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# SOME LOCAL PROPERTIES OF SINGULAR INTEGRAL IN TERMS OF MEAN OSCILLATION

**Abstract**

In this paper some local properties of locally summable functions which are invariant concerning the singular integral operator are considered. These local properties are described in terms of mean oscillation.

Denote by $B(a,r)$ a close ball in the Euclidean space $R^n$ of radius $r>0$ with center at the point $a\in R^n$, i.e. $B(a,r):=\{x\in R^n: \ |x-a|\le r\}$. Apply the process of orthogonalization of relative scalar product

$$(f,g):=\frac{1}{|B(0,1)|}\int\limits_{|B(0,1)|} f(t)g(t)dt$$

to a system of power functions $\{x^v\}$ and $|v|\le k$, arranged in partially lexicographic order (see [1]) where Lebesgue measure of the set $E\subset R^n$ is denoted by $|E|$, $v=(v_1,v_2,...,v_n)$, $x=(x_1,x_2,...,x_n)$, $x^v=x_1^{v_1}\cdot x_2^{v_2}\cdots x_n^{v_n}$, $|v|=v_1+v_2+...+v_n$; $v_1,v_2...v_n$ and $k$ are nonnegative integers. By $\{\varphi_v\}$, $|\nu|\le k$ we denote the result of orthogonalization process.

For $1\le p<\infty$, by $L^p_{loc}(R^n)$ we'll denote a class of all locally summable in $p$-th of power function determined in $R^n$. By $L^\infty_{loc}(R^n)$ we'll denote a class of all functions locally bounded in $R^n$.

Let $k\in N\cup\{0\}$, where $N$ is a set of all positive integers. To each pair $(f,B(a,r))$, where $f\in L^1_{loc}(R^n)$, we associate the following polynomial in $R^n$ power not higher $k$ (see [2], [3]):

$$P_{k,B(a,r)}f(x):=\sum_{|v|\le k}\left(\frac{1}{|B(a,r)|}\int\limits_{B(a,r)} f(t)\varphi_v\left(\frac{t-a}{r}\right)dt\right)\varphi_v\left(\frac{x-a}{r}\right),$$

for $f\in L^P_{loc}(R^n)$ $(1\le p\le\infty)$, $k\in N$, we denote

$$\Omega_k(f,B(a,r))_p:=\left(\frac{1}{|B(a,r)|}\int\limits_{B(a,r)}\left|f(t)-P_{k-1,B(a,r)}f\left(t\right)\right|^p dt\right)^{\frac{1}{p}},\quad (1\le p<\infty),$$

$$\Omega_k(f,B(a,r))_\infty:=es\sup\left\{\left|f\left(t\right)-P_{k-1,B(a,r)}f\left(t\right)\right|: \quad t\in B(a,r)\right\}.$$

$\Omega_k(f,B(a,r))_p$ $\quad(1\le p\le\infty,\quad k\in N)$ is said to be the $k$-th order mean oscillation of the function $f$ in the ball $B\left(a,r\right)$ in the metric $L^P$ [4]. Let's consider the following metric characteristic of the function $f\in L^p_{loc}(R^n)$ [4]:

$$m^k_f(x_0;\delta)_p:=\sup\left\{\Omega_k(f,B(x_0,r))_p: \ r\le\delta\right\}\quad(\delta>0),$$

where $x_0 \in R^n$ is a fixed point, $k \in N$. It is easy to see that the function $m_f^k(x_0;\delta)_p$ takes only non-negative values and monotonically increases with respect to $\delta$ in the interval $(0,+\infty)$.

