Rovshan G. GASANOV

ON INTRINSIC COMPACTNESS PROPERTIES OF GENERALIZED SOLUTIONS OF A FOURTH ORDER OPERATOR-DIFFERENTIAL EQUATION ON THE SEGMENT

Abstract

In the paper we obtain conditions on coefficients of a fourth order operator-differential equation on the finite segment that provide intrinsic compactness of generalized solutions of the given equation.

On a separable Hilbert space H consider the boundary value problem:

$$P\left(d/dt\right)u \equiv \frac{d^4u\left(t\right)}{dt^4} + A^4u\left(t\right) + \sum_{j=1}^{4} A_j u^{(4-j)}\left(t\right) = 0, \quad t \in (0;1)\,, \tag{1}$$

$$u^{(j)}\left(0\right) = \varphi_j, \quad u^{(j)}\left(1\right) = \psi_j, \quad j = 0,1, \tag{2}$$

where $u\left(t\right)$ is a vector-function with values in H, φ_j, ψ_j $(j = 0,1)$ are the known vectors from H, the derivatives are understood in the sense of distributions theory, A and A_j $\left(j = \overline{1,4}\right)$ are linear, generally speaking, unbounded operators.

Let A be a positive-definite self-adjoint operator. By H_γ we denote a Hilbert scale of spaces generated by the operator A i.e

$H_\gamma = D\left(A^\gamma\right)$, $(x,y)_\gamma = (A^\gamma x, A^\gamma y)$, $\gamma \geq 0$, $x, y \in H_\gamma$.

Further we determine the following Hilbert spaces [1].

Let a, $b \in R = (-\infty, \infty)$, $a < b$. Let

$$L_2\left((a,b)\,,H\right) = \left\{ f \middle|\, \|f\|_{L_2((a,b),H)} = \left(\int_a^b \|f\left(t\right)\|^2 dt \right)^{1/2} < \infty \right\},$$

$$W_2^2\left((a,b)\,,H\right) = \left\{ u \middle|\, u'' \in L_2\left((a,b)\,H\right), \quad A^2u \in L_2\left(a,b\right),H \right\}$$

with norm

$$\|u\|_{W_2^2((a,b),H)} = \left(\left\|u''\right\|_{L_2((a,b),H)} + \left\|A^2u\right\|_{L_2((a,b),H)} \right)^{1/2}.$$

By $\overset{\circ}{W}{}_2^2\left((a,b)\,,H\right)$ we denote the subspace

$$\overset{\circ}{W}{}_2^2\left((a,b)\,,H\right) = \left\{ u \middle|\, u \in W_2^2\left((a,b)\,,H\right), \; u^{(j)}\left(a\right) = u^{(j)}\left(b\right) = 0, \quad j = 0,1 \right\}.$$

We similarly determine the Hilbert space

$$W_2^1\left((a,b)\,,H\right) = \left\{ u \middle|\, u' \in L_2\left((a,b)\,,H\right), \; Au \in L_2\left((a,b)\,,H\right) \right\}$$

with norm

$$\|u\|_{W_2^1((a,b),H)} = \left(\|u'\|^2_{L_2((a,b),H)} + \|Au\|^2_{L_2((a,b),H)}\right)^{1/2}.$$

Let $D([a,b], H_4)$ be a linear set of vector-functions with values in H_4 having compact supports on the segment $[a;b]$. Obviously, $D([a,b], H_4)$ is everywhere dense set in the space $W_2^2((a,b), H)$ [1].

$$D^0([a,b], H_4) = \left\{ u|\, u \in D([a,b], H_4),\ u^{(j)}(a) = u^{(j)}(b) = 0, \quad j = 0,1 \right\}$$

is similarly determined. It follows from trace theorem [1] that $D^0([a,b], H_4)$ is everywhere dense set in the space $\overset{\circ}{W}{}_2^2((a,b), H)$.

In the paper [2] it is proved.

