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ON A FREQUENCY RESPONSE OF A PRE-STRAINED MANY-LAYERED SLAB ON A RIGID FOUNDATION

Abstract

Within the framework of the piecewise homogeneous body model with the use of Three-dimensional Linearized Theory of Elastic Waves in Initially Stressed Bodies the frequency response of the pre-strained many-layered slab resting on a rigid foundation is studied. It is assumed that the whole thickness of the slab remains constant for any number of layers from which this slab is composed. Moreover, it is assumed that the slab consists of the packet which contains two layers. The elasticity relations of the layers' materials are described by Treloar potential. The cases where the number of layers (packets) in the slab is 6 (3), 4(2) and 2(1) are analyzed. According to the obtained numerical results the influence of the number of layers and the pre-stretching of these layers on the frequency response of the slab is analyzed.

1. Introduction

As usual, by a frequency response the dependencies between the amplitude and frequency of the external force are understood. The study of the frequency response of the elements of constructions was subject to a lot of investigations. A considerable part of these investigations has been carried out within the framework of the classical linear theory of elasticity. A class of interesting and urgent problems on the frequency response are the ones for initially stressed bodies. Such problems have a great significance in both theoretical and practical sense. Until now there were a few studies in this field; see for example references [1-11].

In the investigations [1-8] it was assumed that the region occupied by the body is semi-infinite. Therefore the results obtained in [1-8] cannot be applied for structural elements whose basic material is covered with the layered ones. Because of these discussions in the papers [9, 10] the investigations carried out in [1-8] were developed for the system which comprises bilayered infinite slab and rigid foundation. However, in many cases the covering slab may consist of more than two layers. In the paper [11] the investigations [9, 10] were developed for the case where the slab contains any number of finite pre-stretching layers. At the same time in [11] it was assumed that the thickness of the slab increases with the increasing the number of layers contained by the slab.

In a practical as well as theoretical sense the case where the number of layers in the slab increases under constant thickness of the slab, i.e. the case where the number of layers in the slab increases by decreasing of the thickness of the layers, has also a great significance. This situation is shown schematically in Fig.1. In connection with this, in the present paper the investigation [11] is developed for the foregoing case. As in [11], it is assumed that a time-harmonic point located normal force acts on the face plane of the upper layer of the slab and axisymmetric stress state in this slab is studied. We suppose that the layers of the slab are finite pre-strained (-stretched) radially and the material of the layers is incompressible neo-Hookean materials. The stress-strain relation for those are given through the Treloar potential. The investigations are carried out within the framework of the piecewise-homogeneous body model by the use of the Three-dimensional Linearized Theory of Elastic Waves in Initially Stressed Bodies (TLTEWISB).

2. Formulation of the problem and solution procedure

We consider the many-layered slab resting on the rigid foundation (Fig. 1). Assume that in the natural state the thicknesses of the layers are $h_1, h_2, ..., h_N$ and $h_1 + h_2 + ... + h_N = H = const$. Here H is a whole thickness of the slab which remains constant for any number the layers (N) from which this slab is composed. In the natural state we determine the position of the points of the layers by the Lagrangian coordinates in the Cartesian system of coordinates $Oy_1y_2y_3$ as well as in the cylindrical system of coordinates $Or\theta y_3$. Assume that the layers of the slab have infinite length in the radial direction. We aim that the layers before the compounding with each other and with a rigid foundation be stretched separately along the radial direction and in each of them the homogeneous axisymmetric initial finite strain appear.