Let's consider the singular integral operator [4]:

$$A_k f(x) = \lim_{\epsilon\to+0} \int\limits_{R^n} \left\{ K_\epsilon(x-y) - \left( \sum_{|\nu|\le k-1} \frac{x^v}{v!} D^v K(-y) \right) X_{\{|t|>1\}}(y) \right\} f(y)dy,$$

where

$$K(x) = \omega(x)\cdot |x|^{-n}, \quad \int\limits_{S^{n-1}} \omega(x)ds = 0, \quad K_\epsilon(x) = K(x)\cdot X_{\{|t|>\epsilon\}}(x),$$

the function $\omega(x)$ is homogeneous of power 0, $X_{\{|t|>\epsilon\}}$ is a characteristic function of the set $\{t\in R^n: \ |t|>\epsilon\}$, $S^{n-1}$ is a unique sphere in the Euclidean space $R^n$; we assume that for $k=1$ the function $K(x)$ is differentiable and has bounded partial derivatives of the first order, and for$k>1$ the function $K(x)$ is $k$ times continuously differentiable on the sphere $S^{n-1}$; $v=(v_1,v_2,...v_n)$, $v_1,v_2,...,v_n$ are non-negative integers, $|v| = v_1+v_2+...+v_n$, $v! = v_1!v_2!...v_n!$ $k\in N$,

$$D^v f := \frac{\partial^{|v|} f}{\partial x_1^{v_1} \partial x_2^{v_2} ... \partial x_n^{v_n}}.$$

Notice that if $f\in L^p(R^n)$ $(1\le p<\infty)$, then singular integral $A_k f$ differs from the singular integral

$$Tf(x) := \lim_{\epsilon\to+0} \int\limits_{R^n} K_\epsilon(x-y) f(y)dy$$

by a polynomial of power not higher $k-1$.

Here we study some local structural properties of singular integral $A_k f$ in terms of metric characteristic $m_f^k(x_0;\delta)_p$. By proving we main result we'll use the following theorem from [4].

**Theorem 1.** *[4]. Let $x_0\in R^n$, $f\in L^p_{loc}(R^n)$, $1<p<\infty$. Then at the convergence of the integral in the right hand side, it is true the inequality*

$$m_{\tilde f}^k(x_0;\delta)_p \le c\cdot\delta^k \int\limits_\delta^\infty \frac{m_f^k(x_0;t)_p}{t^{k+1}} dt \quad (\delta>0) \tag{1}$$

*where $\tilde f := A_k f$, and the constant $c>0$ is independent of $x_0, f$ and $\delta$.*

Some local properties of locally summable functions that remain invariant with respect to singular integral operator are distinguished in the following theorem. These local properties are expressed in terms of mean oscillation. Namely, the following theorem holds frue:

**Theorem 2.** *Let* $1 < p < \infty$, $1 \leq \theta \leq \infty$, $x_0 \in R^n$, $f \in L^p_{loc}(R^n)$, $\tilde{f} := A_k f$, $k \in N$; $\varphi(x)$ *be such a non-negative, monotonically increasing on* $(0, +\infty)$ *function that*

$$\delta^k \int\limits_{\delta}^{\infty} \frac{\varphi(t)}{t^{k+1}} dt = O(\varphi(\delta)) \quad (\delta > 0). \tag{2}$$

*Then if*

$$||f||_{\varphi,\theta,k} := \left( \int\limits_0^{\infty} \left( \frac{m_f^k(x_0;t)_p}{\varphi(t)} \right)^{\theta} \frac{dt}{t} \right)^{\frac{1}{\theta}} < +\infty, \tag{3}$$

*with appropriate modification in the case* $\theta = \infty$, *the condition*

$$\left|\left|\tilde{f}\right|\right|_{\varphi,\theta,k} := \left( \int\limits_0^{\infty} \left( \frac{m_{\tilde{f}}^k(x_0;t)_p}{\varphi(t)} \right)^{\theta} \frac{dt}{t} \right)^{\frac{1}{\theta}} < +\infty \tag{4}$$

*is fulfilled.*

*Besides, there exists such a constant* $c > 0$ *that*

$$\forall f : \quad \left|\left|\tilde{f}\right|\right|_{\varphi,\theta,k} \leq c \cdot ||f||_{\varphi,\theta,k} \,. \tag{5}$$

**Proof.** First we consider the case $\theta = \infty$. Then condition (3) takes the form

$$||f||_{\varphi,\infty,k} := \sup \left\{ \frac{m_f^k(x_0;t)_p}{\varphi(t)} : \quad t > 0 \right\} \; < \; +\infty.$$

Hence $m_f^k(x_0;\delta)_p \leq ||f||_{\varphi,\infty,k} \cdot \varphi(\delta)$ $(\delta > 0)$.