Lemma 1. *Let the following conditions be fulfilled:*

a) A is a positive-definite self-adjoint operator;

b) The operators $B_j = A_j A^{-j}$ $(j = 1,2)$ and $D_j = A^{-2} A_j A^{-1}$ are bounded in H. Then the bilinear functional

$$P(u,g) = P((d/dt)\, u, g)_{L_2((0,1);H)}$$

continues from the space $D([0,1], H_4) + D^0([0,1], H_4)$ to the space $W_2^2((0,1), H) + \overset{\circ}{W}{}_2^2((0,1), H)$ a functional acting in the following way

$$P(u,g) = (u,g)_{W_2^2((0,1),H)} + P_1(u,g) \equiv$$

$$\equiv (u'', g'')_{L_2((0,1),H)} + \left(A^2 u, A^2 g\right)_{L_2((0,1);H)} +$$

$$+ \sum_{j=1}^{2} \left(A_j u^{(2-j)}, g''\right)_{L_2((0,1);H)} + \sum_{j=3}^{4} \left(A_j u^{(4-j)}, g\right)_{L_2((0,1);H)}. \tag{3}$$

Further, in [2] definition of the generalized solution of problem (1), (2) was given and a theorem on its existence was proved.

Definition. *The vector-function $u(t) \in W_2^2((0,1), H)$ is said to be a generalized solution of problem (1), (2), if $u^{(j)}(0) = \varphi_j$, $u^{(j)}(1) = \psi_j$, $(j = 0,1)$ and for any $g \in \overset{\circ}{W}{}_2^2((0,1), H)$ the identity*

$$P(u,g) = (u,g)_{W_2^2((0,1),H)} + P_1(u,g) = 0 \tag{4}$$

holds.

It is proved the following.

Theorem 1. [2] *Let conditions a) and b) from lemma 1 be fulfilled, and it hold the inequality*

$$\alpha = \sum_{j=1}^{2} m_j \|B_j\| + \sum_{j=3}^{4} m_j \|D_j\| < 1, \tag{5}$$

where $m_1 = m_3 = \frac{1}{\sqrt{2}}$, $m_2 = \frac{1}{2}$, $m_4 = 1$. Then for any $\varphi_j \in H_{2-j-\frac{1}{2}}$ and $\psi_j \in H_{2-j-\frac{1}{2}}$ $(j = 0, 1)$ there exists a unique generalized solution that satisfies the inequality

$$P(g, g) \geq (1 - \alpha) \|g\|^2_{W_2^2((0,1),H)} \text{ for any } g \in \overset{\circ}{W}{}_2^2((0,1), H). \tag{6}$$

In the present paper, following the Lax paper [3] we give definition of intrinsic compactness of generalized solutions of problem (1), (2) and under some additional conditions on coefficients of operator-differential equation (1) we show intrinsic compactness of generalized solutions of problem (1), (2).

By $N(P)$ we denote a space of generalized solutions of problem (1), (2) that is a close subspace of the space $W_2^2((0,1), H)$. We complete this space by the norm $\|u\|_{W_2^1((0,1),H)}$ that is weaker than the norm $\|u\|_{W_2^2((0,1),H)}$.

Denote the obtained space by $\widetilde{N}(P)$.

Definition 2. *Let a, b, a_1, b_1 be any numbers satisfying the conditions:*
$0 \leq a < a_1 < b_1 < b \leq 1$. *If any set*

$$\widetilde{N}(P) \left\{ u | \, u \in \widetilde{N}(P), \|u\|_{W_2^1((a,b),H)} \leq M \right\},$$

where M is a constant positive number, compact by the norm of the space $W_2^1((a_1, b_1), H)$, we'll say the space of generalized solutions of problem (1), (2) is intrinsically compact.

It holds the following.

Theorem 2. *Let all the conditions of theorem 1 be fulfilled, the operator A^{-1} be completely continuous, and the operator $C_2 = A^{-1}A_2A^{-1}$ be bounded in H. Then a space of generalized solutions of problem (1), (2) is intrinsically compact.*

Proof. Let $u \in N(P)$, and a scalar function $\varphi(t) \in C_0^\infty(a; b)$
$(0 \leq a < a_1 < b_1 < b \leq 1)$, moreover $\varphi(t) = 1$ for $t \in (a_1; b_1)$. Then it is obvious that the vector-function $\varphi(t)\, u(t) \in \overset{\circ}{W}{}_2^2((0,1), H)$ and by theorem 1 we have

$$(P(\varphi u), \varphi u) \geq (1 - \alpha) \|\varphi u\|^2_{W_2^2((0,1),H)}.$$

