MATHEMATICS

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DETERMINATION OF COEFFICIENT IN THE RIGHT HAND SIDE OF A SYSTEM OF PARABOLIC EQUATIONS

Abstract

In the paper we research matters of correctness of the inverse problem on determination of component of the right hand side, independent of a spatial variable, for the system of reaction-diffusion type parabolic equations. The theorems on the existence, uniqueness and stability of solution are proved.

Accept the following denotation: D' is a bounded area from R^{n-1}, $D = D' \times (a,b) \subset R^n$, a, b are some numbers, $x^{'} = (x_1, ..., x_{n-1})$, $x = (x', x_n)$ are arbitrary points of areas D' and D, respectively, $Q = D \times (0,T]$, $Q' = D' \times (0,T]$, $S = \partial D \times [0,T]$, $0 < T = const$.

Consider a problem of determination of $\{f_k(x',t), u_k(x,t), k = \overline{1,m}\}$ from the conditions:

$$u_{kt} - \Delta u_k = f_k\left(x',t\right) g_k(u), \quad (x,t) \in Q \tag{1}$$

$$u_k(x,0) = \varphi_k(x), \quad x \in \overline{D} = D \cup \partial D, \ u_k(x,t) = \psi_k(x,t), \ (x,t) \in S \tag{2}$$

$$\int\limits_a^b u_k\left(x', x_n, t\right) dx_n = h_k\left(x',t\right), \quad \left(x',t\right) \in \overline{Q}' \tag{3}$$

where $u_{kt} = \dfrac{\partial u_k}{\partial t}$, $u_{kx_i} = \dfrac{\partial u_k}{\partial x_i}$, $\Delta u_k = \sum\limits_{i=1}^{n} \dfrac{\partial^2 u_k}{\partial x_i^2}$ is a Laplace operator, $g_k(\cdot)$, $\varphi_k(\cdot)$, $\psi_k(\cdot)$, $h_k(\cdot)$ are the given functions, $u = (u_1, ..., u_m)$.

Similar problems, as a rule, incorrect in terms of Adamard and were studied in the papers [1-6 and etc.].

If in equation (1) functions $f_k(x',t)$ are given, then, naturally, condition (3) isn't given. Solvability problems of (1)-(2) are considered in more general formulation in the papers [7-9 and etc].

For input data we suppose that:

1^0. $g_k(\cdot) \in Lip_{(loc)}(R^n)$, $|g_k(\cdot)| \geq \nu > 0$, ν is a number;

2^0. $\varphi_k(\cdot) \in C^{2+\alpha}\left(\overline{D}\right)$,$\psi_k(x,t) \in C^{2+\alpha, 1+\alpha/2}(S)$, $\varphi_k(x) = \psi_k(x_k, 0)$, $x \in \partial D$;

3^0. $h_k(x',t) \in C^{2+\alpha, 1+\alpha/2}\left(\overline{Q}'\right)$, $h_k(x',0) = \int\limits_a^b \varphi_k(x,0)\, dx_n$,

$h_k(x,t) = \int\limits_a^b \psi_k(x,t)\, dx_n$, $(x,t) \in S$.

4^0. $[\psi_{kt}(x,0)-\Delta\varphi_k(x)]\cdot\int\limits_a^b g_k(\varphi(x))\,dx_n=$

$=\left[h_{kt}(x',0)-\Delta h_k(x',0)-\psi_{kx_n}(x',b,0)+\psi_{kx_n}(x',a,0)\right]g_k(\varphi(x))$, $x\in\partial D$, $x'\in\partial D'$ (determination of spaces $C^{l+\alpha,(l+\alpha)/2}(\cdot)$, $l=0,1,2$, $0<\alpha<1$ and corresponding norms, see [7, p.16]).