If the function $\varphi$ satisfies condition (2), then by inequality (1) we get

$$m_{\tilde{f}}^k(x_0;\delta)_p \leq c \cdot \delta^k \int\limits_{\delta}^{\infty} \frac{m_f^k(x_0;t)_p}{t^{k+1}} dt \leq$$

$$\leq c \cdot \delta^k \cdot ||f||_{\varphi,\infty,k} \cdot \int\limits_{\delta}^{\infty} \frac{\varphi(t)}{t^{k+1}} dt \leq c_1 \cdot ||f||_{\varphi,\infty,k} \cdot \varphi(\delta), \quad \delta > 0,$$

where the constant $c_1 > 0$ is independent of $x_0, f$ and $\delta$.

And hence, in turn we have

$$\left|\left|\tilde{f}\right|\right|_{\varphi,\infty,k} := \sup \left\{ \frac{m_{\tilde{f}}^k(x_0;t)_p}{\varphi(t)} : \quad t > 0 \right\} \; \leq \; c_1 \cdot ||f||_{\varphi,\infty,k} \,.$$

Thus, the statement of the theorem is true for the case $\theta = \infty$.

Now, let's consider the case $1 \leq \theta < \infty$. Show that if $\varphi$ satisfies condition (2), and the function $f$ satisfies condition (3), then

$$\int\limits_1^{\infty} \frac{m_f^k(x_0;t)_p}{t^{k+1}} dt < +\infty. \tag{6}$$

If condition (3) is fulfilled, for any $\tau\in(0,+\infty)$ we have

$$\left(\int\limits_{\tau}^{\infty}\left(\frac{m_f^k(x_0;t)_p}{\varphi(t)}\right)^{\theta}\frac{dt}{t}\right)^{\frac{1}{\theta}}\geq\left(\int\limits_{\tau}^{2\tau}\left(\frac{m_f^k(x_0;t)_p}{\varphi(t)}\right)^{\theta}\frac{dt}{t}\right)^{\frac{1}{\theta}}\geq\frac{m_f^k(x_0;\tau)_p}{\varphi(2\tau)}\cdot(\ln 2)^{\frac{1}{\theta}}. \quad (7)$$

Now, let's verify that if relation (2) is fulfilled, there exists such a constant $c>0$ that for any $\delta\in(0,+\infty)$ the inequality $\varphi(2\delta)\leq c\varphi(\delta)$ is true. Really, allowing for monotonicity of the function $\varphi(\delta)$ we have

$$\delta^k\int\limits_{\delta}^{\infty}\frac{\varphi(t)}{t^{k+1}}dt\geq\delta^k\int\limits_{2\delta}^{\infty}\frac{\varphi(t)}{t^{k+1}}dt\geq\varphi(2\delta)\cdot\delta^k\int\limits_{2\delta}^{\infty}t^{-k-1}dt=\varphi(2\delta)\cdot\delta^k\left(\frac{t^{-k}}{-k}\Big|_{2\delta}^{\infty}\right)=$$

$$=\varphi(2\delta)\cdot\delta^k\left(0+\frac{1}{k(2\delta)^k}\right)=\frac{1}{k2^k}\cdot\varphi(2\delta).$$

Hence by (2) we get $\exists c>0\ \forall\delta\in(0,+\infty):\varphi(2\delta)\leq c\cdot\varphi(\delta)$.