With the initial state of the layers of the slab we associate the Lagrangian cylindrical system of coordinates $O'r'\theta'y_3'$ and Cartesian system of coordinates $O'y_1'y_2'y_3'$. Assume that the material of the layers is of incompressible neo-Hookean type and the values related to the k-th $(1 \le k \le N)$ layer of the slab are denoted by upper indices (k). Furthermore, we denote the values related to the initial state by upper index 0. Thus, according to the above-stated the initial state in the layers can be determined as follows.

$$u_m^{(k),0} = (\lambda_m^{(k)} - 1)y_m, \quad \lambda_1^{(k)} = \lambda_2^{(k)} \neq \lambda_3^{(k)}, \quad \lambda_m^{(k)} = const,$$

$$\lambda_1^{(\underline{k})}\lambda_2^{(\underline{k})}\lambda_3^{(\underline{k})} = 1, m = 1,\ 2,\ 3,\ \ k = 1,\ 2. \tag{1}$$

where $u_m^{(k),0}$ is a displacement and $\lambda_m^{(k)}$ is the elongation along the Oy_maxis. We introduce the following notation

$$\lambda_1^{(k)} = \lambda_2^{(k)} = \lambda^{(k)}; \quad \lambda_3^{(k)} = (\lambda^{(k)})^{-2}. \tag{2}$$

It follows from (1) that

$$y_i' = \lambda_{\underline{i}}^{(k)} y_{\underline{i}}, \quad r' = \lambda^{(k)} r, \ h_1' = (\lambda^{(1)})^{-2} h_1, \quad h_2' = (\lambda^{2)})^{-2} h_2. \tag{3}$$

Below the values related to the system of coordinates associated with the initial state, i.e. with $O'y_1'y_2'y_3'$ are denoted by upper prime.

According to [12], we write the basic relations of the TLTEWISB for an incompressible body in an axisymmetric stress-strain state. These relations are satisfied within each layer because we use the piecewise-homogeneous body model.

The equations of motion are

$$\frac{\partial}{\partial r'} Q'^{(k)}_{rr} + \frac{\partial}{\partial y_3'} Q'^{(k)}_{r'3} + \frac{1}{r'}(Q'^{(k)}_{r'r'} - Q'^{(k)}_{\theta'\theta'}) = \rho'^{(k)} \frac{\partial^2}{\partial t^2} u'^{(k)}_{r'},$$

$$\frac{\partial}{\partial r'}Q'^{(k)}_{3r'}+\frac{\partial}{\partial y'_3}Q'^{(k)}_{33}+\frac{1}{r'}Q'^{(k)}_{3r'}=\rho'^{(k)}\frac{\partial^2}{\partial t^2}u'^{(k)}_3. \quad (4)$$

The mechanical relations are

$$Q'^{(k)}_{r'r'}=\chi'^{(k)}_{1111}\frac{\partial u'^{(k)}_{r'}}{\partial r'}+\chi'^{(k)}_{1122}\frac{u'^{(k)}_{r'}}{r'}+\chi'^{(k)}_{1133}\frac{\partial u'^{(k)}_3}{\partial y'_3}+p'^{(k)},$$

$$Q'^{(k)}_{\theta'\theta'}=\chi'^{(k)}_{2211}\frac{\partial u'^{(k)}_{r'}}{\partial r'}+\chi'^{(k)}_{2222}\frac{u'^{(k)}_{r'}}{r'}+\chi'^{(k)}_{2233}\frac{\partial u'^{(k)}_3}{\partial y'_3}+p'^{(k)},$$

$$Q'^{(k)}_{33}=\chi'^{(k)}_{3311}\frac{\partial u'^{(k)}_{r'}}{\partial r'}+\chi'^{(k)}_{3322}\frac{u'^{(k)}_{r'}}{r'}+\chi'^{(k)}_{3333}\frac{\partial u'^{(k)}_3}{\partial y'_3}+p'^{(k)},$$

$$Q'^{(k)}_{r'3}=\chi'^{(k)}_{1313}\frac{\partial u'^{(k)}_{r'}}{\partial y'_3}+\chi'^{(k)}_{1331}\frac{\partial u'^{(k)}_3}{\partial r'},\ Q'^{(k)}_{3r'}=\chi'^{(k)}_{3113}\frac{\partial u'^{(k)}_{r'}}{\partial y'_3}+\chi'^{(k)}_{3131}\frac{\partial u'^{(k)}_3}{\partial r'}. \quad (5)$$

In (4) and (5) through $Q'^{(k)}_{r'r'},\ldots,Q'^{(k)}_{3r'}$ the perturbations of the components of Kirchhoff stress tensor are determined. The notation $u'^{(k)}_{r'}$, $u'^{(k)}_3$ shows the perturbations of the components of the displacement vector, $p'^{(k)}=p'^{(k)}(r',y'_3,t)$ is an unknown function (Lagrange multiplier). The constants $\chi'^{(k)}_{1111},\ldots,\chi'^{(k)}_{3333}$ in (4), (5) are determined through the mechanical constants of the layers' materials and through the initial stress state. $\rho'^{(k)}$ is a density of the k-th layer material.