Definition 1. *Functions* $\{f_k(x',t),u_k(x,t),k=1,m\}$ *is said to be a solution of problems (1)-(3), if: 1)* $f_k(x',t)\in C(Q')$*; 2)* $u_k(x,t)\in C^{2,1}(Q)\cap C(\overline{Q})$*; 3) for them correlations (1)-(3) are satisfied.*

Let $K=\left\{(f_k,u_k)\,|f_k(x',t)\in C^{\alpha,\alpha/2}(Q'),\ u_k(x,t)\in C^{2+\alpha,1+\alpha/2}(\overline{Q})\right\}$.

Theorem 1. *Let conditions 1-3 be satisfied. Then, if there exists a solution of problem (1)-(3) and it belongs to the set* K*, then this solution is unique and stability estimation is true:*

$$\|u-\overline{u}\|_0+\left\|f-\overline{f}\right\|_0\leq$$
$$\leq M_1\left[\|g-\overline{g}\|_0+\|\varphi-\overline{\varphi}\|_2+\left\|\psi-\overline{\psi}\right\|_{2,1}+\left\|h-\overline{h}\right\|_{2,1}\right] \qquad (4)$$

where $\|\upsilon\|_l=\sum\limits_{k=1}^m\|\upsilon_k\|_{C^l}$, $\left\{\overline{f}_k(x',t),\overline{u}_k(x,t)\right\}$ *is a solution of problem (1)-(3) from the set* K *with data* $\bar g_k,\bar\varphi_k,\bar\psi_k,\bar h_k$*, which satisfy conditions* 1^0-3^0*, respectively,* $M>0$ *depends on data of problem and on the set* K*.*

Everywhere below we denote by M_i the positive constants, which depend both on data of problem and on sets K, and which depend only on data of problem we denote by N_i.

Proof. Integrating both parts of equation (1) by variable x_n on the interval (a,b) and taking conditions of theorem 1 into account, for function $f_k(x',t)$, $k=\overline{1,m}$ we get:

$$f_k\left(x',t\right)=\left[h_{kt}\left(x',t\right)-\Delta h_k\left(x',t\right)-u_{kx_n}\left(x',b,t\right)+u_{kx_n}\left(x',a,t\right)\right]/$$
$$\int\limits_a^b g_k(u)\,dx_n,\quad \left(x',t\right)\in\overline{Q}' \qquad (5)$$

Determine a function [8, p.87]

$$p_k(x,t)\in C^{2+\alpha,1+\alpha/2}\left(\overline{Q}\right),\ p_k(x,0)=\varphi_k(x),$$
$$x\in\overline{D},\ p_k(x,t)=\psi_k(x,t),\ k=\overline{1,m},\quad (x,t)\in S \qquad (6)$$

Let

$$z_k(x,t)=u_k(x,t)-\bar u_k(x,t),\quad \lambda_k\left(x',t\right)=f_k\left(x',t\right)-\overline{f}_k\left(x',t\right),$$
$$\delta_{1k}(u)=g_k(u)-\bar g_k(u),\quad \delta_{2k}(x)=\varphi_k(x)-\overline{\varphi}_k(x),$$

$$\delta_{3k}(x,t)=\psi_k(x,t)-\overline{\psi}_k(x,t),\quad \delta_{4k}(x',t)=h_k(x',t)-\overline{h}_k(x',t),$$

$$\delta_{5k}(x,t)=p_k(x,t)-\overline{p}_k(x,t),\quad k=\overline{1,m}.$$

It is easy to verify that functions $\{\lambda_k(x',t),\vartheta_\kappa(x,t)=z_k(x,t)-\delta_{5k}(x,t)\}$ satisfy the following system:

$$\vartheta_{\kappa t}-\Delta\vartheta_\kappa=\lambda_k(x',t)\,g_k(u)+F_k(x,t),\ (x,t)\in Q, \tag{7}$$

$$\vartheta_{\kappa t}(x,0)=0, x\in\overline{D};\ \vartheta_k(x,t)=0,\quad (x,t)\in S, \tag{8}$$

$$\lambda_k(x',t)=\left[\delta_{4kt}(x',t)-\Delta\delta_{4k}(x',t)-z_{kx_n}(x',b,t)+z_{kx_n}(x',a,t)\right]/$$
$$/\int_a^b g_k(u)\,dx_n+H_k(x',t),\quad (x',t)\in\overline{Q}', \tag{9}$$

