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FORCED VIBRATIONS OF A SPHERICAL SHELL WHEN TANGENTIAL DISPLACEMENTS AND RADIAL STRAINS DON'T EXIST ON ITS FACE

Abstract

In the paper we analyse non-axial symmetrical dynamic mixed boundary value problem of elasticity theory for a spherical shell. Due to spherical symmetry the general boundary value problem is divided into the potential and whirlwind problems and investigated. Firstly this problems is exactly solved, after this the roots of variance equations of the spectral problems derived from them asymptotically investigated, corresponding different group roots (special value) for setting displacements and strains situation is obtained simple asymptotic formulae of a spherical shell.

Consider the forced vibration of are spherical shell under the homogeneous boundary conditions on its face

$$\sigma_r = 0, \quad u_\theta = 0, \quad u_\varphi = 0 \quad \text{when } r = r_k \ \ (k = 1, 2), \tag{1}$$

and the following boundary conditions are given on the other part of the boundary

$$\sigma_\theta = Q_\theta^{(s)}(r,\varphi)\, e^{i\omega t}, \ \ \tau_{r\theta} = Q_{r\theta}^{(s)}(r,\varphi)\, e^{i\omega t}, \quad \tau_{r\varphi} = Q_{r\varphi}^{(s)}(r,\varphi)\, e^{i\omega t}$$
$$\text{when} \ \ \theta = \theta_s \, (s = 1, 2). \tag{2}$$

Here the other non-homogeneous boundary conditions are possible, too.

Using the results of [1,2,3] we obtain the following boundary problems

$$\begin{cases} L_1(u,\Phi) = 0, \\ L_2(u,\Phi) = 0, \end{cases} \tag{3}$$

$$\begin{cases} \left[(1-\nu)\frac{\partial u}{\partial r} + \frac{2\nu}{r} u\right]_{r=r_k} = 0, \\ \\ [\Phi]_{r=r_k} = 0 \qquad\qquad (k = 1, 2). \end{cases} \tag{4}$$

and

$$L_3(F) = 0, \tag{5}$$

$$[F]_{r=r_k} = 0 \quad (k = 1, 2) \tag{6}$$

where

$$L_1(u,\Phi) = \frac{2(1-\nu)}{1-2\nu}\left(\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} - \frac{2}{r^2}u\right) + \frac{1}{r^2}\Delta_0 u +$$
$$+\frac{1}{1-2\nu}\left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{4\nu-3}{r^2}\right)\Delta_0\Phi + \lambda^2 u,$$
$$L_2(u,\Phi) = \frac{1}{1-2\nu}\frac{1}{r}\left[\frac{\partial u}{\partial r} + \frac{4-4\nu}{r}u\right] + \frac{2(1-\nu)}{1-2\nu}\frac{1}{r^2}\Delta_0\Phi +$$

$$+\frac{\partial^2\Phi}{\partial r^2}+\frac{2}{r}\frac{\partial\Phi}{\partial r}+\lambda^2\Phi,$$

$$L_3(F)=\frac{\partial^2 F}{\partial r^2}+\frac{2}{r}\frac{\partial F}{\partial r}+\frac{1}{r^2}\Delta_0 F+\lambda^2 F,\ \Delta=\frac{\partial^2}{\partial\theta^2}+ctg\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}.$$

Here r,θ,φ are spherical coordinates; u_r,u_θ,u_φ are components of displacement vector; $\sigma_r,\sigma_\theta,\sigma_\varphi,\tau_{r\theta},\tau_{\theta\varphi}$ are components of strain tensor; G is Lame coefficient; E elasticity module of the material; ω is frequency of vibrations; ν is a Poisson coefficient; g density of the material of shell; ε is a small parameter characterizing the thickness of a spherical shell; λ is non-dimensional frequency parameter; z is a spectral parameter. Using the results of [3,4], from (3), (4) and (5), (6) we obtain the following boundary problems

$$\begin{cases}\left(a'+\frac{2}{\rho}a\right)'+\frac{1-2\nu}{2(1-\nu)}\left(\lambda^2-\frac{z^2-\frac14}{\rho^2}\right)a+\frac{1}{2(1-\nu)}\frac{1}{\rho}\left(z^2-\frac14\right)\left[b'-\frac{3-4\nu}{\rho}b\right]=0\\ \frac{1}{1-2\nu}\left(\frac{1}{\rho}a'+\frac{4-4\nu}{\rho^2}a\right)+b''+\frac{2}{\rho}b'\left[\lambda^2-\frac{2(1-\nu)}{1-2\nu}\frac{z^2-\frac14}{\rho^2}\right]b=0.\end{cases}\tag{7}$$

$$\begin{cases}\left[(1-\nu)a'+\frac{2\nu}{\rho}a\right]_{\rho=\rho_k}=0,\\ b|_{\rho=\rho_k}=0\qquad (k=1,2).\end{cases}\tag{8}$$

$$\psi''+\frac{2}{\rho}\psi'+\left[\lambda^2-\frac{1}{\rho^2}\left(z^2-\frac14\right)\right]\psi=0,\tag{9}$$

$$\psi=0\quad\text{when}\quad\rho=\rho_k\,(k=1,2).\tag{10}$$

Boundary problems (7), (8) describe potential motion, boundary problems (9), (10) describe whirlwind motion of a shell [2,4].

As is known the solution of a system of equations (7) and (9) has the following form, respectively:

$$a(\rho)=\frac{1}{\sqrt{\rho}}\left\{C_{1z}\left[\alpha J_z'(\alpha\rho)-\frac{1}{2\rho}J_z(\alpha\rho)\right]+C_{2z}\left[\alpha Y_z'(\alpha\rho)-\frac{1}{2r}Y_z(\alpha\rho)\right]-\right.$$

$$\left.-\frac{1}{\rho}\left(z^2-\frac14\right)J_z(\lambda\rho)C_{3z}-\frac{1}{\rho}\left(z^2-\frac14\right)Y_z(\lambda\rho)C_{4z}\right\},\tag{11}$$

$$b(\rho)=\frac{1}{\sqrt{\rho}}\left\{C_{1z}\frac{1}{\rho}J_z(\alpha\rho)+C_{2z}\frac{1}{\rho}Y_z(\alpha\rho)-C_{3z}\left[\lambda J_z'(\lambda\rho)+\frac{1}{2\rho}J_z(\lambda\rho)\right]-\right.$$

$$\left.-C_{4z}\left[\lambda Y_z'(\lambda\rho)+\frac{1}{2\rho}Y_z(\lambda\rho)\right]\right\}$$

and

$$\psi(\rho)=\frac{1}{\sqrt{\rho}}\left[C_{5z}J_z(\lambda\rho)+C_{6z}Y_z(\lambda\rho)\right].\tag{12}$$

Here $J_z(x)$, $Y_z(x)$ are the Bessel functions of the first and second kind, respectively, $C_{iz}\left(i=\overline{1,6}\right)$ are arbitrary constants, $\rho=r/R_0$ is non-dimensional radial coordinate, $R_0=(r_1+r_2)/2$ is radius of middle surface of a spherical shell.