As in [10, 11], in the present investigation we assume that the elasticity relations of the layers' materials are given by neo-Hookean type (Treloar) potential. This potential is given as follows

$$\Phi=C_{10}\left(I_1-3\right),\ I_1=3+2A_1,\quad A_1=\epsilon_{rr}+\epsilon_{\theta\theta}+\epsilon_{33}, \quad (6)$$

where C_{10} is an elastic constant; A_1 is the first algebraic invariant of the Green's strain tensor, ϵ_{rr}, $\epsilon_{\theta\theta}$ and ϵ_{33} are the components of this tensor.

According to [12], for the considered case the following expressions for the constants $\chi'^{(k)}_{1111},\ldots,\chi'^{(k)}_{3333}$ in (5) are obtained.

$$\chi'^{(k)}_{1111}=2C^{(k)}_{10}\left(\left(\lambda^{(k)}\right)^2+\left(\lambda^{(k)}\right)^{-4}\right),\ \chi'^{(k)}_{1331}=2C^{(k)}_{10}\left(\lambda^{(k)}\right)^2,\ \chi^{(k)}_{1221}=2C^{(k)}_{10}\left(\lambda^{(k)}\right)^2,$$

$$\chi'^{(k)}_{1122}=\chi'^{(k)}_{1133}=\chi'^{(k)}_{2233}=\chi'^{(k)}_{3311}=\chi'^{(k)}_{2211}=\chi'^{(k)}_{3322}=0,\chi'^{(k)}_{3333}=4C^{(k)}_{10}\left(\lambda^{(k)}\right)^{-4},$$

$$\chi'^{(k)}_{1313}=2C^{(k)}_{10}\left(\lambda^{(k)}\right)^{-4},\quad \chi'^{(k)}_{3113}=2C^{(k)}_{10}\left(\lambda^{(k)}\right)^2 \quad (7)$$

It should be noted that to the above-written equations the incompressibility conditions of the layers' materials must be added. These conditions for the considered case can be written as follows:

$$\frac{1}{\lambda^{(k)}}\left(\frac{\partial u'^{(k)}_{r'}}{\partial r'}+\frac{u'^{(k)}_{r'}}{r'}\right)+\left(\lambda^{(k)}\right)^2\frac{\partial u'^{(k)}_3}{\partial y'_3}=0. \quad (8)$$

Thus, the stress state in the many-layered slab will be investigated by the use of the equations (4), (5), (7) and (8). In this case we will assume that the following boundary and contact conditions are satisfied.

$$Q'^{(1)}_{33}\Big|_{y'_3=0} = -P_0\delta(r')e^{i\omega t}\frac{1}{\left(\lambda^{(1)}\right)^2},\quad Q'^{(1)}_{3r'}\Big|_{y'_3=0} = 0,$$

$$\left\{Q'^{(1)}_{33};Q'^{(1)}_{3r'};u'^{(1)}_3;u'^{(1)}_{r'}\right\}\Big|_{y'_3=-h_1/\left(\lambda^{(1)}\right)^2} = \left\{Q'^{(2)}_{33};Q'^{(2)}_{3r'};u'^{(2)}_3;u'^{(2)}_{r'}\right\}\Big|_{y'_3=-h_1/\left(\lambda^{(1)}\right)^2},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\left\{Q'^{(k-1)}_{33};Q'^{(k-1)}_{3r'};u'^{(k-1)}_3;u'^{(k-1)}_{r'}\right\}\Big|_{y'_3=-h_1/\left(\lambda^{(1)}\right)^2-h_2/\left(\lambda^{(2)}\right)^2-\ldots-h_{k-1}/\left(\lambda^{(k-1)}\right)^2} =$$