Hence we get

$$((\varphi u), (\varphi u))_{W_2^2((0,1),H)} + P_1(\varphi u, \varphi u) \geq const \|\varphi u\|^2_{W_2^2((0,1),H)}$$

or

$$\|\varphi u\|^2_{W_2^2((a,b),H)} + P_1(\varphi u, \varphi u) \geq$$
$$\geq const \left(\|\varphi u\|_{W_2^2((a,b),H)} \cdot \|u\|_{W_2^2((a_1,b_1),H)} \right) \geq const \|u\|^2_{W_2^2((a_1,b_1),H)}. \tag{7}$$

Consequently

$$\|u\|_{W_2^2((a_1,b_1),H)} \leq const \|\varphi u\|_{W_2^2((a,b),H)} \tag{8}$$

Further, we upper bound the left hand side of inequality (7). First of all we consider the expression $\|\varphi u\|^2_{W_2^2((a,b),H)}$. Obviously

$$\|\varphi u\|^2_{W_2^2((a,b),H)} = \left((\varphi u)'', (\varphi u)''\right)_{L_2((a,b),H)} + \left(A^2(\varphi u), A^2(\varphi u)\right)_{L_2((a,b),H)} =$$

$$= \left((\varphi u)'', (\varphi u)''\right)_{L_2((a,b),H)} + \left(A^2 u, A^2\left(\varphi^2 u\right)\right)_{L_2((a,b),H)}. \tag{9}$$

Denote $g = \varphi^2 u \in \overset{\circ}{W}{}_2^2\left((0,1), H\right)$. Obviously

$$\left((\varphi u)'', (\varphi u)''\right)_{L_2((a,b),H)} = \left\|\varphi u'' + 2\varphi' u' + \varphi'' u\right\|^2_{L_2((a,b),H)} = \left\|\varphi u''\right\|^2_{L_2((a,b),H)} +$$

$$+2\operatorname{Re}\left(\varphi u'', 2\varphi' u' + \varphi'' u\right)_{L_2((a,b),H)} + \left\|2\varphi' u + \varphi'' u\right\|^2_{L_2((a,b),H)}. \tag{10}$$

Notice that

$$\left\|2\varphi' u' + \varphi'' u\right\|_{L_2((a,b),H)} \le const\, \|u\|^2_{W_2^1((a,b),H)}. \tag{11}$$

On the other hand we have

$$\left\|\varphi u''\right\|^2_{L_2((a,b),H)} = \left(\varphi u'', \varphi u''\right)_{L_2((a,b),H)} = \left(u'', \varphi^2 u''\right)_{L_2((a,b),H)} =$$

$$= \left(u'', \left(\varphi^2 u\right)''\right)_{L_2((a,b),H)} + \left(u'', \varphi^2 u'' - \left(\varphi^2 u\right)''\right)_{L_2((a,b),H)} =$$

$$= \left(u'', g''\right)_{L_2((a,b),H)} - \left(u'', 4\varphi'\varphi' u - \left(\varphi^2\right)'' u\right)_{L_2((a,b),H)} = \left(u'', g''\right)_{L_2((a,b),H)} -$$

$$- \left(\varphi u'', 4\varphi' u'\right)_{L_2((a,b),H)} - \left(u', \left(\left(\varphi^2\right)'' u\right)'\right)_{L_2((a,b),H)}. \tag{12}$$

It is easily seen that

$$\left|\left(u', \left(\left(\varphi^2\right)'' u\right)'\right)_{L_2((a,b),H)}\right| \le const\, \|u\|^2_{W_2^1((a,b),H)}. \tag{13}$$

Now let's consider the second term in (12):

$$\left|\left(\varphi u'', 4\varphi' u'\right)_{L_2((a,b),H)}\right| = \left|\left((\varphi u)'', 4\varphi' u'\right)_{L_2((a,b),H)}\right| +$$

$$+\left|\left(\varphi u'' - (\varphi u)'', 4\varphi' u'\right)_{L_2((a,b),H)}\right| \le \left\|(\varphi u)''\right\|_{L_2((a,b),H)} 4\left\|\varphi' u'\right\|_{L_2((a,b),H)} +$$

$$+\left|\left(2\varphi' u' + \varphi'' u, 4\varphi' u'\right)_{L_2((a,b),H)}\right| \le$$

$$\le const\left(\|(\varphi u)\|_{W_2^2((a,b),H)} \cdot \|u\|_{W_2^1((a,b),H)} + \|u\|^2_{W_2^1((a,b),H)}\right) \tag{14}$$