Taking this into account, from inequality (7) we get

$$m_f^k(x_0;\tau)_p\leq(\ln 2)^{\frac{1}{\theta}}\cdot c\cdot\varphi(\tau)\cdot\left(\int\limits_{\tau}^{\infty}\left(\frac{m_f^k(x_0;t)_p}{\varphi(t)}\right)^{\theta}\frac{dt}{t}\right)^{\frac{1}{\theta}}\leq$$

$$\leq c_1\cdot\varphi(\tau)\cdot||f||_{\varphi,\theta,k}\,,\quad\tau\in(0,+\infty),$$

where $c_1=c\cdot(\ln 2)^{\frac{1}{\theta}}$. Hence we have

$$\int\limits_{1}^{\infty}\frac{m_f^k(x_0;t)_p}{t^{k+1}}dt\leq c_1\cdot||f||_{\varphi,\theta,k}\cdot\int\limits_{1}^{\infty}\frac{\varphi(t)}{t^{k+1}}dt<+\infty,$$

because, if (2) holds, the integral at the right hand side of the last inequality converges. Thus, under our suppositions for the function $f$ inequality (1) holds. If we introduce the denotation

$$F_f^k(\tau):=\tau^k\int\limits_{\tau}^{\infty}\frac{m_f^k(x_0;t)_p}{t^{k+1}}dt,$$

by inequality (1) the proof of the theorem will be complete, if we'll prove that

$$\left(\int\limits_{0}^{\infty}\left(\frac{F_f^k(t)}{\varphi(t)}\right)^{\theta}\frac{dt}{t}\right)^{\frac{1}{\theta}}\leq c\cdot||f||_{\varphi,\theta,k}\,, \quad (8)$$

where $c$ is a positive constant that doesn't depend on $f$

Let $h\in L^{\theta_1}(0,+\infty)$, $h(\tau)\geq 0$, $\frac{1}{\theta_1}+\frac{1}{\theta}=1$. Then, changing the integration order we have

$$\int\limits_{0}^{\infty}\frac{F_f^k(t)}{t^{\frac{1}{\theta}}\cdot\varphi(t)}\cdot h(t)dt=\int\limits_{0}^{\infty}\left(\frac{1}{t^{\frac{1}{\theta}}\cdot\varphi(t)}\cdot t^k\int\limits_{t}^{\infty}\frac{m_f^k(x_0;\tau)}{\tau^{k+1}}d\tau\right)h(t)dt=$$

$$= \int_0^\infty \frac{m_f^k(x_0;\tau)_p}{\tau^{k+1}} \left( \int_0^\tau \frac{t^k h(t)}{t^{\frac{1}{\theta}} \cdot \varphi(t)} dt \right) d\tau. \tag{9}$$

It is known [5] that if $\varphi$ satisfies condition (2), there exists such a number $\alpha \in (0,k)$ that $\frac{\varphi(t)}{t^\alpha}$ almost decreases. Let $\beta = \alpha - k + \frac{1}{\theta}$. Then by means of relation (9) we get

$$\int_0^\infty \frac{F_f^k(t)}{t^{\frac{1}{\theta}} \cdot \varphi(t)} \cdot h(t)\, dt = \int_0^\infty \frac{m_f^k(x_0;\tau)_p}{\tau^{k+1}} \left( \int_0^\tau \frac{h(t)}{\left(\frac{\varphi(t)}{t^\alpha}\right) \cdot t^\beta} dt \right) d\tau \leq$$

$$\leq c \int_0^\infty \frac{m_f^k(x_0;\tau)_p}{\tau^{k+1}} \left( \frac{\tau^\alpha}{\varphi(\tau)} \int_0^\tau h(t) t^{-\beta} dt \right) d\tau =$$

$$= c \cdot \int_0^\infty \frac{m_f^k(x_0;\tau)_p}{\varphi(\tau)} \left( \tau^{\alpha-k-1} \int_0^\tau h(t) t^{-\beta} dt \right) d\tau =$$

$$= c \cdot \int_0^\infty \frac{m_f^k(x_0;\tau)_p}{\tau^{\frac{1}{\theta}} \cdot \varphi(\tau)} \left( \tau^{\beta-1} \int_0^\tau h(t) t^{-\beta} dt \right) d\tau.$$