where

$$F_k(x,t)=\bar{f}_k(x',t)\left[g_k(u)-\bar{g}_k(\bar{u})\right]-\delta_{5kt}(x,t)+\Delta\delta_{5k}(x',t),$$

$$H_k(x',t)=\left\{\left[\bar{h}_{kt}(x',t)-\Delta\bar{h}_k(x',t)-\bar{u}_{kx_n}(x',b,t)+\bar{u}_{kx_n}(x',a,t)\right]\right.$$
$$\left.\int_a^b\left[\bar{g}_k(\bar{u})-g_k(u)\right]dx_n\right\}/\left[\int_a^b g_k(u)\,dx_n\int_a^b\bar{g}_k(\bar{u})\,dx_n\right]$$

By the conditions of the theorem, it follows that coefficients and the right hand side of the equation (7) satisfy Holder condition. So, there exists classical solution of problems (7) and (8) and it can be represented in the form [7, p.468]:

$$\vartheta_\kappa(x,t)=\int_0^t\int_D G_k(x,t;\xi,\tau)\left[\lambda_k(\xi',\tau)\,g_k(u)+F_k(\xi,\tau)\right]d\xi d\tau \tag{10}$$

where $\xi'=(\xi_1,...,\xi_{n-1})$, $\xi=(\xi',\xi_n)$, $d\xi=d\xi_1...d\xi_n, G_k(\cdot)$ is a Green function of problem (7) and (8), for which the following estimations are true [7, ch. IV]:

$$|G_k(x,t;\xi,\tau)|\le N_1(t-\tau)^{-n/2}\exp\left(-N_2|x-\xi|^2/(t-\tau)\right),$$

$$\int_D\left|D_x^lG_k(x,t;\xi,\tau)\right|d\xi\le N_3(t-\tau)^{-(l-\alpha)/2},\quad l=0,1,2,\ k=\overline{1,m} \tag{11}$$

here D_x^l are the various derivatives by $x_i\ (i=\overline{1,n})$ of order l.

Taking $\vartheta_k(x,t)=z_k(x,t)-\delta_{5k}(x,t)$, into account, we get from (10):

$$z_k(x,t)=\delta_{5k}(x,t)+\int_0^t\int_D G_k(x,t;\xi,\tau)\left[\lambda_k(\xi',\tau)\,g_k(u)+F_k(\xi,\tau)\right]d\xi d\tau \tag{12}$$

Suppose $\chi\equiv\|u-\bar{u}\|_0+\left\|f-\bar{f}\right\|_0$.

Subject to conditions of the theorem and from the definition of the set K, and taking estimations (11) into account, from (12) and (9) we get:

$$|z_k(x,t)|\le M_2\left[\|\delta_1\|_0+\|\delta_5\|_{2,1}\right]+M_3\chi t,\ (x,t)\in\overline{Q} \tag{13}$$

$$\left|\lambda_k\left(x',t\right)\right| \leq M_4\left\lfloor\|\delta_1\|_0+\|\delta_4\|_{2,1}+\|\delta_5\|_{2,1}\right\rfloor+M_5\chi t^{(1+\alpha)/2},\ \left(x',t\right)\in\overline{Q}' \qquad (14)$$

Inequalities (13) and (14) are satisfied at for any values of $(x,t)\in\overline{Q}$. Therefore, they should be satisfied also for maximal values of the left hand sides. Hence

$$\chi \leq M_6\left\lfloor\|\delta_1\|_0+\|\delta_4\|_{2,1}+\|\delta_5\|_{2,1}\right\rfloor+M_7\chi t^{(1+\alpha)/2} \qquad (15)$$

Let $T_1\,(0<T_1\leq T)$ be such that $M_7T_1^{(1+\alpha)/2}<1$. Then, form (15) we get, that for $(x,t)\in\overline{D}\times[0,T_1]$ estimation of stability (4) for solution of problem (1)-(3) is true.