Satisfying the homogeneous boundary conditions (8), (10) we obtain variance equations that have the following form, respectively

$$\Delta_1 = \frac{1}{\rho_1\rho_2}\left\langle\left(z^2-\frac{1}{4}\right)\left\{\left(z^2-\frac{1}{4}\right)L_z^{(0,0)}(\alpha)\left\{\left[-4\lambda^2\rho_1\rho_2L_z^{(1,1)}(\lambda)+\right.\right.\right.\right.$$

$$\left.+6\lambda\rho_2L_z^{(0,1)}(\lambda)+6\lambda\rho_1L_z^{(1,0)}(\lambda)-9L_z^{(0,0)}(\lambda)\right]+\nu\left[8\lambda^2\rho_1\rho_2L_z^{(1,1)}(\lambda)-\right.$$

$$\left.-28\lambda\rho_2L_z^{(0,1)}(\lambda)-28\lambda\rho_1L_z^{(1,0)}(\lambda)+42L_z^{(0,0)}(\lambda)\right]-\nu^2\left[4\lambda^2\rho_1\rho_2L_z^{(1,1)}(\lambda)-\right.$$

$$\left.\left.-14\lambda\rho_2L_z^{(0,1)}(\lambda)-14\lambda\rho_1L_z^{(1,0)}(\lambda)+49L_z^{(0,0)}(\lambda)\right]\right\}+\left[2\lambda^2\rho_1\rho_2L_z^{(1,1)}(\lambda)-\right.$$

$$\left.-3\lambda\rho_1L_z^{(1,0)}(\lambda)+\lambda\rho_2L_z^{(0,1)}(\lambda)-\frac{3}{2}L_z^{(0,0)}(\lambda)\right]\left\{\left[\varphi_4(z,\rho_1)L_z^{(0,0)}(\alpha)-\right.\right.$$

$$\left.\left.-4\alpha\rho_1L_z^{(1,0)}(\alpha)\right]-\nu\left[\varphi_5(z,\rho_1)L_z^{(0,0)}(\alpha)-8\alpha\rho_1L_z^{(1,0)}(\alpha)\right]\right\}+$$

$$+\left[2\lambda^2\rho_1\rho_2L_z^{(1,1)}(\lambda)-7\lambda\rho_1L_z^{(1,0)}(\lambda)+\lambda\rho_2L_z^{(0,1)}(\lambda)-\frac{7}{2}L_z^{(0,0)}(\lambda)\right]\times$$

$$\times\left\{\nu^2\left[\varphi_5(z,\rho_1)L_z^{(0,0)}(\alpha)-8\alpha\rho_1L_z^{(1,0)}(\alpha)\right]-\nu\left[\varphi_4(z,\rho_1)L_z^{(0,0)}(\alpha)-\right.\right.$$

$$\left.\left.-\ 4\alpha\rho_1L_z^{(1,0)}(\alpha)\right]\right\}+\left[2\lambda^2\rho_1\rho_2L_z^{(1,1)}(\lambda)-3\lambda\rho_2L_z^{(0,1)}(\lambda)+\lambda\rho_1L_z^{(1,0)}(\lambda)-\right.$$

$$\left.-\frac{3}{2}L_z^{(0,0)}(\lambda)\right]\left\{\left[\varphi_4(z,\rho_2)L_z^{(0,0)}(\alpha)-4\alpha\rho_2L_z^{(0,1)}(\alpha)\right]-\right.$$

$$\left.-\nu\left[\varphi_5(z,\rho_2)L_z^{(0,0)}(\alpha)-8\alpha\rho_2L_z^{(0,1)}(\alpha)\right]\right\}+\left[2\lambda^2\rho_1\rho_2L_z^{(1,1)}(\lambda)-\right. \tag{13}$$

$$\left.-7\lambda\rho_2L_z^{(0,1)}(\lambda)+\lambda\rho_1L_z^{(1,0)}(\lambda)-\frac{7}{2}L_z^{(0,0)}(\lambda)\right]\left\{\nu^2\left[\varphi_5(z,\rho_2)L_z^{(0,0)}(\alpha)-\right.\right.$$

$$\left.\left.-8\alpha\rho_2L_z^{(0,1)}(\alpha)\right]-\nu\left[\varphi_4(z,\rho_2)L_z^{(0,0)}(\alpha)-4\alpha\rho_2L_z^{(0,1)}(\alpha)\right]\right\}-$$

$$\left.-\frac{128(1-2\nu)^2}{\pi^2}\right\}-\left[\lambda^2\rho_1\rho_2L_z^{(1,1)}(\lambda)+\frac{1}{2}\lambda\rho_2L_z^{(0,1)}(\lambda)+\right.$$

$$\left.+\frac{1}{2}\lambda\rho_1L_z^{(1,0)}(\lambda)+\frac{1}{4}L_z^{(0,0)}(\lambda)\right]\left\{\left[16\alpha^2\rho_1\rho_2L_z^{(1,1)}(\alpha)-\right.\right.$$

$$-4\alpha\rho_1\varphi_4(z,\rho_2)L_z^{(1,0)}(\alpha)-4\alpha\rho_2\varphi_4(z,\rho_1)L_z^{(0,1)}(\alpha)+$$

$$\left.+\ \varphi_4(z,\rho_1)\varphi_4(z,\rho_2)L_z^{(0,0)}(\alpha)\right]-\nu\left[64\alpha^2\rho_1\rho_2L_z^{(1,1)}(\alpha)-\right.$$

$$-4\alpha\rho_1\varphi_5(z,\rho_2)L_z^{(1,0)}(\alpha)-8\alpha\rho_2\varphi_4(z,\rho_1)L_z^{(0,1)}(\alpha)-4\alpha\rho_2\varphi_5(z,\rho_1)L_z^{(0,1)}(\alpha)-$$

$$-8\alpha\rho_1\varphi_4(z,\rho_2)L_z^{(1,0)}(\alpha)+(\varphi_4(z,\rho_1)\varphi_5(z,\rho_2)+\varphi_4(z,\rho_2)\varphi_5(z,\rho_1))\times$$

$$\left.\times L_z^{(0,0)}(\alpha)\right]+\nu^2\left[64\alpha^2\rho_1\rho_2L_z^{(1,1)}(\alpha)-8\alpha\rho_1\varphi_5(z,\rho_2)L_z^{(1,0)}(\alpha)-\right.$$

$$\left.\left.\left.-8\alpha\rho_2\varphi_5(z,\rho_1)L_z^{(0,1)}(\alpha)+\varphi_5(z,\rho_1)\varphi_5(z,\rho_2)L_z^{(0,0)}(\alpha)\right]\right\}\right\rangle=0.$$

and

$$\Delta_2 = L_z^{(0,0)}(\lambda) = 0 \tag{14}$$

where $\varphi_4(z,\rho) = 2z^2 + \frac{3}{2} - 2\alpha^2\rho^2$, $\varphi_5(z,\rho) = 2z^2 + \frac{7}{2} - 2\alpha^2\rho^2$, $\alpha^2 = \frac{1-2\nu}{2(1-\nu)}\lambda^2$,
$\lambda^2 = \frac{2(1+\nu)gR_0^2\omega^2}{E}$, $L_z^{(s,l)}(\beta) = J_z^{(s)}(\beta r_1)\, Y_z^{(l)}(\beta r_2) - J_z^{(l)}(\beta r_2)\, Y_z^{(s)}(\beta r_1)$

$(s,\ l = 0,1)$.