$$\left\{Q'^{(k)}_{33};Q'^{(k)}_{3r'};u'^{(k)}_3;u'^{(k)}_{r'}\right\}\Big|_{y'_3=-h_1/\left(\lambda^{(1)}\right)^2-h_2/\left(\lambda^{(2)}\right)^2-\ldots-h_{k-1}/\left(\lambda^{(k-1)}\right)^2},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\left\{u'^{(k)}_3;u'^{(k)}_{r'}\right\}\Big|_{y'_3=-h_1/\left(\lambda^{(1)}\right)^2-h_2/\left(\lambda^{(2)}\right)^2-\ldots-h_{N-1}/\left(\lambda^{(N-1)}\right)^2-h_N/\left(\lambda^{(N)}\right)^2} = 0, \tag{9}$$

where $\delta(r')$ is the Dirac delta function.

It should be noted that, in the case $\lambda^{(k)} = 1$ $(k = 1,...,N)$ equations (4), (5), (8) and conditions (9) for the k-th layer transform to the corresponding relations of the classical linear theory of elasticity for incompressible bodies.

The solution procedure of the formulated problem is the same as in [10, 11]. Therefore we here consider briefly some fragments of this procedure, according to which, for solution to the equations (4), (5), (7) and (8) we use the following representation [12]:

$$u'^{(k)}_{r'} = -\frac{\partial^2}{\partial r'\partial y'_3}X'^{(k)},\ u'^{(k)}_3 = \Delta'_1X'^{(k)},\quad p'^{(k)} = \left[\left(\chi'^{(k)}_{1111} - \chi'^{(k)}_{1133} - \chi'^{(k)}_{1313}\right)\Delta'_1 + \right.$$

$$\left.\chi'^{(k)}_{3113}\frac{\partial^2}{\partial {y'_3}^2} - \rho'^{(k)}\frac{\partial^2}{\partial t^2}\right]\frac{\partial}{\partial y'_3}X'^{(k)}, \tag{10}$$

where

$$\Delta'_1 = \frac{d^2}{dr'^2} + \frac{1}{r'}\frac{d}{dr'} \tag{11}$$

and the function $X'^{(k)}$ satisfies the following equation:

$$\left[\left(\Delta'_1 + \left(\xi'^{(k)}_2\right)^2\frac{\partial^2}{\partial {y'_3}^2}\right)\left(\Delta'_1 + \left(\xi'^{(k)}_3\right)^2\frac{\partial^2}{\partial {y'_3}^2}\right) - \frac{\rho'^{(k)}}{\chi'^{(k)}_{1331}}\left(\Delta'_1 + \frac{\partial^2}{\partial {y'_3}^2}\right)\frac{\partial^2}{\partial t^2}\right]X'^{(k)} = 0, \tag{12}$$

where for the considered case

$$\left(\xi'^{(k)}_2\right)^2 = 1,\ \left(\xi'^{(k)}_3\right)^2 = \left(\lambda^{(k)}\right)^{-6}. \tag{13}$$

Since the point force is harmonic in time, only a stationary case is to be considered. Then all the dependent variables, become harmonic in time and can be represented as

$$\left\{Q'^{(k)}_{r'r'},...,Q'^{(k)}_{33},u'^{(k)}_{r'},u'^{(k)}_{3},p'^{(k)},X'^{(k)}\right\}=\left\{\bar{Q}'^{(k)}_{r'r'},...,\bar{Q}'^{(k)}_{33},\bar{u}'^{(k)}_{r'},\bar{u}'^{(k)}_{3},\bar{p}'^{(k)},\bar{X}'^{(k)}\right\}e^{i\omega t}, \tag{14}$$

where the over bar denotes the amplitude of the corresponding quantity. Hereafter, this over bar will be omitted.