From (9)-(14) we get $\left(g = \varphi^2 u\right)$

$$\|(\varphi u)\|^2_{W_2^2((a,b),H)} = \left(u'', g''\right)_{L_2((a,b),H)} + \left(A^2 u, A^2 g\right)_{L_2((a,b),H)} + K_0(u, \varphi),$$

moreover

$$|K_0(u,\varphi)| \le const\left(\|(\varphi u)\|_{W_2^2((a,b),H)} \cdot \|u\|_{W_2^1((a,b),H)} + \|u\|^2_{W_2^1((a,b),H)}\right). \tag{15}$$

Now let's estimate $P_1(u, \varphi)$.

Obviously

$$\left(A_1 (\varphi u)', (\varphi u)''\right)_{L_2((a,b),H)} = \left(A_1\left(\varphi u' + \varphi' u\right), (\varphi u)''\right)_{L_2((a,b),H)} =$$

$$= \left(A_1 u', \varphi\left(\varphi u\right)''\right)_{L_2((a,b),H)} + \left(A_1 u, \varphi'\left(\varphi u\right)''\right)_{L_2((a,b),H)} =$$

$$= \left(A_1 u', \left(\varphi^2 u\right)''\right)_{L_2((a,b),H)} + \left(A_1 u', \varphi\left(\varphi u\right)'' - \left(\varphi^2 u\right)''\right)_{L_2((a,b),H)} +$$

$$+ \left(A_1 u, \varphi'\left(\varphi u\right)''\right)_{L_2((a,b),H)} = \left(A_1 u', g''\right)_{L_2((a,b),H)} +$$

$$+ \left(A_1 \varphi u', -2\varphi' u' + \varphi'' u\right)_{L_2(R_+,H)} - \left(A_1 u', \left(\varphi^2\right)'' u\right)_{L_2(R_+,H)} +$$

$$+ \left(A_1 u, \varphi'\left(\varphi u\right)''\right)_{L_2((a,b),H)}. \tag{16}$$

Hence we get:

$$\left|\left(A_1 \varphi u', \varphi'' u - 2\varphi' u'\right)_{L_2((a,b),H)}\right| =$$

$$= \left|\left(A_1\left(\left(\varphi u\right)' - \varphi' u\right), \varphi'' u - 2\varphi' u'\right)_{L_2((a,b),H)}\right| \le$$

$$\le \left\|A_1 A^{-1}\right\| \left\|A\left(\varphi u\right)'\right\|_{L_2((a,b),H)} \cdot \left\|\varphi'' u - 2\varphi' u'\right\|_{L_2((a,b),H)} +$$

$$+ \left\|A_1 A^{-1}\right\| \left\|A u\right\|_{L_2((a,b),H)} \left(\left\|\varphi' \varphi'' u\right\|_{L_2((a,b),H)} + \left\|2\varphi'^2 u'\right\|_{L_2((a,b),H)}\right) \le$$

$$\le const \left(\left\|(\varphi u)\right\|_{W_2^2((a,b),H)} \cdot \left\|u\right\|_{W_2^1((a,b),H)} + \left\|u\right\|^2_{W_2^1((a,b),H)}\right). \tag{17}$$

Since

$$\left|\left(A_1 u', \left(\varphi^2\right)'' u\right)_{L_2((a,b),H)}\right| = \left|\left(A_1 u, \left(\left(\varphi^2\right)'' u\right)'\right)_{L_2((a,b),H)}\right| \le$$

$$\le \left\|A_1 A^{-1}\right\| \left\|A u\right\|_{L_2((a,b),H)} \left\|\left(\left(\varphi^2\right)'' u\right)'\right\|_{L_2((a,b),H)} \le const \left\|u\right\|^2_{W_2^1((a,b),H)}$$