The transcendental equations (13) and (14) have a denumerable set of roots z_k, and constants corresponding to them C_{1z_k}, C_{2z_k}, C_{3z_k}, C_{4z_k}, C_{5z_k}, C_{6z_k} proportionately algebraically completive of elements arbitrary rows or system's determine. Solutions of systems we are choosing algebraically completive of elements of first rows. For boundary problems (7), (8) we have

$$C_{1z_k} = -\frac{8(1-2\nu)}{\pi\rho_1}\left(z_k^2 - \frac{1}{4}\right)\Big\{4\alpha\rho_2 Y'_{z_k}(\alpha\rho_2) - \varphi_4(z_k,\rho_2)\, Y_{z_k}(\alpha\rho_2) -$$
$$-\nu\Big[8\alpha\rho_2 Y'_{z_k}(\alpha\rho_2) - \varphi_5(z_k,\rho_2)\, Y_{z_k}(\alpha\rho_2)\Big]\Big\} + \big\{4\alpha\rho_1 Y'_{z_k}(\alpha\rho_1) -$$
$$-\varphi_4(z_k,\rho_1)\, Y_{z_k}(\alpha\rho_1) - \big[8\alpha\rho_1 Y'_{z_k}(\alpha\rho_1) - \varphi_5(z_k,\rho_1)\, Y_{z_k}(\alpha\rho_1)\big]\big\}\times$$
$$\times G_1(z_k,\lambda,\rho_1,\rho_2) + \frac{1}{\rho_1} Y_{z_k}(\alpha\rho_1)\, G_2(z_k,\lambda,\rho_1,\rho_2),$$

$$C_{2z_k} = \frac{8(1-2\nu)}{\pi\rho_1}\left(z_k^2 - \frac{1}{4}\right)\Big\{4\alpha\rho_2 J'_{z_k}(\alpha\rho_2) - \varphi_4(z_k,\rho_2)\, J_{z_k}(\alpha\rho_2) -$$
$$-\nu\big[8\alpha\rho_2 J'_{z_k}(\alpha\rho_2) - \varphi_5(z_k,\rho_2)\, J_{z_k}(\alpha\rho_2)\big]\big\} - \big\{4\alpha\rho_1 J'_{z_k}(\alpha\rho_1) -$$
$$-\varphi_4(z_k,\rho_1)\, J_{z_k}(\alpha\rho_1) - \nu\big[8\alpha\rho_1 J'_{z_k}(\alpha\rho_1) - \varphi_5(z_k,\rho_1)\, J_{z_k}(\alpha\rho_1)\big]\big\}\times$$
$$\times G_1(z_k,\lambda,\rho_1,\rho_2) - \frac{1}{\rho_1} J_{z_k}(\alpha\rho_1)\, G_2(z_k,\lambda,\rho_1,\rho_2),$$

$$C_{3z_k} = \frac{8(1-2\nu)}{\pi\rho_1}\left(z_k^2 - \frac{1}{4}\right)\Big\{2\lambda\rho_2 Y'_{z_k}(\lambda\rho_2) - 3Y_{z_k}(\lambda\rho_2) -$$
$$-\nu\Big[2\lambda\rho_2 Y'_{z_k}(\lambda\rho_2) - 7Y_{z_k}(\lambda\rho_2)\Big]\Big\} + \big\{2\lambda\rho_1 Y'_{z_k}(\lambda\rho_1) - 3Y_{z_k}(\lambda\rho_1) -$$
$$-\nu\big[2\lambda\rho_1 Y'_{z_k}(\lambda\rho_1) - 7Y_{z_k}(\lambda\rho_1)\big]\big\}\, F_1(z_k,\alpha,\rho_1,\rho_2) -$$
$$-\left[\lambda Y'_{z_k}(\lambda\rho_1) + \frac{1}{2\rho_1} Y_{z_k}(\lambda\rho_1)\right] F_2(z_k,\alpha,\rho_1,\rho_2), \tag{15}$$

$$C_{4z_k} = -\frac{8(1-2\nu)}{\pi\rho_1}\left(z_k^2 - \frac{1}{4}\right)\Big\{2\lambda\rho_2 J'_{z_k}(\lambda\rho_2) - 3J_{z_k}(\lambda\rho_2) -$$
$$-\nu\Big[2\lambda\rho_2 J'_{z_k}(\lambda\rho_2) - 7J_{z_k}(\lambda\rho_2)\Big]\Big\} - \big\{2\lambda\rho_1 J'_{z_k}(\lambda\rho_1) - 3J_{z_k}(\lambda\rho_1) -$$
$$-\nu\big[2\lambda\rho_1 J'_{z_k}(\lambda\rho_1) - 7J_{z_k}(\lambda\rho_1)\big]\big\}\, F_1(z_k,\alpha,\rho_1,\rho_2) +$$
$$+\left[\lambda J'_{z_k}(\lambda\rho_1) + \frac{1}{2\rho_1} J_{z_k}(\lambda\rho_1)\right] F_2(z_k,\alpha,\rho_1,\rho_2).$$

Here

$$G_1\left(z_k,\lambda,\rho_1,\rho_2\right)=\left(z_k^2-\frac{1}{4}\right)\left\{2\lambda^2\rho_2 L_{z_k}^{(1,1)}\left(\lambda\right)-3\lambda L_{z_k}^{(1,0)}\left(\lambda\right)+\frac{\rho_2}{\rho_1}\lambda L_{z_k}^{(0,1)}\left(\lambda\right)-\right.$$

$$-\frac{3}{2\rho_1}L_{z_k}^{(0,0)}\left(\lambda\right)-\nu\left[2\lambda^2\rho_2 L_{z_k}^{(1,1)}\left(\lambda\right)-7\lambda L_{z_k}^{(1,0)}\left(\lambda\right)+\right.$$

$$\left.\left.+\frac{\rho_2}{\rho_1}\lambda L_{z_k}^{(0,1)}\left(\lambda\right)-\frac{7}{2\rho_1}L_{z_k}^{(0,0)}\left(\lambda\right)\right]\right\},$$

$$G_2\left(z_k,\lambda,\rho_1,\rho_2\right)=\left(z_k^2-\frac{1}{4}\right)^2\left\{4\lambda^2\rho_1\rho_2 L_{z_k}^{(1,1)}\left(\lambda\right)-6\lambda\rho_2 L_{z_k}^{(0,1)}\left(\lambda\right)-\right.$$

$$-6\lambda\rho_1 L_{z_k}^{(1,0)}\left(\lambda\right)+9L_{z_k}^{(0,0)}\left(\lambda\right)-\nu\left[8\lambda^2\rho_1\rho_2 L_{z_k}^{(1,1)}\left(\lambda\right)-28\lambda\rho_2 L_{z_k}^{(0,1)}\left(\lambda\right)-\right.$$

$$\left.-28\lambda\rho_1 L_{z_k}^{(1,0)}\left(\lambda\right)+42L_{z_k}^{(0,0)}\left(\lambda\right)\right]+\nu^2\left[4\lambda^2\rho_1\rho_2 L_{z_k}^{(1,1)}\left(\lambda\right)-\right.$$

$$\left.\left.-14\lambda\rho_2 L_{z_k}^{(0,1)}\left(\lambda\right)-14\lambda\rho_1 L_{z_k}^{(1,0)}\left(\lambda\right)+49L_{z_k}^{(0,0)}\left(\lambda\right)\right]\right\},$$

$$F_1\left(z_k,\alpha,\rho_1,\rho_2\right)=\left(z_k^2-\frac{1}{4}\right)\left\{-4\alpha\frac{\rho_2}{\rho_1}L_{z_k}^{(0,1)}\left(\alpha\right)+\frac{1}{\rho_1}\varphi_4\left(z_k,\rho_2\right)L_{z_k}^{(0,0)}\left(\alpha\right)-\right.$$

$$\left.-\nu\left[-8\alpha\frac{\rho_2}{\rho_1}L_{z_k}^{(0,1)}\left(\alpha\right)+\frac{1}{\rho_1}\varphi_5\left(z_k,\rho_2\right)L_{z_k}^{(0,0)}\left(\alpha\right)\right]\right\},$$

$$F_2\left(z_k,\alpha,\rho_1,\rho_2\right)=16\alpha^2\rho_1\rho_2 L_{z_k}^{(1,1)}\left(\alpha\right)-4\alpha\rho_2\varphi_4\left(z_k,\rho_1\right)L_{z_k}^{(0,1)}\left(\alpha\right)-$$

$$-4\alpha\rho_1\varphi_4\left(z_k,\rho_2\right)L_{z_k}^{(1,0)}\left(\alpha\right)+\varphi_4\left(z_k,\rho_1\right)\varphi_4\left(z_k,\rho_2\right)L_{z_k}^{(0,0)}\left(\alpha\right)-$$