Considering problem (1)-(3) step by step in cylinders $\overline{D}\times[T_1,2T_1]$, $\overline{D}\times[2T_1,3T_1]$ and etc. for the finite number of steps we'll get estimation of stability (4) on all $\overline{D}\times[0,T]$.

Uniqueness of solution of problem (1)-(3) follows from the estimation of (4) for

$$g_k(u)=\overline{g}_k(u),\ \varphi_k(x)=\overline{\varphi}_k(x),\ \psi_k(x,t)=\overline{\psi}_k(x,t),\ h_k(x,t)=\overline{h}_k\left(x',t\right)$$

Existence of solution of problem (1)-(3), in terms of Definition 1, is proved by convergence method. To find $\left\{f_k^{(s)}\left(x',t\right),u_k^{(s)}(x,t)\right\}$, $k=\overline{1,m}$, $s=1,2,...$ we use the following algorithm:

$$u_{kt}^{(s+1)}-\Delta u_k^{(s+1)}=f_k^{(s)}\left(x',t\right)g_k\left(u^{(s)}\right),(x,t)\in Q \qquad (16)$$

$$u_k^{(s+1)}(x,0)=\varphi_k(x),\ x\in\overline{D};\ u_k^{(s+1)}(x,t)=\psi_k(x,t)\ \ (x,t)\in S \qquad (17)$$

$$f_k^{(s+1)}\left(x',t\right)=\left[h_{kt}\left(x',t\right)-\Delta h_k\left(x',t\right)-u_{kx_n}^{(s+1)}\left(x',b,t\right)+u_{kx_n}^{(s+1)}\left(x',a,t\right)\right]/$$

$$/\int_a^b g_k\left(u^{(s+1)}\right)dx_n,\quad (x,t)\in\overline{D}' \qquad (18)$$

Theorem 2. *Let conditions* 1^0-4^0 *hold and* $\partial D\in C^{2+\alpha}$. *Then problem (1)-(3) has a unique solution for* $(x,t)\in D\times[0,T]$.

Proof. First, prove existence of solution of problem (1)-(3) from the class $f_k\left(x',t\right)\in C\left(\overline{Q}'\right)$, $u_k(x,t)\in C^{2,1}\left(\overline{Q}\right)$, $k=\overline{1,m}$.

It is easy to check, that if we choose $u_k^{(0)}(x,t)\in C^{2+\alpha,1+\alpha/2}\left(\overline{Q}\right)$, $f_k^{(0)}\left(x',t\right)\in C^{\alpha,\alpha/2}\left(\overline{Q}'\right)$, then on conditions of theorem 2 $u_k^{(1)}(x,t)\in C^{2+\alpha,1+\alpha/2}\left(\overline{Q}\right)$ [7,p. 364]. Then, from (18), on conditions of theorem 2, it follows, that $f_k^{(1)}\left(x',t\right)\in C^{\alpha,\alpha/2}\left(\overline{Q}'\right)$. Consequently, we can assert, that functions $f_k^{(k)}\left(x',t\right)$ and $u_k^{(s)}(x,t)$ $k=\overline{1,m}$, obtained from system (16)-(18) for $s=1,2,...$ belong to the functional spaces $C^{\alpha,\alpha/2}\left(\overline{Q}'\right)$ and $C^{2+\alpha,1+\alpha/2}\left(\overline{Q}\right)$, respectively. Show the uniform boundedness of sequences $\left\{f_k^{(s)}\left(x',t\right)\right\}$, $\left\{D_x^lu_k^{(s)}(x,t)\right\}$, $l=0,1,2$, $k=\overline{1,m}$.