and

$$\left|\left(A_1 u, \varphi\left(\varphi u\right)''\right)_{L_2((a,b),H)}\right| \le const \left\|\left(\varphi u\right)''\right\|_{L_2((a,b),H)} \left\|A_1 A^{-1}\right\| \left\|A u\right\|_{L_2((a,b),H)} \le$$

$$\le const \left\|\varphi u\right\|_{W_2^2((a,b),H)} \cdot \left\|u\right\|_{W_2^1((a,b),H)}, \tag{18}$$

then from equality (16) and inequalities (17) and (18) it follows

$$\left(A_1\left(\varphi u\right)', \left(\varphi u\right)''\right)_{L_2((a,b),H)} = \left(A_1 u', g''\right)_{L_2((a,b),H)} + K_1\left(u, \varphi\right), \tag{19}$$

where

$$\left|K_1\left(u, \varphi\right)\right| \le const \left(\left\|(\varphi u)\right\|_{W_2^2((a,b),H)} \cdot \left\|u\right\|_{W_2^1((a,b),H)} + \left\|u\right\|^2_{W_2^1((a,b),H)}\right). \tag{20}$$

Now, let's estimate expression $\left(A_2\left(\varphi u\right), \left(\varphi u\right)''\right)_{L_2((a,b),H)}$.

It is easily verified that

$$\left(A_2\left(\varphi u\right), \left(\varphi u\right)''\right)_{L_2((a,b),H)} = \left(A_2 u, \varphi\left(\varphi u\right)''\right)_{L_2((a,b),H)} =$$

$$= \left(A_2\left(\varphi^2 u\right)''\right)_{L_2((a,b),H)} + \left(A_2 u, \varphi\left(\varphi u\right)'' - \left(\varphi^2 u\right)''\right)_{L_2((a,b),H)} =$$

$$= \left(A_2 u, g''\right)_{L_2((a,b),H)} + \left(A_2 u, \varphi\varphi'' u - 2\varphi'\varphi u' - \left(\varphi^2\right)'' u\right)_{L_2((a,b),H)} =$$

$$= \left(A_2u, g''\right)_{L_2((a,b),H)} + \left(A_2\varphi u, \varphi''u - 2\varphi'u'\right)_{L_2((a,b),H)} -$$

$$- \left(A_2u, \left(\varphi^2\right)'' u\right)_{L_2((a,b),H)} . \tag{21}$$

Since the inequalities

$$\left|\left(A_2\varphi u, \varphi''u - 2\varphi'u'\right)_{L_2((a,b),H)}\right| \leq \left\|A_2A^{-2}\right\| \left\|A^2\left(\varphi u\right)\right\|_{L_2((a,b),H)} \times$$

$$\times \left\|\varphi''u - 2\varphi'u'\right\|_{L_2((a,b),H)} \leq$$

$$\leq const \left\|\varphi u\right\|_{W_2^2((a,b),H)} \cdot \left\|u\right\|_{W_2^1((a,b),H)} , \tag{22}$$

and

$$\left|\left(A_2u, \left(\varphi^2\right)'' u\right)_{L_2((a,b),H)}\right| = \left|\left(A^{-1}A_2A^{-1}Au, \left(\varphi^2\right)'' Au\right)_{L_2((a,b),H)}\right| \leq$$

$$\leq \left\|C_2\right\| const \left\|Au\right\|^2_{L_2((a,b),H)} = const \left\|u\right\|^2_{W_2^1((a,b),H)} ,$$

hold, from (21) it follows that

$$\left(A_2\left(\varphi u\right), \left(\varphi u\right)''\right)_{L_2((a,b),H)} = \left(A_2u, g''\right) + K_2\left(u, \varphi\right) , \tag{23}$$

where

$$\left|K_2\left(u, \varphi\right)\right| \leq const \left(\left\|\left(\varphi u\right)\right\|_{W_2^2((a,b),H)} \cdot \left\|u\right\|_{W_2^1((a,b),H)} + \left\|u\right\|^2_{W_2^1((a,b),H)}\right) . \tag{24}$$

Further we get

$$\left(A_3\left(\varphi u\right)', \varphi u\right)_{L_2((a,b),H)} = \left(A_3\left(\varphi u' + \varphi'u\right)\varphi u\right)_{L_2((a,b),H)} =$$

$$= \left(A_3u', \varphi^2u\right)_{L_2((a,b),H)} + \left(A_3u, \varphi'\left(\varphi u\right)\right)_{L_2((a,b),H)} =$$

$$= \left(A_3u', g\right)_{L_2((a,b),H)} + \left(A_3u, \varphi'\left(\varphi u\right)\right)_{L_2((a,b),H)} . \tag{25}$$