$$-\nu\left[64\alpha^2\rho_1\rho_2 L_{z_k}^{(1,1)}\left(\alpha\right)-4\alpha\rho_1\varphi_5\left(z_k,\rho_2\right)L_{z_k}^{(1,0)}\left(\alpha\right)-8\alpha\rho_2\varphi_4\left(z_k,\rho_1\right)L_{z_k}^{(0,1)}\left(\alpha\right)-\right.$$

$$-4\alpha\rho_2\varphi_5\left(z_k,\rho_1\right)L_{z_k}^{(0,1)}\left(\alpha\right)-8\alpha\rho_1\varphi_4\left(z_k,\rho_2\right)L_{z_k}^{(1,0)}\left(\alpha\right)+$$

$$\left.+\left(\varphi_4\left(z_k,\rho_1\right)\varphi_5\left(z_k,\rho_2\right)+\varphi_4\left(z_k,\rho_2\right)\varphi_5\left(z_k,\rho_1\right)\right)L_{z_k}^{(0,0)}\left(\alpha\right)\right]+$$

$$+\nu^2\left[64\alpha^2\rho_1\rho_2 L_{z_k}^{(1,1)}\left(\alpha\right)-8\alpha\rho_1\varphi_5\left(z_k,\rho_2\right)L_{z_k}^{(1,0)}\left(\alpha\right)-\right.$$

$$\left.-8\alpha\rho_2\varphi_5\left(z_k,\rho_1\right)L_{z_k}^{(0,1)}\left(\alpha\right)+\varphi_5\left(z_k,\rho_1\right)\varphi_5\left(z_k,\rho_2\right)L_{z_k}^{(0,0)}\left(\alpha\right)\right].$$

For boundary problems (9), (10) obtain

$$C_{5z_k}=Y_{z_k}\left(\lambda\rho_2\right),\qquad C_{6z_k}=-J_{z_k}\left(\lambda\rho_2\right).\tag{16}$$

We denote the displacements and strains corresponding to the potential problem by the upper index "1" and to the whirlwind problem - by the upper index "2".

Substituting (15) in (11) and (16) is (12) and according to the generalized Hook's law, we obtain of the homogeneous solutions of the form, respectively.

$$u_\rho^{(1)}=\frac{1}{\sqrt{\rho}}\sum_{k=1}^{\infty}C_k u_{\rho k}T_k\left(\theta,\varphi\right)e^{i\omega t},\qquad u_\theta^{(1)}=\frac{1}{\sqrt{\rho}}\sum_{k=1}^{\infty}C_k u_{\theta k}\frac{\partial T_k}{\partial\theta}e^{i\omega t},\tag{17}$$

$$u_{\varphi}^{(1)}=\frac{1}{\sqrt{\rho}}\sum_{k=1}^{\infty}C_k u_{\theta k}\frac{1}{\sin\theta}\frac{\partial T_k}{\partial\varphi}e^{i\omega t}. \qquad \sigma_{\rho}^{(1)}=\frac{\overline{G}}{\rho^2\sqrt{\rho}}\sum_{k=1}^{\infty}C_k\sigma_{\rho k}T_k\left(\theta,\varphi\right)e^{i\omega t},$$

$$\sigma_{\theta}^{(1)}=\frac{\overline{G}}{\rho^2\sqrt{\rho}}\sum_{k=1}^{\infty}C_k\left[\sigma_{\theta k}T_k\left(\theta,\varphi\right)+\sigma_{\varphi k}\frac{\partial^2 T_k}{\partial\theta^2}\right]e^{i\omega t},$$

$$\sigma_{\varphi}^{(1)}=\frac{\overline{G}}{\rho^2\sqrt{\rho}}\sum_{k=1}^{\infty}C_k\left\{\sigma_{\theta k}T_k\left(\theta,\varphi\right)+\sigma_{\varphi k}\left[\frac{\partial T_k}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2 T_k}{\partial\varphi^2}\right]\right\}e^{i\omega t},$$

$$\tau_{\rho\theta}^{(1)}=\frac{\overline{G}}{\rho^2\sqrt{\rho}}\sum_{k=1}^{\infty}C_k\tau_{\rho k}\frac{\partial T_k}{\partial\theta}e^{i\omega t}, \qquad \tau_{\rho\varphi}^{(1)}=\frac{\overline{G}}{\rho^2\sqrt{\rho}}\sum_{k=1}^{\infty}C_k\tau_{\rho k}\frac{1}{\sin\theta}\frac{\partial T_k}{\partial\varphi}e^{i\omega t}, \tag{18}$$

$$\tau_{\theta\varphi}^{(1)}=\frac{2\overline{G}}{\rho^2\sqrt{\rho}}\sum_{k=1}^{\infty}C_k\sigma_{\varphi k}\frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}\left[\frac{\partial T_k}{\partial\theta}-ctg\theta T_k\left(\theta,\varphi\right)\right]e^{i\omega t}.$$

$$u_{\rho}^{(2)}=0,\ u_{\theta}^{(2)}=\frac{1}{\sqrt{\rho}}\sum_{k=1}^{\infty}C_k V_{z_k}\left(\lambda\rho\right)\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}e^{i\omega t},$$

$$u_{\varphi}^{(2)}=-\frac{1}{\sqrt{\rho}}\sum_{k=1}^{\infty}C_k V_{z_k}\left(\lambda\rho\right)\frac{\partial^2 T_k}{\partial\varphi\partial\theta}e^{i\omega t}. \tag{19}$$

$$\sigma_{\rho}^{(2)}=0,\ \sigma_{\theta}^{(2)}=\frac{2\overline{G}}{\rho\sqrt{\rho}}\sum_{k=1}^{\infty}C_k V_{z_k}\left(\lambda\rho\right)\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}\right)e^{i\omega t},$$

$$\sigma_{\varphi}^{(2)}=-\frac{2\overline{G}}{\rho\sqrt{\rho}}\sum_{k=1}^{\infty}C_k V_{z_k}\left(\lambda\rho\right)\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}\right)e^{i\omega t}, \tag{20}$$

$$\tau_{\rho\theta}^{(2)}=\frac{\overline{G}}{2\rho\sqrt{\rho}}\sum_{k=1}^{\infty}C_k\tau_{\theta k}\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}e^{i\omega t}, \qquad \tau_{\rho\varphi}^{(2)}=-\frac{\overline{G}}{2\rho\sqrt{\rho}}\sum_{k=1}^{\infty}C_k\tau_{\theta k}\frac{\partial^2 T_k}{\partial\theta\partial\varphi}e^{i\omega t},$$

$$\tau_{\theta\varphi}^{(2)}=-\frac{\overline{G}}{\rho\sqrt{\rho}}\sum_{k=1}^{\infty}C_k V_{z_k}\left(\lambda\rho\right)\frac{\partial}{\partial\varphi}\left[\frac{\partial^2 T_k}{\partial\theta^2}+\frac{1}{2}\left(z_k^2-\frac{1}{4}\right)T_k\left(\theta,\varphi\right)\right]e^{i\omega t},$$

where

$$u_{\rho k}=\alpha Z'_{z_k}\left(\alpha\rho\right)-\frac{1}{2\rho}Z_{z_k}\left(\alpha\rho\right)-\frac{1}{\rho}\left(z_k^2-\frac{1}{4}\right)Z_{z_k}\left(\lambda\rho\right),$$

$$u_{\theta k}=\frac{1}{\rho}Z_{z_k}\left(\alpha\rho\right)-\lambda Z'_{z_k}\left(\lambda\rho\right)-\frac{1}{2\rho}Z_{z_k}\left(\lambda\rho\right),$$

