Fakhraddin A. NAMAZOV

THE INVERSE SCATTERING PROBLEM FOR A SYSTEM OF FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS ON A SEMI-AXIS WITH THE SAME INCIDENT WAVES

Abstract

In the paper we study the inverse scattering problem for the system $n \geq 3$ of ordinary differential equations on a semi-axis with $n-1$ same velocities

1. The scattering problem. On a semi-axis $x \geq 0$ let's consider the system $n \geq 3$ of the first order ordinary differential equations of the form

$$\begin{gathered} -i\frac{dy_k(x)}{dx} + \sum_{j=1}^{n} \left(\xi_k - \xi_j\right) c_{kj}(x)\, y_j(x) = \lambda \xi_k y_k(x), \\ k = 1, 2, ..., n; \quad x \geq 0, \quad \xi_1 = \xi_2 = ... = \xi_{n-1} > 0 > \xi_n. \end{gathered} \tag{1}$$

The inverse scattering problem on a semi-axis for a system of Dirac equations in self-adjoint case was studied in the papers [1,2,5], in the self-adjoint case in the paper [4], on the axis - in [3]. The inverse scattering problem on the axis for a system of first order system of equations was studied in the papers [6-10], on a semi-axis for $\xi_k \neq \xi_j$ $(k \neq j)$ (in particular cases) - in [11] and in other papers. When $n = 3$, the inverse scattering problem was investigated in [12].

It is assumed coefficients of system (1) are complex-valued, measurable, bounded functions and satisfy the coefficients

$$|c_{kj}(x)| \leq ce^{-\varepsilon x}, \quad \varepsilon > 0, \quad c = const. \tag{2}$$

Let's consider on the semi-axis $n-1$ the problem: the k-th problem is in funding the solution of system (1) on the given asymptotics

$$\lim_{x \to +\infty} y_j^k(x, \lambda)\, e^{i\lambda \xi_j x} = A_j, \quad j = 1, 2, ..., n-1, \tag{3}$$

and satisfying the boundary condition

$$y_n^k(0, \lambda) = y_k^k(0, \lambda), \qquad k = 1, 2, ..., n-1, \tag{4}$$

The joint consideration of these $n-1$ problems is said to be the scattering problem for system (1) on a semi-axis.

Theorem 1. *Let the coefficients of system (1) satisfy conditions (1) and λ be a real number* $(\operatorname{Im} \lambda = 0)$. *Then there exists a unique bounded solution of the scattering problem on a semi-axis for a system of equations (1).*

Proof. The scattering problem is equivalent to the following system of integral equations

$$y_j^k(x, \lambda) = A_j e^{i\lambda \xi_j x} + i \int_x^{+\infty} \sum_{p=1}^{n} \left(\xi_j - \xi_p\right) c_{jp}(\tau)\, y_j^k(\tau)\, e^{i\lambda \xi_j (x-\tau)} d\tau,$$

$j=1,2,...,n-1$

$$y_n^k(x,\lambda)=B_k e^{i\lambda\xi_n x}+i\int\limits_x^{+\infty}\sum_{p=1}^{n}\left(\xi_n-\xi_p\right)c_{np}(\tau)\,y_p^k(\tau)\,e^{i\lambda\xi_n(x-\tau)}d\tau, \tag{5}$$

where

$$B_k=A_k+i\int\limits_x^{+\infty}[\sum_{p=1}^{n}\left(\xi_k-\xi_p\right)c_{kp}(\tau)\,y_k^k(\tau)\,e^{-i\lambda\xi_k\tau}d\tau-$$

$$-\left(\xi_n-\xi_p\right)c_{np}(\tau)\,y_p^k(\tau)\,e^{-i\lambda\xi_n\tau}\Big]\,d\tau,\quad k=1,2,...,n-1.$$

The existence and uniqueness of the solution of a system of equations (5) in the class of bounded functions follows from the sequential approximation method. The theorem is proved.

By conditions (2) from (5) we get

$$\lim_{x\to+\infty}y_n^k(x,\lambda)\,e^{i\lambda\xi_n x}=B_k,\qquad(k=1,2,...,n-1). \tag{6}$$

By theorem 1, to each vector $A=(A_1,A_2,...,A_{n-1})^t$ there correspond the solution of system (5). By (6) these $n-1$ solutions determine vector $B-(B_1,B_2,...,B_{n-})^t$. The matrix $S(\lambda)=\|S_{ij}(\lambda)\|_{i,j=1}^{n-1}$ transferring the vector A to B will be said to be the scattering matrix for system (1) on the semi-axis

$$S(\lambda)\,A=B. \tag{7}$$

Later on, for simplicity we'll take $\xi_1=...=\xi_{n=1}=-1$, $\xi_n=1$.