Since

$$\left|\left(A_3u, \varphi'\left(\varphi u\right)\right)_{L_2((a,b),H)}\right| = \left|\left(A^{-2}A_3A^{-1}Au, \varphi'A^2\left(\varphi u\right)\right)_{L_2((a,b),H)}\right| \leq$$

$$\leq const \left\|\varphi u\right\|_{W_2^2((a,b),H)} \cdot \left\|u\right\|_{W_2^1((a,b),H)} , \tag{26}$$

it follows from (25) that

$$\left(A_3\left(\varphi u\right)', \left(\varphi u\right)\right)_{L_2((a,b),H)} = \left(A_3u', g\right)_{L_2((a,b),H)} + K_3\left(u, \varphi\right) , \tag{27}$$

where

$$\left|K_3\left(u, \varphi\right)\right| \leq const \left(\left\|\left(\varphi u\right)\right\|_{W_2^2((a,b),H)} \cdot \left\|u\right\|_{W_2^1((a,b),H)}\right) . \tag{28}$$

Finally we consider the expression

$$\left(A_4\left(\varphi u\right), \left(\varphi u\right)\right)_{L_2((a,b),H)} = \left(A_4u, \varphi^2u\right)_{L_2((a,b),H)} = \left(A_4u, g\right)_{L_2((a,b),H)} \tag{29}$$

From equalities (9), (12), (16), (23), (27) and (29) we get

$$P(\varphi u, \varphi u) = P(u, g) + \sum_{i=0}^{3} K_i(u, \varphi).$$

Since $u \in N(P)$, then $P(u, g) = 0$. Therefore by inequalities (15), (20), (24) and (28)

$$|P(\varphi u, \varphi u)| \leq \sum_{i=0}^{3} |K_i(u, \varphi)| \leq$$

$$\leq const \left(\|(\varphi u)\|_{W_2^2((a,b),H)} \cdot \|u\|_{W_2^1((a,b),H)} + \|u\|^2_{W_2^1((a,b),H)} \right). \quad (30)$$

Allowing for inequality (30) in inequality (7) we have

$$\|(\varphi u)\|_{W_2^2((a,b),H)} \cdot \|u\|_{W_2^2((a_1,b_1),H)} \leq$$

$$\leq const \left(\|(\varphi u)\|_{W_2^2((a,b),H)} \cdot \|u\|_{W_2^1((a,b),H)} + \|u\|^2_{W_2^1((a,b),H)} \right). \quad (31)$$

Now, using inequality (8) and (31) we prove the theorem.

Let's consider the two cases:

1) If the set $\left\{ \|\varphi u\|_{W_2^2((a,b),H)}, \;\; u \in \widetilde{N}_0(P) \right\}$ is bounded, it follows from inequality (8) that the set $\left\{ \|u\|_{W_2^2((a_1,b_1),H)}, \;\; u \in \widetilde{N}_0(P) \right\}$ is bounded, i.e $\widetilde{N}_0(P)$ is bounded in $W_2^2((a_1, b_1), H)$. Since A^{-1} is completely continuous operator, $W_2^2((a_1, b_1), H)$ is compactly imbedded into the space $W_2^1((a_1, b_1), H)$ [4, p.81]. Thus, $\widetilde{N}_0(P)$ is a compact space. In this case the theorem is proved.

2) The set $\left\{ \|\varphi u\|_{W_2^2((a,b),H)}, \;\; u \in \widetilde{N}_0(P) \right\}$ is unbounded. Then by inequality (31) we get that $\left\{ \|u\|_{W_2^2((a_1,b_1),H)}, \;\; u \in \widetilde{N}_0(P) \right\}$ is bounded. Further, operating as in case 1) we complete the proof of the theorem.

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Rovshan G.Gasanov
Institute of Mathematics and Mechanics of NAS of Azerbaijan
9, F. Agayev str., AZ1141, Baku, Azerbaijan
Tel.: (99412) 439 47 20 (off.)

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