$$\sigma_{\theta k}=2\alpha\rho Z'_{z_k}\left(\alpha\rho\right)-\left(1+\frac{\nu}{1-\nu}\lambda^2\rho^2\right)Z_{z_k}\left(\alpha\rho\right)-2\left(z_k^2-\frac{1}{4}\right)Z_{z_k}\left(\lambda\rho\right),$$

$$\sigma_{\rho k}=\varphi_2\left(z_k,\rho\right)Z_{z_k}\left(\alpha\rho\right)-4\alpha\rho Z'_{z_k}\left(\alpha\rho\right)+\left(z_k^2-\frac{1}{4}\right)\left(3Z_{z_k}\left(\lambda\rho\right)-2\lambda\rho Z'_{z_k}\left(\lambda\rho\right)\right),$$

$$\tau_{\rho k}=2\alpha\rho Z'_{z_k}\left(\alpha\rho\right)-3Z_{z_k}\left(\alpha\rho\right)+2\lambda\rho Z'_{z_k}\left(\lambda\rho\right)-\varphi_3\left(z_k,\rho\right)Z_{z_k}\left(\lambda\rho\right),$$

$$\sigma_{\varphi k}=2Z_{z_k}\left(\alpha\rho\right)-2\lambda\rho Z'_{z_k}\left(\lambda\rho\right)-Z_{z_k}\left(\lambda\rho\right), \quad \tau_{\theta k}=3V_{z_k}\left(\lambda\rho\right)-2\lambda\rho V'_{z_k}\left(\lambda\rho\right).$$

Here C_k are arbitrary constants, $\varphi_2(z,\rho)=2z^2+\frac{3}{2}-\lambda^2\rho^2$, $V_{z_k}(y)=C_{5_{z_k}}J_{z_k}(y)+C_{6_{z_k}}Y_{z_k}(y)$, $Z_{z_k}(x)=C_{iz_k}J_{z_k}(x)+C_{jz_k}Y_{z_k}(x)$, at $x=\alpha\rho$ $i=1$, $j=2$, at $x=\lambda\rho$ $i=3$, $j=4$, $\overline{G}=G/R_0$.

Now we investigate the roots of the variance equation (13). Apparently it has quite complicated structure. For effective studies of its roots let us do some hypothesize of geometricl parameters of spherical shell [2,6].

$$\rho=1+\varepsilon\eta,\quad -1\le\eta\le1,\quad \rho_1=1-\varepsilon,\quad \rho_2=1+\varepsilon,\quad \varepsilon=\frac{r_2-r_1}{R_0}=\frac{h}{R_0}.$$

Equation (13) expand in series by ε.

$$\begin{gathered}
D(z,\lambda,\varepsilon)=\frac{16}{\pi^2}\varepsilon^2\Big\langle\Big\{(1-2\nu)^2\lambda^4z^2-(1-2\nu)^2\lambda^6+\\
+\frac{(1-2\nu)^2(15+17\nu)}{4(1-\nu)}\lambda^4\Big\}+\frac{1}{3}\Big\{4\left[(1-2\nu)^2\lambda^4+(3+2\nu)\lambda^2+1\right]z^4+\\
+4\left[-\frac{(1-2\nu)^2(7-8\nu)}{4(1-\nu)}\lambda^6+\frac{(1-2\nu)\left(18\nu^3-27\nu^2+16\nu-5\right)}{2(1-\nu)^2}\lambda^4+\right.\\
\left.+\frac{20\nu^2-8\nu-3}{2(1-\nu)}\lambda^2+4(1-2\nu)^2\right]z^2+\frac{(1-2\nu)^2(3-4\nu)}{1-\nu}\lambda^8+\\
+\frac{(1-2\nu)^2\left(20\nu^2+73\nu-61\right)}{4(1-\nu)^2}\lambda^6+\frac{(1-2\nu)\left(94\nu^3-15\nu^2-156\nu+73\right)}{4(1-\nu)^2}\lambda^4+\\
+\frac{1}{4}(3+2\nu)\lambda^2-4(1-2\nu)^2\Big\}\varepsilon^2+\frac{1}{45}\Big\{-60(1-\nu)^2z^8+ \qquad (21)\\
+\left[32(1-2\nu)^2\lambda^4-\frac{2\left(384\nu^3+454\nu^2-271\nu-90\right)}{1-\nu}\lambda^2+750\nu^2-\right.\\
-667\nu+150]z^6+\left[-\frac{36\nu(1-2\nu)^2}{(1-\nu)^2}\lambda^8+\right.\\
+\frac{4(1-2\nu)^2\left(24\nu^3-71\nu^2+70\nu-20\right)}{(1-\nu)^3}\lambda^6-\\
-\frac{1274\nu^4-3102\nu^3+2291\nu^2-1032\nu+211}{(1-\nu)^2}\lambda^4+\frac{1224\nu^3-1122\nu^2+491\nu+370}{2(1-\nu)}\lambda^2-\\
\left.-\frac{1}{4}\left(735\nu^2-4849\nu+135\right)\right]z^4+[...]z^2+...\Big\}\varepsilon^4+...\Big\rangle=0.
\end{gathered}$$

For the equation (21) the following statement is valid.

The equation (21) finite λ has two groups of zeros, where the first group consists of two zeros $z_k=O(1)$ $(k=1,2)$, and second group contains a denumerable set of roots which have the order $O\left(\varepsilon^{-1}\right)$.

For the first case we seek z_k in the form of the expansion

$$z_k=z_{k_0}+\varepsilon^2z_{k_2}+\dots\,. \qquad (22)$$

After substitution (22) in (21) we obtain

$$z_{k_0}^2=\lambda_0^2-\frac{15+17\nu}{4(1-\nu)},$$

$$z_{k_2}^2=-\frac{1}{6(1-2\nu)^2 z_{k_0}}\left\{4\left[(1-2\nu)^2+(3+2\nu)\lambda^{-2}+\lambda^{-4}\right]z_{k_0}^4+\right.$$
$$+4\left[-\frac{(1-2\nu)^2(7-8\nu)}{4(1-\nu)}\lambda^2+\frac{(1-2\nu)\left(18\nu^3-27\nu^2+16\nu-5\right)}{2(1-\nu)^2}+\right.$$
$$\left.+\frac{20\nu^2-8\nu-3}{2(1-\nu)}\lambda^{-2}+4(1-2\nu)^2\lambda^{-4}\right]z_{k_0}^2+ \tag{23}$$
$$+\frac{(1-2\nu)^2(3-4\nu)}{1-\nu}\lambda^4+\frac{(1-2\nu)^2\left(20\nu^2+73\nu-61\right)}{4(1-\nu)^2}\lambda^2+$$
$$\left.+\frac{(1-2\nu)\left(94\nu^3-15\nu^2-156\nu+73\right)}{4(1-\nu)^2}+\frac{1}{4}(3+2\nu)\lambda^{-2}-4(1-2\nu)^2\lambda^{-4}\right\}.$$

From (23) it's obvious that when $\lambda_0^2\geq\frac{15+17\nu}{4(1-\nu)}$ we have two real and when $\lambda_0^2<\frac{15+17\nu}{4(1-\nu)}$ two imaginary roots. Some penetrating solutions correspond to these groups of roots.

For constructing the asymptotic of zeros of the second group we'll seek z_n $(n=1,2,..)$ in the form of

$$z_n=\frac{\delta_n}{\varepsilon}+O(\varepsilon). \tag{24}$$

Substituting (24) in the variance equation (13) and transforming it with the help of asymptotically expansions of the functions $J_z(x)$, $Y_z(x)$ for large z for δ_n we have

$$sh^2 2\delta=0. \tag{25}$$

Apparently this equation has a denumerable set of zeros.