Substituting Eq. (14) in Eqs. (9), (10), (12) and replacing the operator $\frac{\partial^2}{\partial t^2}$ with $-\omega^2$ we obtain the equations and conditions for the amplitude of the quantities sought. Consequently, introducing the dimensionless coordinates $r'\to r'/H$, $y'_3\to y'_3/H$, dimensionless thicknesses $h_i\to h_i/H$ and the dimensionless frequency

$$\Omega^2=\frac{(\omega H)^2\rho'^{(2)}}{2C^{(2)}_{10}} \tag{15}$$

we obtain the following equation for the potential $X'^{(k)}$ from Eqs. (10) and (12).

$$\left[\left(\Delta'_1+\left(\xi'^{(k)}_2\right)^2\frac{\partial^2}{\partial y'^2_3}\right)\left(\Delta'_1+\left(\xi'^{(k)}_3\right)^2\frac{\partial^2}{\partial y'^2_3}\right)-\right.$$
$$\left.-\frac{\Omega^2}{\left(\lambda^{(k)}\right)^{(2)}}\left(\Delta'_1+\frac{\partial^2}{\partial y'^2_3}\right)\frac{C^{(2)}_{10}\rho'^{(k)}}{C^{(k)}_{10}\rho'^{(2)}}\right]X'^{(k)}=0. \tag{16}$$

To solve Eq. (16), we use the Hankel integral representation for the function $X'^{(k)}$.

$$X'^{(k)}=\int\limits_0^\infty F^{(k)}_1 e^{\gamma^{(k)}y'_3}J_0(sr)sds, \tag{17}$$

In this way the solution to equation (17) is determined as

$$X'^{(k)}=\int\limits_0^\infty\left[F^{(k)}_1e^{sy'_3}+F^{(k)}_2e^{-sy'_3}+F^{(k)}_3e^{\gamma^{(k)}_2y'_3}+F^{(k)}_4e^{-\gamma^{(k)}_2y'_3}\right]J_0(sr')sds. \tag{18}$$

Thus, the unknowns $F^{(k)}_1(s),...,F^{(k)}_4(s)$ are found from boundary and contact conditions (9), after which the stresses and displacements can be calculated by using the corresponding integral expressions. In calculating the integrals, we employ the algorithm used in [10, 11].

Now we will analyze some numerical results obtained within the framework of the solution procedure utilized and clarify the influence of the number (N) and the pre-stretching of the layers under the same constant thickness of the slab on the dependencies between Q'_{33} (at $y'_3=-1$) and dimensionless frequency Ω.

3. Numerical results and discussions

We consider the cases where $N=6,4$ and 2. The selected cases are indicated in Fig.1 by numbers $1,2$ and 3 respectively. Assume that the slab consists of the alternating layers of two materials, i.e., for example, for $N=6$ the following relations occur:

$C_{10}^{(1)} = C_{10}^{(3)} = C_{10}^{(5)}$; $C_{10}^{(2)} = C_{10}^{(4)} = C_{10}^{(6)}$; $\rho'^{(1)} = \rho'^{(3)} = \rho'^{(5)}$; $\rho'^{(2)} = \rho'^{(4)} = \rho'^{(6)}$; $\lambda^{(1)} = \lambda^{(3)} = \lambda^{(5)}$; $\lambda^{(2)} = \lambda^{(4)} = \lambda^{(6)}$; $h_i = 1./N$. Introduce the notation

$$e = \frac{C_{10}^{(1)}}{C_{10}^{(2)}},\ q_{33} = \left(\frac{Q_{33}'^{(N)} H^2}{P}\right)_{y_3' = -\sum\limits_{i=1}^{N} h_i/\left(\lambda^{(i)}\right)^2}. \tag{19}$$

Assume that $C_{10}^{(2)}\rho'^{(1)}/\left(C_{10}^{(1)}\rho'^{(2)}\right) = 1$. As it follows from Fig.1 that the cases 1., 2. and 3. ($1'$., $2'$. and $3'$.) correspond to $e > 1$ ($e < 1$).