2. Transformation operators. We can express the bounded solution of system (1) on a semi-axis by the following vector-function:

$$g^1(x,\lambda)=\left\{y_1(0,\lambda)\,e^{i\lambda x},...,y_{n-1}(0,\lambda)\,e^{i\lambda x},y_n(0,\lambda)\,e^{-i\lambda x}\right\},$$

$$g^k(x,\lambda)=\{A_1e^{i\lambda x},...,A_{k-1}e^{i\lambda x},y_k(0,\lambda)\,e^{i\lambda x},...$$

$$....,y_{n-1}(0,\lambda)\,e^{i\lambda x},y_n(0,\lambda)\,e^{-i\lambda x}\}$$

$$k=2,3,...,n;$$

$$g^{n+1}(x,\lambda)=\left\{A_1e^{i\lambda x},...,A_{n-1}e^{i\lambda x},Be^{-i\lambda x}\right\},$$

$$g^{n+k}(x,\lambda)=\{y_1(0,\lambda)\,e^{i\lambda x},...$$

$$....,y_{k-1}(0,\lambda)\,e^{i\lambda x},A_ke^{i\lambda x},...,A_{n-1}e^{i\lambda x},Be^{-i\lambda x}\}$$

$$k=2,...,n-1;$$

$$g^{2n}(x,\lambda)=\left\{y_1(0,\lambda)\,e^{i\lambda x},...,y_{n-1}(0,\lambda)\,e^{i\lambda x},Be^{-i\lambda x}\right\}, \tag{8}$$

Lemma 1. *Let the coefficients of the system of equations (1) be bounded functions and satisfy conditions (2). Then for each bounded solution it holds the integral representation*

$$y_p(x,\lambda)=g_p^1(x,\lambda)+\sum_{j=1}^{n}\int\limits_{-x}^{x}G_{pj}^1(x,\tau)\,e^{i\lambda\tau}d\tau y_j(0,\lambda), \tag{9}$$

$$y_p(x,\lambda) = g_p^k(x,\lambda) + \\ + \int_{-\infty}^{x} \sum_{j=1}^{n} G_{pj}^1(x,\tau)\, g_j^k(\tau,\lambda)\, d\tau, \left(2 \le k \le n-1\right), \tag{10}$$

$$y_p(x,\lambda) = g_p^n(x,\lambda) + \\ + \int_{-\infty}^{x} \sum_{j=1}^{n-1} G_{pj}^n(x,\tau)\, g_j^n(\tau,\lambda)\, d\tau + \int_{-\infty}^{x} G_{pn}^n(x,\tau)\, g_n^n(\tau,\lambda)\, d\tau, \tag{11}$$

$$y_p(x,\lambda) = g_p^{n+1}(x,\lambda) + \int_{x}^{+\infty} \sum_{j=1}^{n-1} G_{pj}^{n+1}(x,\tau)\, g_j^{n+1}(\tau,\lambda)\, d\tau + \\ + \int_{-\infty}^{-x} G_{pn}^{n+1}(x,\tau)\, g_n^{n+1}(\tau,\lambda)\, d\tau, \tag{12}$$

$$y_p(x,\lambda) = g_p^{n+k}(x,\lambda) + \int_{x}^{+\infty} \sum_{j=1}^{n-1} G_{pj}^{n+k}(x,\tau)\, g_j^{n+k}(\tau,\lambda)\, d\tau + \\ + \int_{-x}^{+\infty} G_{pn}^{n+k}(x,\tau)\, g_n^{n+k}(\tau,\lambda)\, d\tau, \qquad (k = 2, ..., n-1); \tag{13}$$

$$y_p(x,\lambda) = g_p^{2n}(x,\lambda) + \\ + \int_{x}^{+\infty} \sum_{j=1}^{n-1} G_{pj}^{2n}(x,\tau)\, g_j^{2n}(\tau,\lambda)\, d\tau + \int_{x}^{+\infty} G_{pn}^{n}(x,\tau)\, g_n^{2n}(\tau,\lambda)\, d\tau, \tag{14}$$

The proof of this lemma is carried out similar to the paper [11].