Using functions $p_k(x,t)$, defined in (6), and presentation of solution by Green function [7, c.468], we find expression of solution of problem on determination of $u_k^{(s+1)}(x,t)$ from the conditions (16), (17)

$$u_k^{(s+1)}(x,t) = p_k(x,t) + \int_0^t \int_D G_k(x,t,\xi,\tau) \times$$

$$\times \left[f_k^{(s)}(\xi',\tau)\, g_k\left(u^{(s)}\right) + \Delta p_k - p_{k\tau} \right] d\xi d\tau \tag{19}$$

Acting as in proving theorem 1 and taking estimations (11) and conditions of theorem 2 into account, from (18) and (19) we get:

$$\left| D_x^l u_k^{(s+1)}(x,t) \right| \le N_4 \|p\|_{2,1} + N_5 t^{(2+\alpha-k)/2} \times \left| f_k^{(s)}(x',t) \right|, \quad l=0,1,2, \ (x,t) \in \overline{Q},$$

$$\left| f_k^{(s+1)}(x',t) \right| \le N_6 \|h\|_{2,1} + N_7 t^{\alpha/2} \sum_{l=0}^{2} \left| D_x^l u_k^{(s+1)}(x,t) \right|, (x,t) \in \overline{Q}',$$

or

$$\gamma^{(s+1)} \le N_8 \left[\|h\|_{2,1} + \|p\|_{2,1} \right] + N_9 t^{\alpha/2} \gamma^{(s)}$$

where $\gamma^{(s)} = \sum_{l=0}^{2} \left\| D_x^l u^{(s)} \right\|_0 + \left\| f^{(s)} \right\|_0$

From the last inequality we have:

$$\gamma^{(s+1)} \le N_8 \left[\|h\|_{2,1} + \|p\|_{2,1} \right] (1-q^s)/(1-q) + q^s \gamma^{(0)}, q = N_9 t^{\alpha/2}$$

Let $T_2\ (0 < T_2 \le T)$ be such that $N_9 T_2^{\alpha/2} < 1$. Then we get that subsequences $\left\{ f_k^{(s)} \right\}, \left\{ D_k^l u_k^{(s)} \right\}$, $l=0,1,2$, $k=\overline{1,m}$, are uniformly (by norm C) bounded at $(x,t) \in \overline{D} \times [0,T_2]$.

Analogously, as it was shown in proving theorem 1, we prove uniform boundedness of the sequences $\left\{ f_k^{(s)} \right\} \left\{ D_k^l u_k^{(s)} \right\}$, $l=0,1,2$, $k=\overline{1,m}$ at all $t \in [0,T]$.

Equicontinuity property of the sequences $\left\{ D_x^l u_k^{(s)} \right\}$, $l=0,1,2$ follows from the inequality

$$\left| D_x^l u_k^{(s+1)}(x,t) - D_x^l u_k^{(s+1)}(\bar{x},\bar{t}) \right| \le \left| D_x^l u_k^{(s+1)}(x,t) - D_x^l u_k^{(s+1)}(\bar{x},t) \right| +$$

$$+ \left| D_x^l u_k^{(s+1)}(\bar{x},t) - D_x^l u_k^{(s+1)}(\bar{x},\bar{t}) \right| \le \left| D_x^l p_k(x,t) - D_x^l p_k(\bar{x},t) \right| +$$

$$+ \left| D_x^l p_k(\bar{x},t) - D_x^l p_k(\bar{x},\bar{t}) \right| \le \int_0^t \int_D \left| D_x^l G_k(x,t;\xi,\tau) - D_x^l G_k(\bar{x},t;\xi,\tau) \right| \times$$

$$\times \left| F_k^{(s)}(\xi,\tau) \right| d\xi d\tau + \int_0^{\bar{t}} \int_D \left| D_x^l G_k(\bar{x},t;\xi,\tau) - D_x^l G_k(\bar{x},\bar{t};\xi,\tau) \right| \times$$

$$\times \left| F_k^{(s)}(\xi,\tau) \right| d\xi d\tau + \int_{\bar{t}}^{t} \int_D \left| D_x^l G_k(\bar{x},t;\xi,\tau) \right| \left| F_k^{(s)}(\xi,\tau) \right| d\xi d\tau$$

