed in [1,2] were used in [3] by planning roads and tranffic flows in Moscow.

Investigation of moving particles without overtaking across close trajectories leads to complex mathematical models that are widely used in queuing systems [4] and other applications. Mathematical models of moving particles on a ring were constructed and researched in [5,6], where fact on a random walk of a separately considered particle was proved and a class of distributions arising between moving particles was described as well.

In the present paper we consider models of moving particles without overtaking on a ring that are continuation of researches conducted in [5,6].

The goal of the paper is:

- the investigation of mathematical models of movement of particles on a ring without overtaking and finding necessary and sufficient conditions when a separately considered particle performs a random binomial walk,

- detection of random binomial walk effect for models with great number of particles,

- construction of examples demonstrating the results.

2. Description of the model

Consider a model of movement of S particles sequentially numbered (clockwise) on n equidistant points of a circumference. Movement can occur on any closed trajectory, but for convenience a circumference is considered.

The movement occurs at discrete instant time $t \in T = \{0, h, 2h, ...\}$ counterclockwise. We'll assume that the distance between points on circumference is unit and $h > 0$.

Introduce the notation: $\xi_{i,t}$ is coordinate of particle i at time t; $\rho_{i,t}$ is distance from the particle i to the previous particle in the line of movement at instant time t which is defined as

$$\rho_{i,t} = \begin{cases} \xi_{i+1,t} - \xi_{i,t} \ , & \text{if} \quad \xi_{i+1,t} > \xi_{i,t} \\ n + \left(\xi_{i+1,t} - \xi_{i,t}\right), & \text{if} \quad \xi_{i+1,t} < \xi_{i,t} \end{cases} .$$

$\varepsilon_{i,t} = \left|\xi_{i,t+n} - \xi_{i,t}\right|$ defines the movement of the particle i at time t, i.e. $\varepsilon_{i,t} = 1$, if particle i performed jumping at instant time t, otherwise $\varepsilon_{i,t} = 0$.

3. Movement of two particles. Asymmetric model

Let the parameters of movement of one particle $r, l\,(r + l = 1, 0 < r < 1)$, and the parameters of movement of the other particle depend on distance to the previous particle, i.e.

$$P\left\{\varepsilon_{2,t} = 1 \middle| \rho_{2,t} = k\right\} = r_k, \quad P\left\{\varepsilon_{2,t} = 0 \middle| \rho_{2,t} = k\right\} = l_k,$$

$$\left(k = \overline{2, n-1}\right), \quad r_k + l_k = 1;$$

$$P\left\{\varepsilon_{2,t} = 1 \middle| \rho_{2,t} = 1, \varepsilon_{1,t} = 1\right\} = r_1,$$

$$P\left\{\varepsilon_{2,t} = 0 \middle| \rho_{2,t} = 1, \varepsilon_{2,t} = 1\right\} = l_1, \ r_1 + l_1 = 1;$$

$$P\left\{\varepsilon_{2,t} = 1 \middle| \rho_{2,t} = 1, \varepsilon_{1,t} = 0\right\} = 0, \quad P\left\{\varepsilon_{2,t} = 0 \middle| \rho_{2,t} = 1, \ \varepsilon_{1,t} = 0\right\} = 1;$$

$$P\left\{\varepsilon_{1,t} = 1 \middle| \rho_{2,t} = k\right\} = r, \quad P\left\{\varepsilon_{1,t} = 0 \middle| \rho_{2,t} = k\right\} = l, \quad r + l = 1, \quad k = \overline{1, n-2};$$

$$P\left\{\varepsilon_{1,t} = 1 \middle| \rho_{2,t} = n-1, \varepsilon_{2,t} = 1\right\} = r,$$

$$P\left\{\varepsilon_{1,t} = 0 \middle| \rho_{2,t} = n-1, \ \varepsilon_{2,t} = 1\right\} = l;$$

$$P\left\{\varepsilon_{1,t} = 1 \middle| \rho_{1,t} = n-1, \varepsilon_{2,t} = 0\right\} = 0,$$

$$P\left\{\varepsilon_{1,t} = 0 \middle| \rho_{2,t} = n-1, \varepsilon_{2,t} = 0\right\} = 1; \quad 0 < r_i < 1 \left(i = \overline{1, n-1}\right). \tag{3.1}$$

From descripition of the model it follows that random variables $\rho_{2,t}$ form ergodic Markov chain with finite number of states. Consequently, in [7] there exists a unique stationary distribution $\rho_{2,t}$,

$$\lim_{t\to\infty} P\left\{\rho_{2,t} = k\right\} = a_k, \quad \sum_{k=1}^{n-1} a_k = 1.$$

Using formula of complete probabilities and relation (3,1) for stationary probabilities of distribution of distance between the second and first particles we write the recurrence equations

$$a_1 = a_1\left(rr_1 + l\right) + a_2 l r_2;$$

$$a_k = a_{k-1} r l_{k-1} + a_k \left(r r_k + l l_k\right) a_{k+1} l r_{k+1} \left(k = \overline{2, n-2}\right); \tag{3.2}$$

$$a_{n-1} = a_{n-2} r l_{n-2} + a_{n-1} \left(r r_{n-1} + l_{n-1}\right).$$

Let

$$A_k = \left(\frac{r}{l}\right)^{k-1} \frac{l_1 ... l_{k-1}}{r_2 ... r_k}, \quad A_1 = 1, \; A = \sum_{j=1}^{n-1} A_j \tag{3.3}$$

By immediate substitution we are convinced that $a_k = \frac{A_k}{A}$ is a solution of (3.2). From (3.3) we obtain

$$a_k r l_k = a_{k+1} l r_{k+1}. \tag{3.4}$$

It is obvious that

$$P\left\{\varepsilon_{1,t} = 1\right\} = \sum_{k=1}^{n-2} r a_k + r r_{n-1} a_{n-1} \tag{3.5}$$

$$P\left\{\varepsilon_{2,t} = 1\right\} = \sum_{k=2}^{n-1} r_k a_k + r_1 r a_1 \tag{3.6}$$

Lemma1. *For the models defined by relation (3.1) be fulfilled*

$$P\left\{\varepsilon_{1,t} = 1\right\} = P\left\{\varepsilon_{2,t} = 1\right\} = r$$

it is necessary and sufficient that $r_{n-1} = 1$.