Further, applying the Holder inequality, hence we get

$$\int_0^\infty \frac{F_f^k(t)}{t^{\frac{1}{\theta}} \cdot \varphi(t)} \cdot h(t) dt \leq c \left( \int_0^\infty \left( \frac{m_f^k(x_0;\tau)_p}{\varphi(\tau)} \right)^\theta \frac{d\tau}{\tau} \right)^{\frac{1}{\theta}} \times$$

$$\times \left( \int_0^\infty \left( \tau^{\beta-1} \int_0^\tau h(t) t^{-\beta} dt \right)^{\theta_1} d\tau \right)^{\frac{1}{\theta_1}} \tag{10}$$

with appropriate modification in the case $\theta_1 = \infty$ (i.e. $\theta = 1$), where $c$ is a positive constant.

By our denotation $\beta = \alpha + \frac{1}{\theta} - k < k + \frac{1}{\theta} - k = \frac{1}{\theta} \leq 1$, i.e. $\beta < 1$. Therefore, for $\theta_1 = \infty$ we immediately get

$$\int_0^\infty \frac{F_f^k(t)}{\varphi(t) t^{\frac{1}{\theta}}} \cdot h(t) dt \leq c \cdot ||f||_{\varphi,\theta,k} \cdot ||h||_{L^{\theta_1}(0,+\infty)} . \tag{11}$$

Now, let $1 < \theta_1 < \infty$. Let's consider the Hardy operator

$$Pf(x) = \int_0^x f(t) dt.$$

It is known (see [6]), that if $w(x) = x^a$, $v(x) = x^{a+p}$, $1 \leq p < \infty$ and $a < -1$, it is valid the inequality

$$\left( \int_0^\infty |Pg(x)|^p\, w(x) dx \right)^{\frac{1}{p}} \leq c \left( \int_0^\infty |g(x)|^p\, v(x) dx \right)^{\frac{1}{p}}, \tag{12}$$

where the constant $c > 0$ is independent of $g$.

If we denote $a = (\beta - 1)\theta_1$, $p = \theta_1$, we have

$$a + p = (\beta - 1)\theta_1 + \theta_1 = \beta\theta_1; \quad a = (\beta - 1)\theta_1 =$$
$$= \left(\alpha + \frac{1}{\theta} - k - 1\right)\theta_1 < (k + \frac{1}{\theta} - k - 1)\theta_1 =$$
$$= \left(\frac{1}{\theta} - 1\right)\theta_1 = \left(-\frac{1}{\theta_1}\right) \cdot \theta_1 = -1, \text{ i.e. } a < 1.$$

Then for the function $g(x) = h(x) \cdot x^{-\beta}$ from (12) we have

$$\left(\int\limits_0^\infty x^{(\beta-1)\theta_1} \left(\int\limits_0^x h(t)t^{-\beta}dt\right)^{\theta_1} dx\right)^{\frac{1}{\theta_1}} \leq c \cdot \left(\int\limits_0^\infty (h(x) \cdot x^{-\beta})^{\theta_1} \cdot x^{\beta\theta_1}dx\right)^{\frac{1}{\theta_1}} =$$
$$= c\left(\int\limits_0^\infty (h(x))^{\theta_1}dx\right)^{\frac{1}{\theta_1}} = c \cdot ||h||_{L^{\theta_1}(0,+\infty)} .$$

Taking this into account, from (10) we get

$$\int\limits_0^\infty \frac{F_f^k(t)}{\varphi(t)t^{\frac{1}{\theta}}} h(t)dt \leq c \cdot ||f||_{\varphi,k,\theta} \cdot ||h||_{L^{\theta_1}(0,+\infty)} , \qquad (13)$$

where $c > 0$ is a constant that doesn't depend on $f$ and $h$.

Inequalities (11) and (13) show that for $1 \leq \theta < \infty$ it holds the inequality

$$\left(\int\limits_0^\infty \left(\frac{F_f^k(t)}{\varphi(t)}\right)^\theta \frac{dt}{t}\right)^{\frac{1}{\theta}} \leq c \cdot ||f||_{\varphi,\theta,k} ,$$

where the constant $c > 0$ is independent of $f$. The theorem is proved.