We studied the asymptotical properties z of variance equation, assuming that the frequency parameter λ is finite when $\varepsilon\to 0$. Consider the case when λ unboundedly increases when $\varepsilon\to 0$. For such λ the following statement is valid.

If $\lambda\to\infty$ when $\varepsilon\to 0$, then for equation (21) λ the following limit cases are possible

$$\begin{aligned} a)\ \lambda\varepsilon\to const\ \text{ when } \varepsilon\to 0,\\ b)\ \lambda\varepsilon\to\infty\ \text{ when } \varepsilon\to 0,\end{aligned} \tag{26}$$

under which zeros of (21) unboundedly increase.

a) In this case ($z\sim\lambda$, $\lambda\varepsilon\to const$, $z\varepsilon\to const$ when $\varepsilon\to 0$) we'll seek z_k and λ in the form of

$$z_n=\frac{\delta_n}{\varepsilon}+O(\varepsilon),\qquad \lambda=\frac{\lambda_0}{\varepsilon}+O(\varepsilon). \tag{27}$$

Substituting (27) in the variance equation (13) and transforming it with the help of asymptotical expansions of the functions $J_z(x)$, $Y_z(x)$ for large z and x for δ_n we have [2,6]

$$\beta_n\gamma_n sh2\beta_n sh2\gamma_n=0, \tag{28}$$

where

$$\beta_n = \sqrt{\delta_n^2 - \frac{1-2\nu}{2(1-\nu)}\lambda_0^2}, \quad \gamma_n = \sqrt{\delta_n^2 - \lambda_0^2}.$$

For the given λ the equation (28) is denumerable set z_n.

In case of b), as in [2,6], denoting $\varsigma_k\varepsilon$ as x_k, $\lambda\varepsilon$ as y and using asymptotical expansions of Bessel's functions, for the first member of asymptotics we can represent equation (13) as following variance

$$\beta_k\gamma_k sh2\beta_k sh2\gamma_k = 0, \tag{29}$$

where

$$\beta_k = \sqrt{x_k^2 - \frac{1-2\nu}{2(1-\nu)}y^2}, \quad \gamma_k = \sqrt{x_k^2 - y^2}.$$

As is seen the equation (28) is valid in case b), too.

Now we pass to the investigation of the equation (14) [2].

The equation (14) for arbitrary finite $\lambda\,[\lambda = O(1)\ \ when\ \varepsilon \to 0]$ has a denumerable set of roots.

For to prove this statement, we seek z_k in the following form

$$z_n = \frac{\delta_n}{\varepsilon} + O(\varepsilon). \tag{30}$$

Substituting (30) in the variance equation (14) and transforming it with the help of asymptotical expansions of the functions $J_z(x)$, $Y_z(x)$ for large z and x for δ_n we have [2].

$$sh2\delta_n = 0. \tag{31}$$

As it is seen, the equation (310 has adenumerable set of roots. Let's note that equation (31) coincides with equation defining boundary effects of Saint-Venant in whirlwind problem of theory of thick plate.

Let's consider the case, when $\lambda\varepsilon \to 0$ for $\varepsilon \to 0$. In this case, seeking z_n and λ in the form

$$z_n = i\left[\frac{\delta_n}{\varepsilon} + O(\varepsilon)\right], \tag{32}$$

$$\lambda = \lambda_0\varepsilon^{-h}, \quad \lambda_0 = O(1), \quad 0 \le h < 1,$$

substituting (32) in (14) and transforming it with the help of asymptoical expansions of the functios $J_z(x)$, $Y_z(x)$ for large z and x for δ_n we obtain

$$\sin 2\delta_n = 0. \tag{33}$$

Now let's consider case, when $\lambda\varepsilon \to const$ for $\varepsilon \to 0$ and $z\varepsilon \to const$ for $\varepsilon \to 0$. For this case we seek z_k in the following form

$$z_k = i\left[\frac{\delta_k}{\varepsilon} + O(\varepsilon)\right], \quad \lambda = \frac{\lambda_0}{\varepsilon} + O(\varepsilon). \tag{34}$$

After substituting (34) in (14) and transforming it with the help of asymptotical expansions of the functions $J_z(x)$, $Y_z(x)$ for large z and x for δ_n we obtain

$$\sin 2\mu_k = 0, \tag{35}$$

where $\mu_k = \sqrt{\delta_k^2 + \lambda_0^2},\ i = \sqrt{-1}$.

The equation (35) determines the denumerable roots when λ is given.

Using formulae (17), (18) we represent asymptotically formulae for displacenents and strains corresponding to different group roots of the equation (13). At finite λ some penetrating solutions correspond to first group roots

$$u_\rho = 0,\ u_\theta = \varepsilon \sum_{k=1}^{4} \left[C_k + O(\varepsilon)\right] \frac{\partial T_k}{\partial \theta} e^{i\omega t},$$

$$u_\varphi = \varepsilon \sum_{k=1}^{4} \left[C_k + O(\varepsilon)\right] \frac{1}{\sin\theta} \frac{\partial T_k}{\partial \varphi} e^{i\omega t}. \tag{36}$$

$$\sigma_\rho = -\varepsilon 2\overline{G} \sum_{k=1}^{4} C_k \left[\frac{\nu}{(1-2\nu)} \left(\lambda_0^2 - \frac{15+17\nu}{4(1-\nu)}\right) + O(\varepsilon)\right] T_k(\theta,\varphi) e^{i\omega t},$$

$$\sigma_\theta = \varepsilon 2\overline{G} \sum_{k=1}^{4} C_k \left[\frac{\nu}{(1-2\nu)} \left(\lambda_0^2 - \frac{15+17\nu}{4(1-\nu)}\right) T_k(\theta,\varphi) + \frac{\partial^2 T_k}{\partial \theta^2} + O(\varepsilon)\right] e^{i\omega t},$$

$$\sigma_\varphi = \varepsilon 2\overline{G} \sum_{k=1}^{4} C_k \left[\frac{\nu}{4(1-2\nu)} \left[15 + 17\nu - 4(1-\nu)\lambda_0^2\right] \times \right.$$

$$\left. \times T_k(\theta,\varphi) - \frac{\partial^2 T_k}{\partial \theta^2} + O(\varepsilon)\right] e^{i\omega t}, \tag{37}$$

$$\tau_{\rho\theta} = O(\varepsilon),\ \tau_{\rho\varphi} = O(\varepsilon),\ \tau_{\theta\varphi} = \varepsilon\overline{G} \sum_{k=1}^{4} C_k \left[\frac{\partial^2 T_k}{\partial\theta\partial\varphi} - ctg\theta \frac{\partial T_k}{\partial\varphi} + O(\varepsilon)\right] \frac{2}{\sin\theta} e^{i\omega t}.$$

Here C_k are arbitrary constants.