Now we analyze the dependencies between q_{33} and Ω for various number of layers in the slab for the case where the initial stretching in the layers is absent. Figs. 2, 3 and 4 show the graphs of these dependencies for N =6, 4 and 2 respectively, for various values of e. It follows from these graphs that for the considered change in range of Ω, i.e. for $0 < \Omega \leq 3.5$, under $N = 6$ for all considered e there exists such a value of Ω (denote it by Ω_*) for which $dq_{33}/d\Omega = 0$; such situations are also observed for N =4 under $e \geq 1$ and $e = 1./1.5$. However, for $N = 2$ this situation occurs only for $e = 1.$and 1.5. For $e = 3$ and 5 under a certain value of Ω the resonance jumping (discontinuity) appears. Consequently, the location sequence of the soft and stiff layers in the slab can change significantly the character of the frequency response of that.

The explanation of the occurring of the case where $dq_{33}/d\Omega = 0$ is given in [7-11] and this explanation agrees with the corresponding results in [13-16]. Therefore we here do not stop on this question.

Consider the influence of the pre-stretching of the layers on the character of the investigated dependencies. For this purpose we select the case where $N = 6$. The graphs of the mentioned dependencies for various values of $\lambda^{(1)}$ and $\lambda^{(2)}$ are given in Figs. 5 and 6 for $e = 3$ and 1./3., respectively. According to these graphs, it can be concluded that the absolute values of $q_{33}|_{\Omega=\Omega_*}$ decrease but the values of Ω_* increase with the pre-stretching of the stiff layers. However, the pre-stretching of the soft layers causes the case for which $dq_{33}/d\Omega = 0$ to disappear and to appear the resonance jumping in the values of q_{33} to appear under the considered change range of Ω.

4. Conclusions

In the present paper the investigation carried out in [10, 11] was developed for the many-layered slab in the case where the whole thickness of the slab remains constant for any number of layers from which this slab is composed. The investigations carried out within the framework of the piecewise homogeneous body model with the use of the TLTEWISB. It is assumed that the slab consists of the packet which contains two layers. The elasticity relations of the layers' materials are described by Treloar potential. The cases where the number of layers (packets) in the slab is 6 (3), 4(2) and 2(1) are analyzed.

From the results obtained, the following conclusions can be drawn:

- the location sequence of the soft and stiff layers in the slab can change significantly the character of the frequency response of that;

- the absolute values of $q_{33}|_{\Omega=\Omega_*}$ (19) decrease but the values of Ω_* (for which $dq_{33}/d\Omega = 0$) increase with the pre-stretching of the stiff layers, but the pre-stretching of the soft layers causes the case where $dq_{33}/d\Omega = 0$ to disappear and to appear the resonance jumping in the values of q_{33} (19) to appear under considered change range of Ω.

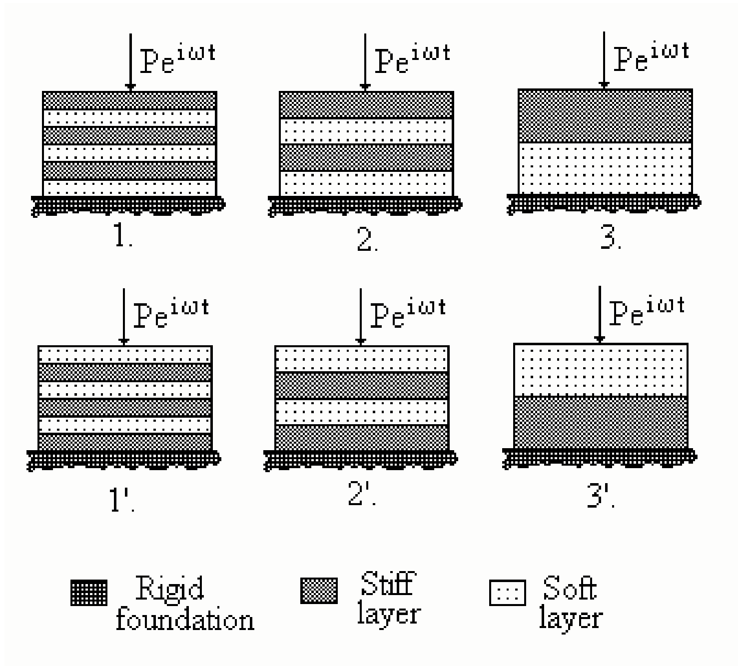


Fig.1.