(here $F_k^{(s)}(x,t) = f_k^{(s)}(x',t)\, g_k\left(u^{(s)}\right) + \Delta p_k - p_{kt}$) subject to uniform boundedness $\left\{ f_k^{(s)}(x,t) \right\}$, continuity and boundedness of input data, estimations (11) and the following [7, p.469]

$$\left| D_x^l G_k(x,t;\xi,\tau) - D_{\bar{x}}^l G_k(\bar{x},t;\xi,\tau) \right| \le N_{10} \left| x-\bar{x} \right|^{\alpha} (t-\tau)^{-(n+2+\alpha)/2} \times$$

$$\times \exp\left(-N_{11} \left| x-\xi \right|^2 / (t-\tau) \right);$$

$$\left| D_x^l G_k(x,t;\xi,\tau) - D_x^l G_k(x,\bar{t};\xi,\tau) \right| \le N_{12} \left| t-\bar{t} \right|^{(2+\alpha-k)/2} (\bar{t}-\tau)^{-(n+2+\alpha)/2} \times$$

$$\times \exp\left(-N_{13} \left| x-\xi \right|^2 / (t-\tau) \right), \ k=\overline{1,m}, \ l=0,1,2.$$

Equicontinuity property of sequence $\left\{ f_k^{(s)}(x',t) \right\}$ follows from the inequality:

$$\left| f_k^{(s)}(x',t) - f_k^{(s)}(\bar{x}',\bar{t}) \right| \le \left| f_k^{(s)}(x',t) - f_k^{(s)}(\bar{x}',t) \right| + \left| f_k^{(s)}(\bar{x}',t) - f_k^{(s)}(\bar{x}',\bar{t}) \right| \le$$

$$\le \left[\left| h_{kt}(x',t) - h_{kt}(\bar{x}',\bar{t}) \right| + \left| \Delta h_k(x',t) - \Delta h_k(\bar{x}',t) \right| + \right.$$

$$\left. + \left| u_{kx_n}^{(s+1)}(x',b,t) - u_{kx_n}^{(s+1)}(\bar{x}',b,t) \right| + \left| u_{kx_n}^{(s+1)}(x',a,t) - u_{kx_n}^{(s+1)}(\bar{x}',a,t) \right| \right] /$$

$$/ \left| \int_a^b g_k\left(u^{(s+1)}(x',x_n,t) \right) dx_n \right| + \left\{ \left| h_{kt}(\bar{x}',t) - \Delta h_k(\bar{x}',t) - \right. \right.$$

$$\left. - u_{kx_n}^{(s+1)}(\bar{x}',a,t) - u_{kx_n}^{(s+1)}(\bar{x}',a,t) \right| \left| \int_a^b \left[g_k\left(u^{(s+1)}(x',x_n,t) \right) - \right. \right.$$

$$\left. \left. - g_k\left(u^{(s+1)}(\bar{x}',x_n,t) \right) \right] dx_n \right| \Big\} / \left| \int_a^b g_k\left(u^{(s+1)}(x',x_n,t) \right) dx_n \times \right.$$

$$\left. \times \int_a^b g_k\left(u^{(s+1)}(\bar{x}',x_n,t) \right) dx_n \right| + \left[\left| h_{kt}(\bar{x}',t) - h_{kt}(\bar{x}',\bar{t}) \right| + \right.$$

$$+ \left| \Delta h_k(\bar{x}',t) - \Delta h_k(\bar{x}',\bar{t}) \right| + \left| u_{kx_n}^{(s+1)}(\bar{x}',b,t) - u_{kx_n}^{(s+1)}(\bar{x}',b,\bar{t}) \right| +$$

$$\left. + \left| u_{kx_n}^{(s+1)}(\bar{x}',a,t) - u_{kx_n}^{(s+1)}(\bar{x}',a,\bar{t}) \right| \right] / \left| \int_a^b g_k\left(u^{(s+1)}(\bar{x}',x_n,t) \right) dx_n \right| +$$