Using zero approach for two solutions, corresponding to roots of $sh^2\delta = 0$ and $ch^2\delta = 0$ equations, we build the asymptotical expression for displacements and strains for the second groups zero ($z\varepsilon \to const$ when $\varepsilon \to 0$). For the series of roots $sh^2\delta = 0$ we obtain.

$$u_\rho = \sum_{n=1}^{\infty} C_n \delta_n \left[ch\delta_n\eta + O(\varepsilon)\right] T_n(\theta,\varphi) e^{i\omega t},$$

$$u_\varphi = \varepsilon \sum_{n=1}^{\infty} C_n \left[sh\delta_n\eta + O(\varepsilon)\right] \frac{1}{\sin\theta} \frac{\partial T_n}{\partial\varphi} e^{i\omega t}, \tag{38}$$

$$u_\theta = \varepsilon \sum_{n=1}^{\infty} C_n \left[sh\delta_n\eta + O(\varepsilon)\right] \frac{\partial T_n}{\partial\theta} e^{i\omega t}.$$

$$\sigma_\rho = \frac{2\overline{G}}{\varepsilon} \sum_{n=1}^{\infty} \delta_n^2 C_n \left[sh\delta_n\eta + O(\varepsilon)\right] T_n(\theta,\varphi) e^{i\omega t},$$

$$\sigma_\theta = O(\varepsilon),\quad \sigma_\varphi = \frac{2\overline{G}}{\varepsilon} \sum_{n=1}^{\infty} \delta_n^2 C_n \left[-sh\delta_n\eta + O(\varepsilon)\right] T_n(\theta,\varphi) e^{i\omega t}, \tag{39}$$

$$\tau_{\rho\theta} = 2\overline{G}\sum_{n=1}^{\infty}\delta_n C_n\left[ch\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\theta}e^{i\omega t},$$

$$\tau_{\rho\varphi} = 2\overline{G}\sum_{n=1}^{\infty}\delta_n C_n\left[ch\delta_n\eta + O(\varepsilon)\right]\frac{1}{\sin\theta}\frac{\partial T_n}{\partial\varphi}e^{i\omega t},$$

$$\tau_{\theta\varphi} = \varepsilon\overline{G}\sum_{n=1}^{\infty}C_n\left[sh\delta_n\eta + O(\varepsilon)\right]\frac{\partial}{\partial\varphi}\left[\frac{\partial T_n}{\partial\theta} - ctg\theta\ T_n(\theta,\varphi)\right]\frac{2}{\sin\theta}e^{i\omega t}.$$

For the series of roots $ch^2\delta = 0$ we have

$$u_\rho = \sum_{n=1}^{\infty}C_n\delta_n\left[sh\delta_n\eta + O(\varepsilon)\right]T_n(\theta,\varphi)e^{i\omega t},$$

$$u_\varphi = \varepsilon\sum_{n=1}^{\infty}C_n\left[ch\delta_n\eta + O(\varepsilon)\right]\frac{1}{\sin\theta}\frac{\partial T_n}{\partial\varphi}e^{i\omega t}, \tag{40}$$

$$u_\theta = \varepsilon\sum_{n=1}^{\infty}C_n\left[ch\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\theta}e^{i\omega t}.$$

$$\sigma_\rho = \frac{2\overline{G}}{\varepsilon}\sum_{n=1}^{\infty}\delta_n^2 C_n\left[ch\delta_n\eta + O(\varepsilon)\right]T_n(\theta,\varphi)e^{i\omega t},$$

$$\sigma_\theta = O(\varepsilon),\quad \sigma_\varphi = \frac{2\overline{G}}{\varepsilon}\sum_{n=1}^{\infty}\delta_n^2 C_n\left[-ch\delta_n\eta + O(\varepsilon)\right]T_n(\theta,\varphi)e^{i\omega t}, \tag{41}$$

$$\tau_{\rho\theta} = 2\overline{G}\sum_{n=1}^{\infty}\delta_n C_n\left[sh\delta_n\eta + O(\varepsilon)\right]\frac{\partial T_n}{\partial\theta}e^{i\omega t},$$

$$\tau_{\rho\varphi} = 2\overline{G}\sum_{n=1}^{\infty}\delta_n C_n\left[sh\delta_n\eta + O(\varepsilon)\right]\frac{1}{\sin\theta}\frac{\partial T_n}{\partial\varphi}e^{i\omega t},$$

$$\tau_{\theta\varphi} = \varepsilon\overline{G}\sum_{n=1}^{\infty}C_n\left[ch\delta_n\eta + O(\varepsilon)\right]\frac{\partial}{\partial\varphi}\left[\frac{\partial T_n}{\partial\theta} - ctg\theta\ T_n(\theta,\varphi)\right]\frac{2}{\sin\theta}e^{i\omega t}.$$

Here C_n are arbitrary constants.

Now we are building the asymptotically formulae for displacements and strains corresponding to the homogeneous solutions to the whirlwind problem (5), (6), for roots of equation $\sin 2\delta = 0$ in case $\lambda\varepsilon \to 0$ when $\varepsilon \to 0$. As it was equation $\sin 2\delta = 0$ has two series roots. Therefore vectors of displacements and strains break into two part, respectively each of them is defined on the form the asymptotically expand in series by ε. From (19), (20) using two approaches for vector of displacements and tensor of strains we obtained the following asymptotically formulae [2,5].

1) For the series of roots $\sin\delta = 0$ we have

$$u_\rho = 0,\ u_\theta = \sum_{k=1}^{\infty}C_k\left\{\sin\delta_k\eta + \varepsilon^2\left[\frac{1}{4}\left(5-\eta^2\right)\sin\delta_k\eta +\right.\right.$$

$$+\frac{\delta_k}{2}\left(\eta^3-\eta\right)\cos\delta_k\eta\Big]+o\left(\varepsilon^2\right)\Big\}\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}e^{i\omega t}, \qquad (42)$$

$$u_\varphi=-\sum_{k=1}^{\infty}C_k\left\{\sin\delta_k\eta+\varepsilon^2\left[\frac{1}{4}\left(5-\eta^2\right)\sin\delta_k\eta+\right.\right.$$

$$\left.\left.+\frac{\delta_k}{2}\left(\eta^3-\eta\right)\cos\delta_k\eta\right]+o\left(\varepsilon^2\right)\right\}\frac{\partial^2 T_k}{\partial\varphi\partial\theta}e^{i\omega t}.$$

$$\sigma_\rho=0,\ \sigma_\theta=2\overline{G}\sum_{k=1}^{\infty}C_k\left\langle\sin\delta_k\eta+\varepsilon^2\left\{\frac{1}{4}\left(3\eta^2+5\right)\sin\delta_k\eta+\right.\right.$$

$$\left.\left.+\frac{\delta_k}{2}\left(\eta^3-\eta\right)\cos\delta_k\eta\right\}+O\left(\varepsilon^3\right)\right\rangle\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}\right)e^{i\omega t},$$

$$\sigma_\varphi=-2\overline{G}\sum_{k=1}^{\infty}C_k\left\langle\sin\delta_k\eta+\varepsilon^2\left\{\frac{1}{4}\left(3\eta^2+5\right)\sin\delta_k\eta+\right.\right.$$

$$\left.\left.+\frac{\delta_k}{2}\left(\eta^3-\eta\right)\cos\delta_k\eta\right\}+O\left(\varepsilon^3\right)\right\rangle\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}\right)e^{i\omega t},$$

$$\tau_{\rho\theta}=\overline{G}\sum_{k=1}^{\infty}C_k\left\langle\frac{1}{\varepsilon}\delta_k\cos\delta_k\eta+\varepsilon\left\{\frac{1}{2}\left[\left(1+\delta_k^2\right)\eta-\delta_k^2\eta^3\right]\sin\delta_k\eta+\right.\right.$$

$$\left.\left.+\left[\frac{\delta_k}{4}\left(5\eta^2+3\right)\right]\cos\delta_k\eta\right\}+O\left(\varepsilon^2\right)\right\rangle\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}e^{i\omega t}, \qquad (43)$$

$$\tau_{\rho\varphi}=-\overline{G}\sum_{k=1}^{\infty}C_k\left\langle\frac{1}{\varepsilon}\delta_k\cos\delta_k\eta+\varepsilon\left\{\frac{1}{2}\left[\left(1+\delta_k^2\right)\eta-\delta_k^2\eta^3\right]\sin\delta_k\eta+\right.\right.$$

$$\left.\left.+\left[\frac{\delta_k}{4}\left(5\eta^2+3\right)\right]\cos\delta_k\eta\right\}+O\left(\varepsilon^2\right)\right\rangle\frac{\partial^2 T_k}{\partial\varphi\partial\theta}e^{i\omega t},$$

$$\tau_{\theta\varphi}=\overline{G}\sum_{k=1}^{\infty}C_k\left\langle\frac{1}{\varepsilon^2}\delta_k^2\sin\delta_k\eta\frac{\partial T_k}{\partial\varphi}+\left\{\left[\frac{1}{4}\left(3\delta_k^2\eta^2+\delta_k^2-4\lambda_0^2\right)\sin\delta_k\eta+\right.\right.\right.$$

$$\left.\left.\left.+\frac{\delta_k^3}{2}\left(\eta^3-\eta\right)\cos\delta_k\eta\right]\frac{\partial T_k}{\partial\varphi}-\frac{\partial^3 T_k}{\partial\varphi\partial\theta^2}\sin\delta_k\eta\right\}+O\left(\varepsilon\right)\right\rangle e^{i\omega t}.$$

Here C_k are arbitrary constants.