Theorem 2. *Let the coefficients of system (1) satisfy conditions (2). Then the scattering matrix $S(\lambda)$ admits the analytical factorization*

$$S(\lambda) = \begin{pmatrix} 1 + G_{n,n-}^{n+1}(\lambda) - G_{1,n-}^{n+1}(\lambda) & 0 & ... & ... & 0 \\ 0 & 1 + G_{n,n-}^{n+1}(\lambda) - G_{2,n-}^{n+1}(\lambda) & ... & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & ... & 1 + G_{n,n-}^{n+1}(\lambda) - G_{n-1,n-}^{n+1}(\lambda) \end{pmatrix} \times$$

$$\times \begin{pmatrix} 1 + G_{11+}^{n+1}(\lambda) - G_{n1+}^{n+1}(\lambda) & G_{12+}^{n+1}(\lambda) - G_{n2+}^{n+1}(\lambda) & ... & ... & G_{1,n-1+}^{n+1}(\lambda) - G_{n,n-1+}^{n+1}(\lambda) \\ G_{21+}^{n+1}(\lambda) - G_{n1+}^{n+1}(\lambda) & 1 + G_{22+}^{n+1}(\lambda) - G_{n,2+}^{n+1}(\lambda) & ... & ... & G_{2,n-1+}^{n+1}(\lambda) - G_{n,n-1+}^{n+1}(\lambda) \\ ... & ... & ... & ... & ... \\ G_{n-1,1+}^{n+1}(\lambda) - G_{n1+}^{n+1}(\lambda) & G_{n-1,2+}^{n+1}(\lambda) - G_{n,2+}^{n+1}(\lambda) & ... & ... & 1 + G_{n-1,n-1+}^{n+1}(\lambda) - G_{n,n-1-}^{n+1}(\lambda) \end{pmatrix}, \tag{15}$$

and the function $S_{11}(\lambda)$ *admits the factorization*

$$S_{11}(\lambda)=\left(1+C_{n,n-}^{n+1}(\lambda)-G_{1,n-}^{n+1}(\lambda)\right)^{-1}\left(1+G_{11+}^{n+1}(\lambda)-G_{n1+}^{n+1}(\lambda)\right), \tag{16}$$

$$S_{11}(\lambda)=\left(1+C_{nn+}^{n+2}(\lambda)\right)^{-1}\left(1-G_{n1+}^{n+2}(\lambda)\right)\left(1-G_{nn-}^{n+2}(\lambda)\right)\left(1+G_{11-}^{n}(\lambda)\right), \tag{17}$$

where

$$G_{-}(\lambda)=\int_{-\infty}^{0}G(0,t)\,e^{i\lambda t}dt,$$

$$G_{+}(\lambda)=\int_{0}^{+\infty}G(0,t)\,e^{i\lambda t}dt.$$

Proof. From integral representation (12), allowing for boundary conditions (4), we have:

$$B_1=\left(1+G_{n,n-}^{n+1}(\lambda)-G_{1,n-}^{n+1}(\lambda)\right)^{-1}[\left(1+G_{11+}^{n+1}(\lambda)-G_{n1+}^{n+1}(\lambda)\right)A_1+$$

$$+\left(G_{12+}^{n+1}(\lambda)-G_{n2+}^{n+1}(\lambda)\right)A_2+...+\left(G_{1,n-1+}^{n+1}(\lambda)-G_{n,n-1+}^{n+1}(\lambda)\right),$$

....

$$B_k=\left(1+G_{n,n-}^{n+1}(\lambda)-G_{k,n-}^{n+1}(\lambda)\right)^{-1}[\left(G_{k1+}^{n+1}(\lambda)-G_{n1+}^{n+1}(\lambda)\right)A_1+$$

$$+\left(G_{k2+}^{n+1}(\lambda)-G_{n2+}^{n+1}(\lambda)\right)A_2+...+\left(1+G_{kk+}^{n+1}(\lambda)-G_{nk+}^{n+1}(\lambda)\right)A_k+$$

$$+...+\left(G_{k,n-1+}^{n+1}(\lambda)-G_{n,n-1+}^{n+1}(\lambda)\right)A_{n-1}],$$

....

$$B_{n-1}=\left(1+G_{n,n-}^{n+1}(\lambda)-G_{n-1,n-}^{n+1}(\lambda)\right)^{-1}[\left(G_{n-1,1+}^{n+1}(\lambda)-G_{n,1+}^{n+1}(\lambda)\right)A_1+$$

$$+\left(G_{n-1,2+}^{n+1}(\lambda)-G_{n2+}^{n+1}(\lambda)\right)A_2+...+\left(1+G_{n-1,n-1+}^{n+1}(\lambda)-G_{n,n-1+}^{n+1}(\lambda)\right)A_{n-1}],$$