Proof.

$$r = r \sum_{k=1}^{n-1} \left(r_k + l_k\right) a_k = \sum_{k=1}^{n-1} r r_k a_k + \sum_{k=1}^{n-1} r l_k a_k =$$

$$= \sum_{k=1}^{n-1} r r_k a_k + r l_{n-1} a_{n-1} + \sum_{k=1}^{n-2} r l_k a_k =$$

$$= r l_{n-1} a_{n-1} + \sum_{k=1}^{n-1} r r_k a_k + \sum_{k=1}^{n-2} l r_{k+1} a_{k+1} =$$

$$= r l_{n-1} a_{n-1} + r r_1 a_1 + \sum_{k=2}^{n-1} r r_k a_k + \sum_{k=2}^{n-1} l r_k a_k =$$

$$= r l_{n-1} a_{n-1} + P\left\{\varepsilon_{2,t} = 1\right\}$$

$$r = P\left\{\varepsilon_{2,t} = 1\right\} + r l_{n-1} a_{n-1} \tag{3.7}$$

$$P\left\{\varepsilon_{1,t} = 1\right\} = \sum_{k=1}^{n-2} r a_k + r r_{n-1} a_{n-1} = r\left(1 - a_{n-1}\right) + r r_{n-1} a_{n-1} = r - r l_{n-1} a_{n-1}$$

$$r = P\left\{\varepsilon_{1,t} = 1\right\} + r a_{n-1} l_{n-1} \tag{3.8}$$

From equalities (3.7) and (3.8) we obtain the statement of lemma 1

We introduce the following notation

$$b\left(\varepsilon_1, ..., \varepsilon_m\right) = P\left\{\varepsilon_{2,t+h} = \varepsilon_1, ..., \varepsilon_{2,t+mh} = \varepsilon_m\right\} =$$

$$=\sum_{k=1}^{n-1}P\left\{\varepsilon_{2,t+h}=\varepsilon_1,...,\varepsilon_{2,t+mh}=\varepsilon_m,\rho_{2,t+mh}=k\right\},$$

$$\varepsilon_m^+=\sum_{j=1}^{m}\varepsilon_j,\varepsilon_m^-=m-\varepsilon_m^+,\qquad \varepsilon_j=0\qquad \text{or}\qquad 1$$

Theorem 1. *In order that* $b(\varepsilon_1,...,\varepsilon_m)=r^{\varepsilon_m^+}l^{\varepsilon_m^-}\quad(r+l=1)$ *it is necessary and sufficient that* $r_{n-1}=1$.

Proof. Sufficiency. We prove the statement of the theorem by mathematical induction method with respect to m. For $m=1$ the statement of the theorem follows from lemma 1. Let the statement be valid for m steps. We prove it for $m+1$ using (3.4) and lemma 1. For definiteness we assume that $\varepsilon_{m+1}=1$,

$$b(\varepsilon_1,...,\varepsilon_{m+1})=\sum_{k=1}^{n-1}P\left\{\varepsilon_{2,t+h}=\varepsilon_1,...,\varepsilon_{2,t+(m+1)h}=k\right\}=$$

$$=\sum_{k=1}^{n-1}P\left\{\varepsilon_{2,t+h}=\varepsilon_1,...,\varepsilon_{2,t+(m+1)h}=1,\rho_{2,t+mh}=k,\varepsilon_{1,t+(m+1)h}=1\right\}+$$

$$+\sum_{k=1}^{n-2}P\left\{\varepsilon_{2,t+h}=\varepsilon_1,...,\varepsilon_{2,t+(m+1)h}=1,\rho_{2,t+mh}=k+1,\varepsilon_{1,t+(m+1)h}=0\right\}=$$

$$=\sum_{k=1}^{n-1}r_kra_kb(\varepsilon_1,...,\varepsilon_m)+\sum_{k=1}^{n-2}r_{k+1}la_{k+1}b(\varepsilon_1,...,\varepsilon_m)=$$

$$=b(\varepsilon_1,...,\varepsilon_m)\left(\sum_{k=1}^{n-1}r_kra_k+\sum_{k=1}^{n-2}r_{k+1}a_{k+1}l\right)=$$

$$=b(\varepsilon_1,...,\varepsilon_m)\left(rr_1a_1+\sum_{k=2}^{n-1}r_kra_k+\sum_{k=2}^{n-1}r_ka_kl\right)=$$

$$=b(\varepsilon_1,...,\varepsilon_m)\left(rr_1a_1+\sum_{k=2}^{n-1}r_ka_k\right)=b(\varepsilon_1,...,\varepsilon_m)\cdot r.$$

The case $\varepsilon_{m+1}=0$ is analogously proved.

Necessity. Let $b(\varepsilon_1,...,\varepsilon_m)=P\{\varepsilon_{2,t+h}=\varepsilon_1,...,\varepsilon_{2,t+mh}=\varepsilon_m\}=r^{\varepsilon_m^+}l^{\varepsilon_m^-}$. For simplicity we assume $\varepsilon_{m+1}=1$. Then we have

$$b(\varepsilon_1,...,\varepsilon_{m+1})=b(\varepsilon_1,...,\varepsilon_m)P\{\varepsilon_{2,t}=1\}.$$

On the other hand $b(\varepsilon_1,...,\varepsilon_m)=r^{\varepsilon_{m+1}^+}l^{\varepsilon_{m+1}^-}=r^{\varepsilon_m^+}l^{\varepsilon_m^-}\cdot r=b(\varepsilon_1,...,\varepsilon_m)\cdot r$. Hence it follows $P\{\varepsilon_{2,t}=1\}=r$. From lemma 1 it follows $r_{n-1}=1$. Theorem 1 is proved.

It is necessary to note that the case of motion of two and more particles entails great difficulties, since it is not succeeded to transfer the results obtained for the motion of two particles to the case of three particles. Obviously, it is connected with the fact that in model of motion of three particles two particles effect on motion of each particle that leads to difficulties of calculation of the required characteristics.

4. Movement of three particles

Let's consider the model of motion of three particles of sequentially numbered by (clockwise) n equidistant points of circumference.