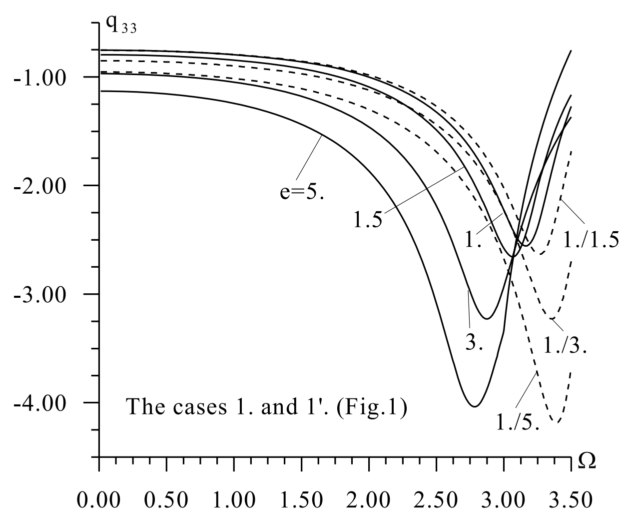
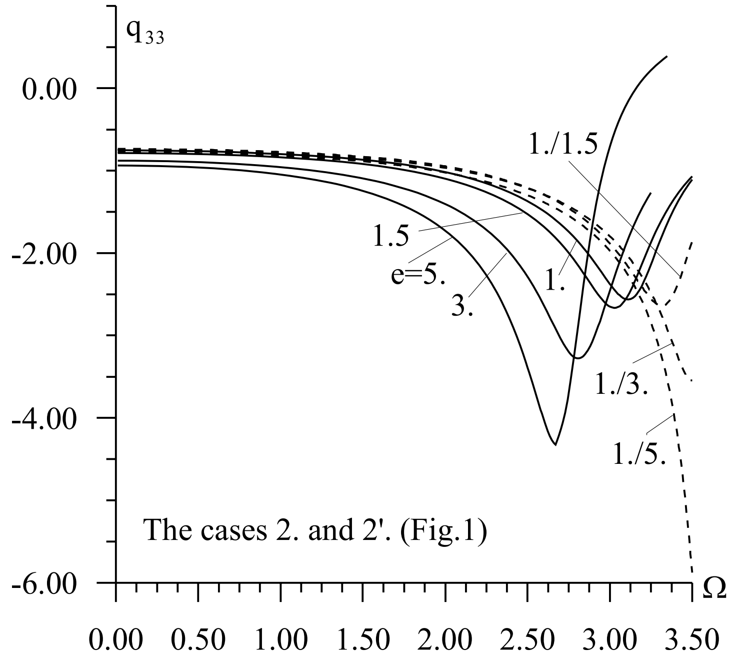


Fig.2.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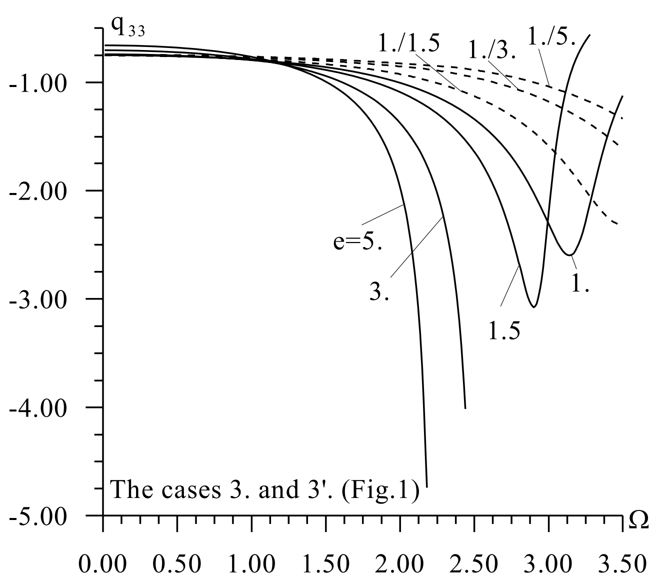
-2.00
-4.00
-6.00
1./1.5
1.5
e=5.
3.
1.
1./3.
1./5.
The cases 2. and 2'. (Fig.1)
Ω
0.00 0.50 1.00 1.50 2.00 2.50 3.00 3.50