Then from definition of scattering matrix (7) we get equality (15). Equality (16) follows from (15). Factorization formula (17) is obtained from representations (11) and (13) (for $k=2$). Indeed, from these representations for $A_2=A_3=...=A_{n-1}=0=0$ and $y_n^1(0,\lambda)=y_1^1(0,\lambda)$ we have:

$$y_1^1(0,\lambda)=\left(1-G_{nn-}^{n}(\lambda)\right)^{-1}\left(1+G_{11-}^{n}(\lambda)\right)A_1,$$

$$y_1^1(0,\lambda)=\left(1-G_{n1+}^{n+2}(\lambda)\right)^{-1}\left(1+G_{nn+}^{n+2}(\lambda)\right)B_1,$$

Consequently

$$B_1=\left(1+G_{nn+}^{n+2}(\lambda)\right)^{-1}\left(1-G_{n1+}^{n+2}(\lambda)\right)\left(1-G_{nn-}^{n}(\lambda)\right)^{-1}\left(1-G_{11-}^{n}(\lambda)\right)A_1,$$

i.e.

$$S_{11}(\lambda)=\left(1+G_{nn+}^{n+2}(\lambda)\right)^{-1}\left(1-G_{n1+}^{n+2}(\lambda)\right)\left(1-G_{nn-}^{n}(\lambda)\right)^{-1}\left(1-G_{11-}^{n}(\lambda)\right).$$

The theorem is proved.

3. The inverse scattering problem. The inverse scattering problem for system (1) is in the restoration of coefficients of equations by the known scattering matrix $S(\lambda)$ of the problem on a semi-axis.

The inverse scattering problem on a semi-axis is reduced the inverse problem for system (1) on the axis with additional condition that the coefficients equal zero for $x < 0$.

Theorem 3. *Let $S(\lambda)$ be the scattering matrix for system (1) with coefficients $c_{kn}(x)$, $c_{nk}(x)$ $(k = 1, 2, ..., n-1)$ satisfying conditions (2) and the functions* $\det S(\lambda)$, $1 + G^{n+1}_{nn-}(\lambda) - G^{n+1}_{kn-}(\lambda)$ $(k = 1, 2, ..., n-1)$ $1 - G^{n}_{nn-}(\lambda)$, $1 + G^{n}_{11-}(\lambda)$, $1 - G^{n+2}_{n1+}(\lambda)$, $1 + G^{n+2}_{nn+}(\lambda)$ *have no zero (the zeros of the problem are absent).*

Thus, the scattering matrix on the axis

$$\begin{pmatrix} 1 + G^{n+1}_{11+}(\lambda) & G^{n+1}_{12+}(\lambda) ... G^{n+1}_{1,n-1+}(\lambda) & G^{n+1}_{1n-}(\lambda) \\ G^{n+1}_{21+}(\lambda) & 1 + G^{n+1}_{22+}(\lambda) ... G^{n+1}_{2,n-1+}(\lambda) & G^{n+1}_{2n-}(\lambda) \\ ... & ... & ... \\ G^{n+1}_{n1+}(\lambda) & G^{n+1}_{n2+}(\lambda) ... G^{n+1}_{n,n-1+}(\lambda) & 1 + G^{n+1}_{nn-}(\lambda) \end{pmatrix} \tag{18}$$

is uniquely determined by the elements of scattering matrix $S(\lambda)$ and factorization elements.

Proof. From relation (15) by the Riemann Hilbert problem we find the factorization multipliers

$$G^{n+1}_{nn-}(\lambda) - G^{n+1}_{kn-}(\lambda) = g_{kn-}(\lambda),$$

$$G^{n+1}_{k1+}(\lambda) - G^{n+1}_{n1+}(\lambda) = g_{k1+}(\lambda), \qquad G^{n+1}_{k2+}(\lambda) - G^{n+1}_{n2+}(\lambda) = g_{k2+}(\lambda),$$

$$G^{n+1}_{k,n-1+}(\lambda) - G^{n+1}_{n,n-1+}(\lambda) = g_{k,n-1+}(\lambda), \quad k = 1, 2, ..., n-1. \tag{19}$$