Let the motion happen by the following rule:

$$P\left\{\varepsilon_{1,t}=1\,(0)\,/\rho_{1,t}=i\right\}=r\,(l)\,,\quad r+l=1,\quad i=\overline{1,n-2};$$

$$P\left\{\varepsilon_{2,t}=1\,(0)\,/\rho_{2,t}=k\right\}=r_k\,(l_k)\,,\quad r_k+l_k=1,\quad k=\overline{2,n-3};$$

$$P\left\{\varepsilon_{2,t}=1/\rho_{2,t}=n-2\right\}=1;\;\; P\left\{\varepsilon_{2,t}=1,\varepsilon_{1,t}=1/\rho_{2,t}=1\right\}=rr_1;$$

$$P\left\{\varepsilon_{2,t}=0,\varepsilon_{1,t}=1/\rho_{2,t}=1\right\}=rl_1;\;\; P\left\{\varepsilon_{2,t}=0,\varepsilon_{1,t}=0/\rho_{2,t}=1\right\}=l;$$

$$P\left\{\varepsilon_{2,t}=1/\varepsilon_{1,t}=0,\rho_{2,t}=1\right\}=0;\;\; P\left\{\varepsilon_{2,t}=1,\varepsilon_{1,t}=0/\rho_{2,t}=1\right\}=0;$$

$$P\left\{\varepsilon_{2,t}=1\,(0)\,/\rho_{2,t}=1,\varepsilon_{1,t}=1\right\}=r_1\,(l_1)\,,r_1+l_1=1;$$

$$P\left\{\varepsilon_{3,t}=1\,(0)\,/\rho_{3,t}=k,\rho_{1,t}\neq 1\right\}=r_k\,(l_k)\,,r_k+l_k=1,\;\; k=\overline{2,n-3};$$

$$P\left\{\varepsilon_{3,t}=1/\rho_{2,t}=n-2\right\}=1;\;\; P\left\{\varepsilon_{3,t}=1/\rho_{3,t}=n-2\right\}=1;$$

$$P\left\{\varepsilon_{3,t}=1/\rho_{1,t}=1\right\}=1;\quad 0<r_i<1,\;\; i=\overline{1,n-2}. \tag{4.1}$$

From the introduced above relations it follows that the first particle is a leading one since any of particles can prevent its motion. On the other hand, if a leading particle arrivsr at minimal admissible distance to the other particle, it pushes it forward in the direction of movement.

It is clear that for the model determined by relations (4.1) there exists a unique stationary distribution

$$a_k=\lim_{t\to\infty}P\left\{\rho_{2,t}=k\right\},\quad \sum_{k=1}^{n-2}a_k=1 \tag{4.2}$$

for which it holds the recurrence formula

$$a_k r l_k=a_{k+1} l r_{k+1}. \tag{4.3}$$

Whence a_k is determined as

$$a_k=\frac{A_k}{A},\quad A_1=1,\;\; A_k=\left(\frac{r}{l}\right)^{k-1}\frac{l_1...l_{k-1}}{r_2...r_k},\;\; \sum_{k=1}^{n-2}A_k=A. \tag{4.4}$$

For probabilities of jumps of the second particle we have

$$P\left\{\varepsilon_{2,t}=1\right\}=\sum_{k=2}^{n-3}r_k a_k+r_1 r a_1+a_{n-2}. \tag{4.5}$$

Lemma 2. *For the model determined by relations (4.1)*

$$P\left\{\varepsilon_{2,t}=1\right\}=r. \tag{4.6}$$

is fulfilled.

Proof.

$$r=r\sum_{k=1}^{n-2}(r_k+l_k)\,a_k=\sum_{k=1}^{n-2}rr_ka_k+\sum_{k=1}^{n-2}rl_ka_k=\sum_{k=2}^{n-2}rr_ka_k+rr_1a_1+$$

$$+rl_{n-2}a_{n-2}+\sum_{k=1}^{n-3}rl_ka_k=rr_1a_1+\sum_{k=2}^{n-2}rr_ka_k+\sum_{k=1}^{n-3}lr_{k+1}a_{k+1}=rr_1a_1+$$

$$+\sum_{k=2}^{n-2}rr_ka_k+\sum_{k=2}^{n-2}lr_ka_k=rr_1a_1+\sum_{k=2}^{n-2}r_ka_k\,(r+l)=rr_1a_1+\sum_{k=2}^{n-3}r_ka_k+$$

$$+r_{n-2}a_{n-2}=rr_1a_1+\sum_{k=2}^{n-3}r_ka_k+a_{n-2}=P\left\{\varepsilon_{2,t}=1\right\}.$$

The lemma is proved.

Theorem 2. *For this model it is fulfilled,*

$$b\,(\varepsilon_1,...,\varepsilon_m)=P\left\{\varepsilon_{2,t+h}=\varepsilon_1,...,\varepsilon_{2,t+mh}=\varepsilon_m\right\}=r^{\varepsilon_m^+}l^{\varepsilon_m^-}\quad (r+l=1)\,,$$

where

$$\varepsilon_m^+=\sum_{j=1}^{m}\varepsilon_j,\varepsilon_m^-=m-\varepsilon_m^+,\quad \varepsilon_j=0\quad \text{or}\quad 1.$$

Proof. For $m=1$ the assertion of the theorem follows from lemma 2. Assume that the assertion is true for m steps. Let's prove its truth for $m+1$. Consider the case $\varepsilon_{m+1}=1$. Using (4.3) and lemma 2, we have

$$b\,(\varepsilon_1,...,\varepsilon_{m+1})=\sum_{k=1}^{n-2}P\left\{\varepsilon_{2,t+h}=\varepsilon_1,...,\varepsilon_{2,t+mh}=\varepsilon_m,\varepsilon_{2,t+(m+1)h}=\right.$$

$$\left.=\varepsilon_{m+1},\rho_{2,t+(m+1)h}=h\right\}=\sum_{k=1}^{n-2}P\left\{\varepsilon_{2,t+h}=\varepsilon_1,...,\varepsilon_{2,t+mh}=\varepsilon_m,\varepsilon_{2,t+(m+1)h}=\right.$$

$$\left.=1,\rho_{2,t+mh}=k,\varepsilon_{1,t+(m+1)h}=1\right\}+\sum_{k=1}^{n-3}P\left\{\varepsilon_{2,t+h}=\varepsilon_1,...,\varepsilon_{2,t+mh}=\right.$$

$$\left.=\varepsilon_m,\varepsilon_{2,t+(m+1)h}=1,\rho_{2,t+mh}=k+1,\varepsilon_{1,t+(m+1)h}=0\right\}=\sum_{k=1}^{n-2}r_kra_kb\,(\varepsilon_1,...,\varepsilon_m)+$$

$$+\sum_{k=1}^{n-3}r_{k+1}la_{k+1}b\,(\varepsilon_1,...,\varepsilon_m)=b\,(\varepsilon_1,...,\varepsilon_m)\left(\sum_{k=1}^{n-2}r_kra_k+\sum_{k=1}^{n-3}r_{k+1}a_{k+1}l\right)=$$