The asymptotical expression for displacements and strains, corresponding to the roots of $\cos\delta=0$ equation is obtained from (42), (43) replacing $\cos\delta_k\eta$ for - $\sin\delta_k\eta$ (with the negetive *sign*) and $\sin\delta_k\eta$ to $\cos\delta_k\eta$, respectively.

The asymptotical formulae for displacemets and strains, correspomding (in case of $\lambda\varepsilon\to const$ when $\varepsilon\to 0$) to the roots of $\sin 2\mu=0$ equation, have the following form [2,5].

1) For the series of roots $sin\mu=0$ we have

$$u_\rho=0,\ u_\theta=\sum_{k=1}^{\infty}C_k\left\langle\sin\mu_k\eta+\varepsilon^2\left\{\frac{1}{4\mu_k^2}\left[4\mu_k^2+\delta_k^2-\left(3\delta_k^2-2\mu_k^2\right)\eta^2\right]\sin\mu_k\eta+\right.\right.$$

$$+\frac{1}{\mu_k}\left[\frac{\delta_k^2}{2}\left(\eta^3-\eta\right)\right]\cos\mu_k\eta\right\}+o\left(\varepsilon^2\right)\right\rangle\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}e^{i\omega t},$$

$$u_\varphi=-\sum_{k=1}^{\infty}C_k\left\langle\sin\mu_k\eta+\varepsilon^2\left\{\frac{1}{4\mu_k^2}\left[4\mu_k^2+\delta_k^2-\left(3\delta_k^2-2\mu_k^2\right)\eta^2\right]\sin\mu_k\eta+\right.\right.$$

$$\left.\left.+\frac{1}{\mu_k}\left[\frac{\delta_k^2}{2}\left(\eta^3-\eta\right)\right]\cos\mu_k\eta\right\}+o\left(\varepsilon^2\right)\right\rangle\frac{\partial^2 T_k}{\partial\varphi\partial\theta}e^{i\omega t}. \qquad (44)$$

$$\sigma_\rho=0,\ \sigma_\theta=2\overline{G}\sum_{k=1}^{\infty}C_k\left\langle\sin\mu_k\eta+\varepsilon^2\left\{\frac{1}{4\mu_k^2}\left[4\mu_k^2+\delta_k^2-3\left(\delta_k^2-2\mu_k^2\right)\eta^2\right]\times\right.\right.$$

$$\left.\left.\times\sin\mu_k\eta+\frac{1}{\mu_k}\left[\frac{\delta_k^2}{2}\left(\eta^3-\eta\right)\right]\cos\mu_k\eta\right\}+O\left(\varepsilon^3\right)\right\rangle\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}\right)e^{i\omega t},$$

$$\sigma_\varphi=-2\overline{G}\sum_{k=1}^{\infty}C_k\left\langle\sin\mu_k\eta+\varepsilon^2\left\{\frac{1}{4\mu_k^2}\left[4\mu_k^2+\delta_k^2-3\left(\delta_k^2-2\mu_k^2\right)\eta^2\right]\sin\mu_k\eta+\right.\right.$$

$$\left.\left.+\frac{1}{\mu_k}\left[\frac{\delta_k^2}{2}\left(\eta^3-\eta\right)\right]\cos\mu_k\eta\right\}+O\left(\varepsilon^3\right)\right\rangle\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}\right)e^{i\omega t},$$

$$\tau_{\rho\theta}=\overline{G}\sum_{k=1}^{\infty}C_k\left\langle\frac{1}{\varepsilon}\mu_k\cos\mu_k\eta+\varepsilon\left\{\left[\left(2+\frac{\delta_k^2}{2}-\frac{3\delta_k^2}{2\mu_k^2}\right)\eta-\frac{\delta_k^2}{2}\eta^3\right]\sin\mu_k\eta+\right.\right.$$

$$\left.\left.+\frac{1}{4\mu_k}\left[4\mu_k^2-\delta_k^2+\left(3\delta_k^2+2\mu_k^2\right)\eta^2\right]\cos\mu_k\eta\right\}+O\left(\varepsilon^2\right)\right\rangle\frac{1}{\sin\theta}\frac{\partial^2 T_k}{\partial\varphi^2}e^{i\omega t}, \qquad (45)$$

$$\tau_{\rho\varphi}=-\overline{G}\sum_{k=1}^{\infty}C_k\left\langle\frac{1}{\varepsilon}\mu_k\cos\mu_k\eta+\varepsilon\left\{\left[\left(2+\frac{\delta_k^2}{2}-\frac{3\delta_k^2}{2\mu_k^2}\right)\eta-\frac{\delta_k^2}{2}\eta^3\right]\sin\mu_k\eta+\right.\right.$$

$$\left.\left.+\frac{1}{4\mu_k}\left[4\mu_k^2-\delta_k^2+\left(3\delta_k^2+2\mu_k^2\right)\eta^2\right]\cos\mu_k\eta\right\}+O\left(\varepsilon^2\right)\right\rangle\frac{\partial^2 T_k}{\partial\varphi\partial\theta}e^{i\omega t},$$

$$\tau_{\theta\varphi}=\overline{G}\sum_{k=1}^{\infty}C_k\left\langle\frac{1}{\varepsilon^2}\frac{\partial T_k}{\partial\varphi}\delta_k^2\sin\mu_k\eta+\left\{\left[\left(\left(\frac{3\delta_k^2}{2\mu_k^2}-\delta_k^2-\frac{3}{2}+\delta_k^2\eta^2\right)-\right.\right.\right.\right.$$

$$\left.-\frac{\delta_k^2}{4\mu_k^2}\left(\left(3\delta_k^2-2\mu_k^2\right)\eta^2-\delta_k^2\right)\right)\sin\mu_k\eta+\frac{\delta_k^4}{2\mu_k}\left(\eta^3-\eta\right)\cos\mu_k\eta\right]\frac{\partial T_k}{\partial\varphi}-$$

$$\left.\left.-\frac{\partial^3 T_k}{\partial\varphi\partial\theta^2}\sin\mu_k\eta\right\}+O\left(\varepsilon\right)\right\rangle e^{i\omega t}.$$

Here C_k are arbitrary constants, $\mu_k^2=\delta_k^2+\lambda_0^2$.

The asymptotical expression for displacements and strains, corresponding to the roots of $\cos\mu=0$ equation is obtained from (44), (45) replacing $\cos\mu_k\eta$ for $\sin\mu_k\eta$ (with the negative *sign*) and $\sin\mu_k\eta$ to $\cos\mu_k\eta$, respectively.

Thus on the basis of asymptotical investigation of variance equations for different asymptotic formulae roots are obtained for displacements and strains when small parameter ε approachies zero.

The problem on satisfaction of boundary conditions on end-walls of shell with the help of a class of homogeneous solutions isn't considered in the present paper.

This question is considered in [4] where with the help of the Hamilton variational principle, the boundary value problem is reduced to a solution of an infinite system of linear algebraic equations.

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