Fig.3.


q33
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1./3.
1./5.
-1.00
-2.00
-3.00
-4.00
-5.00
e=5.
3.
1.5
1.
The cases 3. and 3'. (Fig.1)
Ω
0.00 0.50 1.00 1.50 2.00 2.50 3.00 3.50

Fig.4.

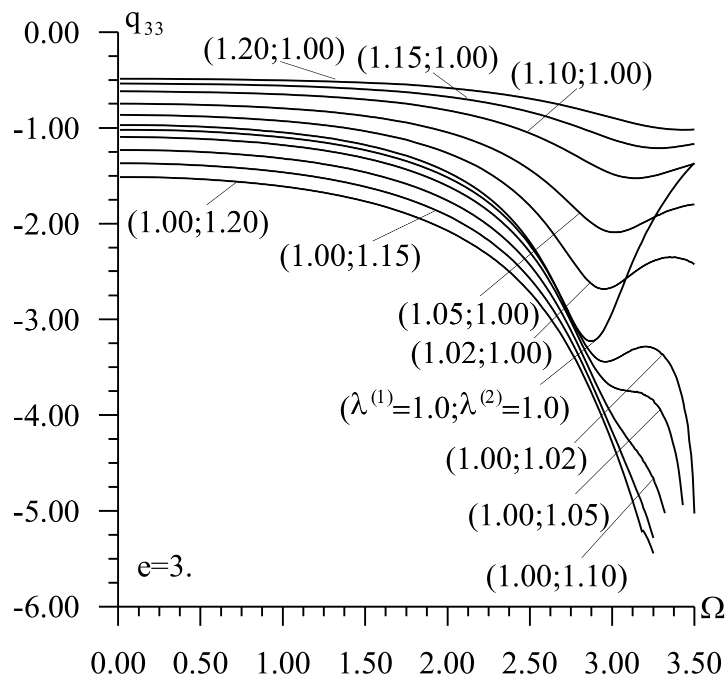

Fig.5.

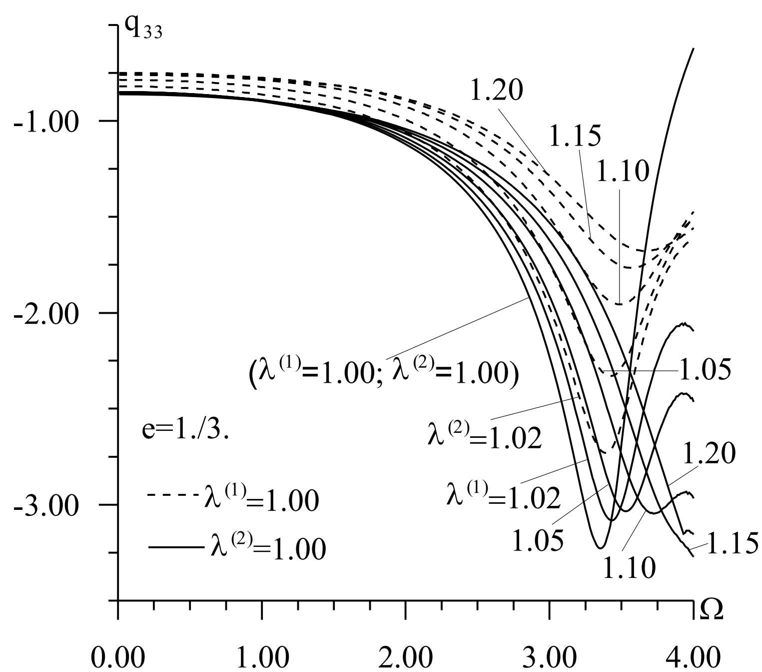

Fig.6.

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