$$=b\,(\varepsilon_1,...,\varepsilon_m)\left(rr_1a_1+\sum_{k=2}^{n-2}r_kra_k+\sum_{k=2}^{n-2}r_ka_kl\right)=b\,(\varepsilon_1,...,\varepsilon_m)\left(rr_1a_1+\sum_{k=2}^{n-2}r_ka_k\right)=$$

$$=b\,(\varepsilon_1,...,\varepsilon_m)\left(rr_1a_1+\sum_{k=2}^{n-3}r_ka_k+r_{n-2}a_{n-2}\right)=$$

$$= b(\varepsilon_1,...,\varepsilon_m)\left(rr_1a_1+\sum_{k=2}^{n-3}r_ka_k+a_{n-2}\right)=$$
$$= b(\varepsilon_1,...,\varepsilon_m)\,P\{\varepsilon_{2,t}=1\}=b(\varepsilon_1,...,\varepsilon_m)\cdot r.$$

The case $\varepsilon_{m+1}=0$ is proved analogously.

Introduce the following notation and conditional probabilities:

$$r_*^i=P\left\{\varepsilon_{3,t}=1/\rho_{1,t}=i\right\},\quad l_*^i=P\left\{\varepsilon_{3,t}=0/\rho_{1,t}=i\right\},$$
$$r_*^{n-2}=P\left\{\varepsilon_{3,t}=1/\rho_{1,t}=n-2,\varepsilon_{1,t}=1\right\},\quad r_*^1=P\left\{\varepsilon_{3,t}=1/\rho_{1,t}=1\right\}=1,$$
$$l_*^1=P\left\{\varepsilon_{3,t}=0/\rho_{1,t}=1\right\}=0,\quad r_*^i+l_*^i=1,\ i=\overline{1,n-2};$$
$$P\left\{\rho_{2,t}=k/\rho_{1,t}=i\right\}=a_k^i,\quad k=\overline{1,n-i-1},\quad i=\overline{1,n-2},\quad \sum_{k=1}^{n-i-1}a_k^i=1,$$
$$a_k^i=\frac{A_k^i}{A^i},\quad A_1^i=1,\quad A_k^i=\left(\frac{r}{l}\right)^{k-1}\frac{l_1...l_{k-1}}{r_2...r_k},$$
$$k=\overline{1,n-i-1},\quad i=\overline{1,n-2},\ A^i=\sum_{j=1}^{n-i-1}A_j^i.$$

Theorem 3. *For the model determined by relations (4.1) there exists a unique stationary distribution*

$$c_i=\lim_{t\to\infty}P\left\{\rho_{1,t}=i\right\},\quad \sum_{i=1}^{n-2}c_i=1, \tag{4.7}$$

for which it holds the recurrent formula

$$c_ilr_*^i=c_{i+1}rl_*^{i+1}. \tag{4.8}$$

Whence c_i is determined as

$$c_i=\frac{C_i}{C},\quad C_1=1,\quad C_i=\left(\frac{l}{r}\right)^{i-1}\frac{r_*^1...r_*^{i-1}}{l_*^2...l_*^i},\quad C=\sum_{i=1}^{n-2}C_i. \tag{4.9}$$

Proof. The random variables $\rho_{1,t}$ form the ergodic Markov chain with finite number of states. Consequently, [7] there exists a unique stationary distribution $\rho_{1,t}$. Using formula of complete probabilities and relation (4.1) for stationary probabilities of distribution of distance between the leading and third particle we write out the recurrent equations

$$\begin{gathered}c_1=c_1r+c_2l_*^2r,\\ c_i=c_{i-1}r_*^{i-1}+c_i\left(rr_*^i+ll_*^i\right)+c_{i+1}l_*^{i+1}r,\quad \left(i=\overline{2,n-3}\right);\\ c_{n-2}=c_{n-2}\left(rr_*^{n-2}+l\right)+c_{n-3}lr_x^{n-3}.\end{gathered} \tag{4.10}$$

Whence r_*^i and l_*^i are determined in the following form

$$r_*^i=\sum_{k=1}^{n-i-2}a_k^ir_{n-k-i}+a_{n-i-1}^ir_{n-i-1}r_1,\quad i=\overline{1,n-2}; \tag{4.11}$$

$$l_*^i = \sum_{k=1}^{n-i-2} a_k^i l_{n-k-i} + a_{n-i-1}^i l_{n-i-1} + a_{n-i-1}^i r_{n-i-1} l_1, \quad i = \overline{1, n-2}. \qquad (4.12)$$

From (4.10) is follows recurrent formula (4.8).

By recurrent formula (4.8) we consecutively find that (4.9) is a solution of (4.10). The theorem is proved.

For probabilities of jumps of particle three we have

$$P\left\{\varepsilon_{3,t} = 1\right\} = \sum_{i=2}^{n-3} c_i r_*^i + c_1 + c_{n-2} r r_*^{n-2} \qquad (4.13)$$

Lemma 3. *For model determined by relations (4.1) it is fulfilled*

$$P\left\{\varepsilon_{3,t} = 1\right\} = r. \qquad (4.14)$$

Proof.

$$l = l \sum_{i=1}^{n-2} \left(r_*^i + l_*^i\right) c_i = \sum_{i=1}^{n-2} l r_*^i c_i + \sum_{k=1}^{n-2} l l_*^i c_i = l r_*^{n-2} c_{n-2} + \sum_{i=1}^{n-3} l r_*^i c_i +$$

$$+ \sum_{i=2}^{n-2} l l_*^i c_i + l l_*^1 c_1 = l r_*^{n-2} c_{n-2} + \sum_{i=1}^{n-3} r l_*^{i+1} c_{i+1} + \sum_{i=2}^{n-2} l l_*^i c_i = l r_*^{n-2} c_{n-2} +$$

$$+ \sum_{i=2}^{n-2} r l_*^i c_i + \sum_{i=2}^{n-2} l l_*^i c_i = l r_*^{n-2} c_{n-2} + \sum_{i=2}^{n-2} l_*^i c_i.$$

$$l = l_*^{n-2} c_{n-2} + \sum_{i=2}^{n-2} l_*^i c_i \Rightarrow 1 - r = l_*^{n-2} c_{n-2} + \sum_{i=2}^{n-2} l_*^i c_i \Rightarrow r = 1 - l r_*^{n-2} c_{n-2} -$$

$$- \sum_{i=2}^{n-2} l_*^i c_i = \sum_{i=1}^{n-2} c_i - l r_*^{n-2} c_{n-2} - \sum_{i=2}^{n-2} l_*^i c_i = \sum_{i=1}^{n-2} c_i \left(1 - l_*^i\right) + c_1 - l r_*^{n-2} c_{n-2} =$$

$$= \sum_{i=1}^{n-3} c_i r_*^i + c_{n-2} r_*^{n-2} + c_1 - l r_*^{n-2} c_{n-2} = \sum_{i=2}^{n-3} c_i r_*^i + c_{n-2} r_*^{n-2} (1 - l) + c_1 =$$

$$= \sum_{i=2}^{n-3} c_i r_*^i + c_{n-2} r_*^{n-2} r + c_1 = P\left\{\varepsilon_{3,t} = 1\right\}.$$

The lemma is proved.