$$+ \left\{ \left| h_{kt}(\bar{x}',\bar{t}) - \Delta h_k(\bar{x}',\bar{t}) - u_{kx_n}^{(s+1)}(\bar{x}',b,\bar{t}) + u_{kx_n}^{(s+1)}(\bar{x}',a,\bar{t}) \right| \times \right.$$

$$\left. \times \left| \int_a^b \left[g_k\left(u^{(s+1)}(\bar{x}',x_n,t) \right) - g_k\left(u^{(s+1)}(\bar{x}',x_n,\bar{t}) \right) \right] dx_n \right| \right\} /$$

$$/ \left| \int_a^b g_k\left(u^{(s+1)}(\bar{x}',x_n,t) \right) dx_n \times \int_a^b g_k\left(u^{(s+1)}(\bar{x}',x_n,\bar{t}) \right) dx_n \right|.$$

subject to uniform boundedness and equicontinuity property of subsequences $\left\{ D_x^l u_k^{(s)} \right\}$, $l=0,1,2$, continuity and boundedness of input data.

Uniform boundedness and equicontinuity property of subsequence $\left\{u_{kt}^{(s)}\right\}$ follows from (16).

By Arzela theorem [8, p.84] from subsequences $\left\{u_{kt}^{(s)}\right\}$, $\left\{D_x^l u_k^{(s)}\right\}$, $l=0,1,2$, $\left\{f_k^{(s)}\right\}$, $k=\overline{1,m}$ we can choose subsequences, which converge to some functions $\{u_{kt}^*\}$, $\{D_x^l u_k^*\}$, $l=0,1,2$, $\{f_k^*\}$, respectively and $u_k^*(x,t)\in C^{2,1}\left(\overline{Q}\right)$, $f_k^*(x,t)\in C\left(\overline{Q}'\right)$, $k=\overline{1,m}$.

Then, passing to the limit at $s\to\infty$ in correlations (16)-(18), it is easy to show, that function $\left\{f_k^*(x',t),\ u_k^*(x,t),\ k=\overline{1,m}\right\}$ satisfy conditions (1)-(3).

Thus, existence of solution of problems (1)-(3) from the class $f_k(x',t)\in C\left(\overline{Q}'\right)$, $u_k(x,t)\in C^{2,1}\left(\overline{Q}\right)$ is proved. We could assert, that such solution belongs to the set K. Indeed, from conditions 1^0, 3^0 and correlation (5) it follows, that $f_k(x',t)\in C^{\alpha,\alpha/2}\left(\overline{Q}'\right)$, $k=\overline{1,m}$. Then, for conditions of theorem 2, $u_k(x,t)\in C^{2+\alpha,1+\alpha/2}\left(\overline{Q}\right)$, $k=\overline{1,m}$ [7, p.364]. Thus, conditions of theorem 1 are satisfied, and consequently, solution of problem (1)-(3) is unique. The theorem is proved completely.

References

[1]. Iskenderov A.D. Izv. AN Azerb. SSR, ser. of phys. Tech. and math. sci., 1976, No2, pp.58-63. (Russian)

[2]. Isakov V.M. DAN SSSR, 1982, v.263, No6, pp.1296-1299. (Russian)

[3]. Prilepko A.I., Kostin A.B. *Math. sb.*, 1992, v.183, No4, pp.49-68. (Russian)

[4]. Savateev E.G. DAN, 1995, v.344, No5, pp.597-598. (Russian)

[5]. Solovyev V.V. *Differential. uravn.* 1996, v.32, No4, pp.536-544. (Russian)

[6]. Akhundov A.Ya. Proceedings of Institute of Mathematics and Mechanics, 2002, v.XVII, p.3-9.

[7]. Ladyzhenskaya O.A., Solonikov V.A., Uraltseva N.I. *Linear and quasilinear parabolic equations.* M.: 1967. (Russian)

[8]. Fridman A. *Partial equations of parabolic type.* M.: 1968. (Russian)

[9]. Henry D. *Geometric theory of semilinear parabolic equations.* M.: 1985. (Russian)

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