The following corollary is obtained from theorem 2.

**Corollary.** *Let* $1 < p < \infty$, $1 \leq \theta \leq \infty$, $x_0 \in R^n$, $f \in L^p_{loc}(R^n)$, $\tilde{f} = A_k f$, $k \in N$; $\psi(x)$*-be a non-negative monotonically increasing on* $(0, +\infty)$ *function, such that*

$$\delta \int\limits_\delta^\infty \frac{\psi(t)}{t^2}dt = O(\psi(\delta)), \quad \delta > 0. \qquad (14)$$

*Then, if*

$$\left(\int\limits_0^\infty \left(\frac{m_f^k(x_0;t)_p}{t^{k-1}\psi(t)}\right)^\theta \frac{dt}{t}\right)^{\frac{1}{\theta}} < +\infty,$$

*with appropriate modification in the case* $\theta = \infty$, *the condition*

$$\left(\int\limits_0^\infty \left(\frac{m_{\tilde{f}}^k(x_0;t)_p}{t^{k-1}\psi(t)}\right)^\theta \frac{dt}{t}\right)^{\frac{1}{\theta}} < +\infty$$

*is fulfilled.*

**Proof.** Let $\psi(x)$ be a non-negative, monotonically increasing on $(0,+\infty)$ function satisfying condition (14) and $\varphi(x) = x^{k-1}\psi(x)$, $x \in (0,+\infty)$. Then we have

$$\delta^k \int_\delta^\infty \frac{\varphi(t)}{t^{k+1}}dt = \delta^k \int_\delta^\infty \frac{t^{k-1}\psi(t)}{t^{k+1}}dt = \delta^{k-1} \cdot \delta \int_\delta^\infty \frac{\psi(t)}{t^2}dt = \delta^{k-1} \cdot O(\psi(\delta)) =$$

$$= O(\delta^{k-1}\psi(\delta)) = O(\varphi(\delta)), \quad \delta > 0.$$

So the function $\varphi(x) = x^{k-1}\psi(x)$ satisfies condition (2). Therefore, the required statement is obtained from theorem 2. The corollary is proved.

Notice that the statement of Corollary at $\theta = \infty$ for Hilbert transformation was earlier announced in the author's paper [7].

Let $\varphi(x)$-be a non-negative, monotonically increasing on $(0,+\infty)$ function, $k \in N$, $1 \le p \le \infty$, $1 \le \theta \le \infty$, $x_0 \in R^n$. By $MO^{k,p}_{\varphi,\theta}(x_0)$ we denote a class of all functions $f \in L^p_{loc}(R^n)$, for which $\|f\|_{\varphi,\theta,k} = \|f\|_{\varphi,\theta,k,p,x_0} < +\infty$, where

$$\|f\|_{\varphi,\theta,k,p,x_0} := \left(\int_0^\infty \left(\frac{m^k_f(x_0;t)_p}{\varphi(t)}\right)^\theta \frac{dt}{t}\right)^{1/\theta}, \quad \text{for} \quad 1 \le \theta < \infty,$$

$$\|f\|_{\varphi,\theta,k,p,x_0} := \sup\left\{\frac{m^k_f(x_0;t)_p}{\varphi(t)} : \quad t > 0\right\}, \quad \text{for} \quad \theta = \infty.$$

If we consider the class $MO^{k,p}_{\varphi,\theta}(x_0)$ as a sub-set in the factor-space $L^P_{loc}(R^n)/\{\text{constants}\}$, then $||\cdot||_{\varphi,\theta,k}$ is a norm in $MO^{k,p}_{\varphi,\theta}(x_0)$.

Theorem 2 affirms that if $1 < p < \infty$, $1 \le \theta \le \infty$, $x_0 \in R^n$, $k \in N$, $\varphi$ is a non-negative monotonically increasing on $(0,+\infty)$ function satisfying condition (2), the singular integral operator $A_kf$ boundedly acts on the space $MO^{k,p}_{\varphi,\theta}(x_0)$.

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