Theorem 4. *For this model it is fulfilled*

$$b\left(\varepsilon_1, ..., \varepsilon_m\right) = P\left\{\varepsilon_{3,t+h} = \varepsilon_1, ..., \varepsilon_{3,mh} = \varepsilon_m\right\} = r^{\varepsilon_m^+} l^{\varepsilon_m^-} \qquad (r + l = 1),$$

where

$$\varepsilon_m^+ = \sum_{j=1}^{m} \varepsilon_j, \varepsilon_m^- = m - \varepsilon_m^+, \quad \varepsilon_j = 0 \quad \text{or} \quad 1.$$

Proof. For $m=1$ the assertion of the theorem follows from lemma 3. Assume that the assertion is true for m steps. Let's prove its validity for $m+1$. Consider the case $\varepsilon_{m+1}=1$. Using (4.8) and lemma 3 we have

$$b\left(\varepsilon_1,...,\varepsilon_{m+1}\right)=\sum_{i=1}^{n-2}P\left\{\varepsilon_{3,t+h}=\varepsilon_1,...,\varepsilon_{3,t+mh}=\varepsilon_m,\varepsilon_{3,t+(m+1)h}=\right.$$

$$\left.=\varepsilon_{m+1},\rho_{1,t+(m+1)h}=i\right\}=\sum_{i=1}^{n-2}P\left\{\varepsilon_{3,t+h}=\varepsilon_1,...,\varepsilon_{3,t+mh}=\varepsilon_m,\varepsilon_{3,t+(m+1)h}=\right.$$

$$\left.=1,\rho_{1,t+mh}=i,\varepsilon_{1,t+(m+1)h}=1\right\}+\sum_{i=1}^{n-3}P\left\{\varepsilon_{3,t+h}=\varepsilon_1,...,\varepsilon_{3,t+mh}=\right.$$

$$\left.=\varepsilon_m,\varepsilon_{3,t+(m+1)h}=1,\rho_{1,t+mh}=i,\varepsilon_{1,t+(m+1)h}=0\right\}=\sum_{i=1}^{n-2}r_*^i rc_i b\left(\varepsilon_1,...,\varepsilon_m\right)+$$

$$+\sum_{i=1}^{n-3}r_*^i lc_i b\left(\varepsilon_1,...,\varepsilon_m\right)=b\left(\varepsilon_1,...,\varepsilon_m\right)\left(\sum_{i=1}^{n-3}r_*^i rc_i+r_*^{n-2}rc_{n-2}+\sum_{i=1}^{n-3}r_*^i c_i l\right)=$$

$$=b\left(\varepsilon_1,...,\varepsilon_m\right)\left(\sum_{i=1}^{n-3}r_*^i c_i+r_*^{n-2}rc_{n-2}\right)=b\left(\varepsilon_1,...,\varepsilon_m\right)\times$$

$$\times\left(\sum_{i=2}^{n-3}r_*^i c_i+r_*^1c_1+r_*^{n-2}rc_{n-2}\right)=b\left(\varepsilon_1,...,\varepsilon_m\right)\left(\sum_{i=2}^{n-3}r_*c_i+c_1+r_*^{n-2}rc_{n-2}\right)=$$

$$=b\left(\varepsilon_1,...,\varepsilon_m\right)P\left\{\varepsilon_{3,t}=1\right\}=b\left(\varepsilon_1,...,\varepsilon_m\right)\cdot r.$$

The case $\varepsilon_{m+1}=0$ is proved analogously.

We can interpret the assertion of theorem 1 and 4 in the following way. If we make one particle visible, and the others invisible, then a visible particle performs a random walk with parameters (r,l).

Example 4.1. Let $n=6,\ S=4,\ r_1=r_2=r,\ l_1=l_2=l\ \ (r+l=1)$ (S is the number of particle)

Note that $(S-1)$ –dimensional random quantity $\left\{\rho_{1,t},\rho_{2,t},...,\rho_{S-1}\right\}$ forms ergodic Markov chain with a unique stationary distribution

$$\pi_{k_1,k_2,...,k_{S-1}}=\lim_{t\to\infty}P\left\{\rho_{1,t}=k_1,...,\rho_{S-1}=k_{S-1}\right\}.$$

Since

$$\sum_{k_1+...+k_{S-1}=B}P\left\{\rho_{1,t}=k_1,...,\rho_{S-1,t}=k_{S-1}\right\}=P\left\{\rho_{s,t}=N-\sum_{i=1}^{S-1}k_i\right\},$$

then

$$\sum_{k_1+...+k_{S-1}=B}\pi_{k_1,...,k_{S-1}}=\pi_{N-B}.$$

For $\pi_{k_1,...,k_{S-1}}$ we can write recurrence relations which will connect $\pi_{k_1,...,k_{S-1}}$ only by r_k,l_k and r,l. Composition of recurrence relations isn't complicated but bulky. Method of finding $\pi_{k_1,\ldots,k_{S-1}}$ is laborious but it is easily realized by using computers.

Then for this example the recurrence equations for
$\pi_{k_1,k_2,k_3} = \lim\limits_{t\to\infty} P\left\{\rho_{1,t} = k_1, \rho_{4,t} = k_2, \rho_{3,t} = k_3\right\}$ have the following form:
$\pi_{1,1,1} = \pi_{1,1,2}r + \pi_{2,1,1}r + \pi_{1,2,1}r,$
$\pi_{1,2,1}\left(1 + r + r^2\right) = \pi_{2,1,1}r^3 + \pi_{1,3,1}r + \pi_{2,1,2}r^2l + \pi_{2,2,1}rl + \pi_{1,2,2}r^2,$
$\pi_{1,1,2,}\left(1 + r\right) = \pi_{1,2,1}r^2 + \pi_{2,1,1}r^2 + \pi_{1,1,3}r + \pi_{2,1,2}lr + \pi_{1,2,2}rl,$
$\pi_{2,1,1}\left(2r + r^3 + r^2\right) = \pi_{1,1,1} + \pi_{1,1,2}l + \pi_{2,1,2}r^3 + \pi_{3,1,1}r + \pi_{2,2,1}r^2 + \pi_{1,2,1}l,$
$\pi_{1,1,3}\left(1 + r\right) = \pi_{2,1,2}r^2 + \pi_{1,2,2}r^2,$
$\pi_{1,3,1}\left(1 + r + r^2\right) = \pi_{2,2,1}r^3,$
$\pi_{3,1,1}\left(1 + r^2 + r\right) = \pi_{2,1,2}r + \pi_{2,1,1}r^2 + \pi_{2,2,1},$
$\pi_{2,1,2}\left(2r + r^3 + r^2\right) = \pi_{1,1,2}+\pi_{1,1,3}+\pi_{2,1,1}lr+\pi_{3,1,1}r^2+\pi_{2,2,1}r^3+\pi_{1,2,2}l+\pi_{1,2,1}rl,$
$\pi_{2,2,1}\left(2r + r^3 + r^2\right) = \pi_{2,1,2}lr + \pi_{2,1,1}lr^2 + \pi_{3,1,1}r^3 + \pi_{1,2,1}r^2 + \pi_{1,3,1} + \pi_{1,2,2}r,$
$\pi_{1,2,2}\left(1 + r + r^2\right) = \pi_{2,1,2}r^3 + \pi_{2,2,1}r^2l + \pi_{1,3,1}r^2,$
If $r = l = 0,5$, then computations give
$\pi_{1,1,1} = 0,2,\quad \pi_{1,1,2} \approx 0,087892,\quad \pi_{1,1,3} \approx 0,026906,\quad \pi_{2,1,2} \approx 0,144542,$
$\pi_{2,1,1} \approx 0,26715,\quad \pi_{3,1,1} \approx 0,125444,\quad \pi_{2,2,1} \approx 0,080468,\quad \pi_{1,2,1} \approx 0,044957,$
$\pi_{1,3,1} \approx 0,005748,\quad \pi_{1,2,2} \approx 0,016893,$
$P\left\{\varepsilon_{2,t} = 1\right\} = \pi_{1,1,1} + \left(\pi_{1,1,2} + \pi_{2,1,1} + \pi_{1,2,1}\right)r+$
$+\left(\pi_{2,1,2} + \pi_{1,3,1} + \pi_{1,2,2} + \pi_{1,1,3} + \pi_{3,1,1} + \pi_{2,2,1}\right)r^2 = 0,49999975,$
$P\left\{\varepsilon_{3,t} = 1\right\} = \pi_{1,1,1} + \pi_{1,1,2} + \pi_{1,1,3} + \left(\pi_{2,1,2} + \pi_{1,2,2}\right)r+$
$+\left(\pi_{2,1,1} + \pi_{1,2,1}\right)r^2 + \left(\pi_{1,3,1} + \pi_{3,1,1} + \pi_{2,2,1}\right)r^3 \approx 0,49999975,$
$P\left\{\varepsilon_{4,t} = 1\right\} = \pi_{1,1,1} + \pi_{1,1,2} + \pi_{1,1,3} + \pi_{1,2,1} + \pi_{1,3,1} + \pi_{1,2,2}+$
$+\pi_{2,2,1}r + \pi_{2,1,2}r^2 + \pi_{2,1,1}r^3 + \pi_{3,1,1}r^4 \approx 0,49999975$
$P\left\{\varepsilon_{2,t} = 1, \varepsilon_{2,t+h} = 1\right\} = \left(r + lr\right)\pi_{1,1,1} + \left(r^3 + r^3l\right)\left(\pi_{1,2,1} + \pi_{2,1,1} + \pi_{1,1,2}\right)+$
$+\left(\pi_{2,1,2} + \pi_{1,3,1} + \pi_{1,2,2} + \pi_{1,1,3} + \pi_{3,1,1} + \pi_{2,2,1}\right)r^4 \approx 0,25000063,$
$P\left\{\varepsilon_{3,t} = 1, \varepsilon_{3,t+h} = 1\right\} = \left(r + lr^2\right)\pi_{1,1,1} + \left(r^5 + lr^5\right)\left(\pi_{1,2,1} + \pi_{2,1,1}\right)+$
$+\left(r^4 + 2r^4l\right)\left(\pi_{2,1,2} + \pi_{1,2,2}\right) + r^2\left(\pi_{1,2,3} + \pi_{2,1,3}\right) + r^4\left(\pi_{2,1,1} + \pi_{1,2,1}\right)+$
$+r^6\left(\pi_{1,3,1} + \pi_{3,1,1} + \pi_{2,2,1}\right) + \left(r + r^2l + r^2l^2\right)\pi_{1,1,2}+$
$+\left(r + lr\right)\pi_{1,1,3} \approx 0,24372182,$
$P\left\{\varepsilon_{4,t} = 1, \varepsilon_{4,t+h} = 1\right\} = \left(r + lr^3\right)\pi_{1,1,1} + \left(r + 2l^2r^3\right)\pi_{1,2,1}+$
$\left(r^7 + r^7l\right)\pi_{2,1,1} + \left(r^6 + 2r^6l\right)\pi_{2,1,2} + \left(r + rl\right)\pi_{1,3,1}+$
$+\left(r + l^2r^2 + lr^2\right)\pi_{1,2,2} + \left(r + l^2r^3 + lr^3\right)\pi_{1,1,2}+$
$+\left(r + lr^2\right)\pi_{1,1,3} + \left(r^5 + 3r^5l\right)\pi_{2,2,1} + \pi_{3,1,1}r^8 \approx 0,23713956.$

Thus, hence we obtain that if $S > 3$, $N \geq S + 2$, then the fact on random walk of a separately taken particle with the parameter (r, l) doesn't hold.

5. Movement of S particles sequentially numbered (clockwise) on $n = S + 1$ equidistant points of a circumference

It seems that a random walk holds when $n = S + 1$. But, unfortunately, we can't prove it for general case. However, numerical calculations conducted for many cases give rise to hope that the fact on random walk holds.

Example 5.1. Let $n = 5$, $S = 4$, $r_1 = r$, $l_1 = l\,(r + l = 1)$. The recurrence equations for $\pi_{k_1,k_2,k_3} = \lim\limits_{t\to\infty} P\left\{\rho_{1,t} = k_1, \rho_{4,t} = k_2, \rho_{3,t} = k_3\right\}$ have the following form:
$\pi_{1,1,1} = \pi_{1,1,1}r + \pi_{2,1,1}rl + \pi_{1,1,2}rl + \pi_{1,2,1}rl,$
$\pi_{2,1,1} = \pi_{2,1,1}\left(l + r^4\right) + \pi_{1,1,2}l + \pi_{1,2,1}l + \pi_{1,1,1}l,$
$\pi_{1,1,2} = \pi_{1,1,2}r^2 + \pi_{2,1,1}r^2l + \pi_{1,2,1}r^2l,$

$\pi_{1,2,1} = \pi_{1,2,1}r^3 + \pi_{2,1,1}r^3 l.$

Simple calculations give.

If we denote $t = 1 + r + r^2$, then $\pi_{1,1,1} = \frac{r}{1+r}$, $\pi_{2,1,1} = \frac{1}{1+rt}$, $\pi_{1,1,2} = \frac{r^2}{(1+r)t}$, $\pi_{1,2,1} = \frac{r^3}{t(1+rt)}$.

$$P\{\varepsilon_{2,t} = 1\} = \pi_{1,1,1} + (\pi_{2,1,1} + \pi_{1,1,2} + \pi_{1,2,1})\, r^2 = r,$$

$$P\{\varepsilon_{3,t} = 1\} = \pi_{1,1,1} + \pi_{1,1,2} + (\pi_{2,1,1} + \pi_{1,2,1})\, r^3 = r,$$

$$P\{\varepsilon_{4,t} = 1\} = \pi_{1,1,1} + \pi_{1,1,2} + \pi_{1,2,1} + \pi_{2,1,1}r^4 = r,$$

$$P\{\varepsilon_{2,t} = 1, \varepsilon_{2,t+h} = 1\} = \pi_{1,1,1}\left(r + lr^2\right) + (\pi_{2,1,1} + \pi_{1,2,1} + \pi_{1,1,2})\, r^4 = r^2,$$

$$P\{\varepsilon_{3,t} = 1, \varepsilon_{3,t+h} = 1\} = (\pi_{1,1,1} + \pi_{1,1,2})\left(r + lr^3\right) + (\pi_{2,1,1} + \pi_{1,2,1})\, r^6 = r^2,$$

$$P\{\varepsilon_{4,t} = 1, \varepsilon_{4,t+h} = 1\} = (\pi_{1,1,1} + \pi_{1,1,2} + \pi_{1,2,1})\left(r + lr^4\right) + \pi_{2,1,1}r^8 = r^2.$$

Example 5.2. Let $n = 6$, $S = 5$, $r_1 = r$, $l_1 = l\,(r + l = 1)$. The recurrence equations for $\pi_{k_1,k_2,k_3,k_4} = \lim_{t\to\infty} P\left\{\rho_{1,t} = k_1, \rho_{5,t} = k_2, \rho_{4,t} = k_3, \rho_{3,t} = k_4\right\}$ have the following form:

$\pi_{1,1,1,1} = \pi_{1,1,1,1}r + \pi_{2,1,1,1}rl + \pi_{1,1,1,2}rl + \pi_{1,1,2,1}rl + \pi_{1,2,1,1}rl,$

$\pi_{2,1,1,1} = \pi_{2,1,1,1}\left(r^5 + l\right) + \pi_{1,1,1,1}l + \pi_{1,1,1,2}l + \pi_{1,1,2,1}l + \pi_{1,2,1,1}l,$

$\pi_{1,1,1,2} = \pi_{1,1,1,2}r^2 + \pi_{2,1,1,1}r^2 l + \pi_{1,1,2,1}r^2 l + \pi_{1,2,1,1}r^2 l,$

$\pi_{1,1,2,1} = \pi_{1,1,2,1}r^3 + \pi_{2,1,1,1}r^3 l + \pi_{1,2,1,1}r^3 l,$

$\pi_{1,2,1,1} = \pi_{1,2,1,1}r^4 + \pi_{2,1,1,1}r^4 l.$

If we denote $k = 1 + r + r^2 + r^3$, then

$\pi_{1,1,1,1} = \frac{r}{1+r}$, $\pi_{2,1,1,1} = \frac{1}{k+r^4}$, $\pi_{1,1,1,2} = \frac{r^2}{k+r+r^2}$,

$\pi_{1,1,2,1} = \frac{r^3}{k(k-r^3)}$, $\pi_{1,2,1,1} = \frac{r^4}{k(k+r^4)}$.

$P\{\varepsilon_{2,t} = 1\} = \pi_{1,1,1,1} + (\pi_{2,1,1,1} + \pi_{1,1,1,2} + \pi_{1,1,2,1} + \pi_{1,2,1,1})\, r^2 = r,$

$P\{\varepsilon_{3,t} = 1\} = \pi_{1,1,1,1} + \pi_{1,1,1,2} + (\pi_{2,1,1,1} + \pi_{1,1,2,1} + \pi_{1,2,1,1})\, r^3 = r,$

$P\{\varepsilon_{4,t} = 1\} = \pi_{1,1,1,1} + \pi_{1,1,1,2} + \pi_{1,1,2,1} + (\pi_{2,1,1,1} + \pi_{1,2,1,1})\, r^4 = r,$

$P\{\varepsilon_{5,t} = 1\} = \pi_{1,1,1,1} + \pi_{1,1,1,2} + \pi_{1,1,2,1} + \pi_{1,2,1,1} + \pi_{2,1,1,1}r^5 = r,$

$P\{\varepsilon_{2,t} = 1, \varepsilon_{2,t+h} = 1\} = \pi_{1,1,1,1}\left(r + lr^2\right) +$
$+ (\pi_{2,1,1,1} + \pi_{1,1,1,2} + \pi_{1,1,2,1} + \pi_{1,2,1,1})\, r^4 = r^2,$

$P\{\varepsilon_{3,t} = 1, \varepsilon_{3,t+h} = 1\} = (\pi_{1,1,1,1} + \pi_{1,1,1,2})\left(r + lr^3\right) +$
$+ (\pi_{2,1,1,1} + \pi_{1,1,2,1} + \pi_{1,2,1,1})\, r^6 = r^2,$

$P\{\varepsilon_{4,t} = 1, \varepsilon_{4,t+h} = 1\} = (\pi_{1,1,1,1} + \pi_{1,1,1,2} + \pi_{1,1,2,1})\left(r + lr^4\right) +$
$+ (\pi_{2,1,1,1} + \pi_{1,2,1,1})\, r^8 = r^2,$

$P\{\varepsilon_{5,t} = 1, \varepsilon_{5,t+h} = 1\} = (\pi_{1,1,1,1} + \pi_{1,1,1,2} + \pi_{1,1,2,1} + \pi_{1,2,1,1}) \times$
$\times \left(r + lr^5\right) + \pi_{2,1,1,1}r^{10} = r^2.$

Example 5.3. Let $n = 7$, $S = 6$, $r_1 = r$, $l_1 = l\,(r + l = 1)$. The recurrence equations for

$$\pi_{k_1,k_2,k_3,k_4,k_5} = \lim_{t\to\infty} P\left\{\rho_{1,t} = k_1, \rho_{6,t} = k_2, \rho_{5,t} = k_3, \rho_{4,t} = k_4, \rho_{3,t} = k_5\right\}$$

have the following form:

$\pi_{1,1,1,1,1} = \pi_{1,1,1,1,1}r + \pi_{2,1,1,1,1}rl + \pi_{1,2,1,1,1}rl + \pi_{1,1,2,1,1}rl + \pi_{1,1,1,2,1}rl + \pi_{1,1,1,1,2}rl,$

$\pi_{2,1,1,1,1} = \pi_{1,1,1,1,1}l + \pi_{2,1,1,1,1}\left(r^6 + l\right) + \pi_{1,2,1,1,1}l + \pi_{1,1,2,1,1}l + \pi_{1,1,1,2,1}l + \pi_{1,1,1,1,2}l,$

$\pi_{1,2,1,1,1} = \pi_{1,2,1,1,1}r^5 + \pi_{2,1,1,1,1}r^5 l,$

$\pi_{1,1,2,1,1} = \pi_{2,1,1,1,1}r^4 l + \pi_{1,2,1,1,1}r^4 l + \pi_{1,1,2,1,1}r^4,$

$\pi_{1,1,1,2,1} = \pi_{2,1,1,1,1}r^3l + \pi_{1,2,1,1,1}r^3l + \pi_{1,1,2,1,1}r^3l + \pi_{1,1,1,2,1}r^3,$
$\pi_{1,1,1,1,2} = \pi_{2,1,1,1,1}r^2l + \pi_{1,2,1,1,1}r^2l + \pi_{1,1,2,1,1}r^2l + \pi_{1,1,1,2,1}r^2l + \pi_{1,1,1,1,2}r^2.$
The calculations give:
$\pi_{1,1,1,1,1} = \frac{r}{1+r},\ \ \pi_{2,1,1,1,1} = \frac{1}{1-r^6},\ \pi_{1,2,1,1,1} = \frac{r^5l^2}{(1-r^5)(1-r^6)},$
$\pi_{1,1,2,1,1} = \frac{r^4l^2}{(1-r^4)(1-r^5)},\ \ \pi_{1,1,1,2,1} = \frac{r^3l^2}{(1-r^3)(1-r^4)},$
$\pi_{1,1,1,1,2} = \frac{r^2l^2}{(1-r^2)(1-r^3)}.$
$P\{\varepsilon_{2,t} = 1\} = \pi_{1,1,1,1,1}\left(1 - r^2\right) + r^2 = \pi_{1,1,1,1,1}l\,(1 + r) + r^2 = r,$
$P\{\varepsilon_{3,t} = 1\} = \pi_{1,1,1,1,1} + \pi_{1,1,1,1,2} + \left(\pi_{2,1,1,1,1} + \pi_{1,2,1,1,1} + \pi_{1,1,2,1,1}\right) r^3 = r,$
$P\{\varepsilon_{4,t} = 1\} = \pi_{1,1,1,1,1} + \pi_{1,1,1,1,2} + \left(\pi_{2,1,1,1,1} + \pi_{1,2,1,1,1} + \pi_{1,1,2,1,1}\right) r^4 = r,$
$P\{\varepsilon_{5,t} = 1\} = \pi_{1,1,1,1,1} + \pi_{1,1,2,1,1} + \pi_{1,1,1,2,1} + \pi_{1,1,1,1,2}+$
$+\left(\pi_{2,1,1,1,1} + \pi_{1,2,1,1,1}\right) r^5 = r,$
$P\{\varepsilon_{6,t} = 1\} = 1 - \pi_{2,1,1,1,1}\left(1 - r^6\right) = r,$
$P\{\varepsilon_{2,t} = 1, \varepsilon_{2,t+h} = 1\} = \pi_{1,1,1,1,1}\left(r + lr^2\right) +$
$+r^4\left(\pi_{2,1,1,1,1} + \pi_{1,2,1,1,1} + \pi_{1,1,1,2,1} + \pi_{1,1,1,1,2} + \pi_{1,1,2,1,1}\right) = r^2,$
$P\{\varepsilon_{3,t} = 1, \varepsilon_{3,t+h} = 1\} = \left(\pi_{1,1,1,1,1} + \pi_{1,1,1,1,2}\right)\left(r + lr^3\right) +$
$+\left(\pi_{2,1,1,1,1} + \pi_{1,2,1,1,1} + \pi_{1,1,2,1,1} + \pi_{1,1,1,2,1}\right) r^6 = r^2,$
$P\{\varepsilon_{4,t} = 1, \varepsilon_{4,t+h} = 1\} = \left(\pi_{1,1,1,1,1} + \pi_{1,1,1,2,1} + \pi_{1,1,1,1,2}\right)\left(r + lr^4\right) +$
$+\left(\pi_{2,1,1,1,1} + \pi_{1,2,1,1,1} + \pi_{1,1,2,1,1}\right) r^8 = r^2,$
$P\{\varepsilon_{5,t} = 1, \varepsilon_{5,t+h} = 1\} = \left(\pi_{1,1,1,1,1} + \pi_{1,1,2,1,1} + \pi_{1,1,1,2,1} + \pi_{1,1,1,1,2}\right) \times$
$\times\left(r + lr^5\right) + \left(\pi_{1,2,1,1,1} + \pi_{2,1,1,1,1}\right) r^{10} = r^2,$
$P\{\varepsilon_{6,t} = 1, \varepsilon_{6,t+h} = 1\} = \left(1 - \pi_{2,1,1,1,1}\right)\left(r + lr^6\right) + \pi_{2,1,1,1,1}r^{12} = r^2.$

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