Their total number is $(n-1)n$. From (19) we have:

$$G^{n+1}_{kn-}(\lambda) = G^{n+1}_{nn-}(\lambda) - g_{kn-}(\lambda),$$

$$G^{n+1}_{n1+}(\lambda) = G^{n+1}_{11+}(\lambda) - g_{11+}(\lambda),$$

$$G^{n+1}_{k1+}(\lambda) = G^{n+1}_{11+}(\lambda) + g_{k1+}(\lambda) - g_{11+}(\lambda), \quad k = 2, ..., n-1 \qquad (k \neq 1, n),$$

$$G^{n+1}_{n2+}(\lambda) = G^{n+1}_{22+}(\lambda) - g_{22+}(\lambda), \tag{20}$$

$$G^{n+1}_{k2+}(\lambda) = G^{n+1}_{22+}(\lambda) + g_{k2+}(\lambda) - g_{22+}(\lambda), \quad k = 1, 3, ..., n-1 \qquad (k \neq 2, n),$$

....

$$G^{n+1}_{n,n-1+}(\lambda) = G^{n+1}_{n-1,n-1+}(\lambda) - g_{n-1,n-1-}(\lambda),$$

$$G^{n+1}_{k,n+}(\lambda) = G^{n+1}_{n-1,n-1+}(\lambda) + g_{k,n-1+}(\lambda) - g_{n-1,n-1+}(\lambda),$$

Taking these values into account in equality (12), we get

$$k \in \{1, 2, ..., n\} \setminus \{n-1, n\}.$$

$$y_1(0, \lambda) = \left(1 + G^{n+1}_{11+}(\lambda)\right) A_1 + \left(G^{n+1}_{22+}(\lambda) + g_{12+}(\lambda) - g_{22+}(\lambda)\right) A_2 + ...+$$

$$+\left(G^{n+1}_{n-1,n-1+}(\lambda)+g_{1,n-1+}(\lambda)-g_{n-1,n-1+}(\lambda)\right)A_{n-1}+\left(G^{n+1}_{nn-}(\lambda)-g_{1n-}(\lambda)\right)B,$$

....

$$\begin{aligned} y_k(0,\lambda)=&\left(G^{n+1}_{11+}(\lambda)+g_{k1+}(\lambda)-g_{11+}(\lambda)\right)A_1+\\ &+\left(G^{n+1}_{22+}(\lambda)+g_{k2+}(\lambda)-g_{22+}(\lambda)\right)+...+\\ &+\left(G^{n+1}_{n-1,n-1+}(\lambda)+g_{k,n-1+}(\lambda)-g_{n-1,n-1+}(\lambda)\right)A_{n-1}+\\ &+\left(G^{n+1}_{nn-}(\lambda)-g_{kn-}(\lambda)\right)B, \end{aligned} \tag{21}$$

....

$$y_n(0,\lambda)=\left(G^{n+1}_{11+}(\lambda)-g_{11+}(\lambda)\right)A_1+\left(G^{n+1}_{22+}(\lambda)-g_{22+}(\lambda)\right)A_2+...+$$
$$+\left(G^{n+1}_{n-1,n-1+}(\lambda)-g_{n-1,n-1+}(\lambda)\right)A_{n-1}+\left(1-G^{n+1}_{nn-}(\lambda)\right)B.$$

1. Let $A_2=...=A_{n-1}=0$. We get from (21)

$$y^1_1(0,\lambda)=\left(1+G^{n+1}_{11+}(\lambda)\right)A_1+\left(G^{n+1}_{nn-}(\lambda)-g_{1n-}(\lambda)\right)B_1,$$

and from representations (19)

$$y^1_1(0,\lambda)=\left(1-G^{n}_{nn-}(\lambda)\right)^{-1}\left(1+G^{n+1}_{11-}(\lambda)\right)A_1\equiv\left(1+\bar{G}^{n}_{11-}(\lambda)\right)A_1.$$

Comparing the last two equalities we have

$$1+G^{n+1}_{11-}(\lambda)-\left(G^{n+1}_{nn-}(\lambda)-g_{1n-}(\lambda)\right)S_{11}(\lambda)=1+\bar{G}^{n}_{11-}(\lambda)$$

or

$$G^{n+1}_{11+}(\lambda)+\left(G^{n+1}_{nn-}(\lambda)-g_{1n-}(\lambda)\right)S_{11}(\lambda)=\bar{G}^{n}_{11-}(\lambda). \tag{22}$$

Since

$$S_{11}(\lambda)=\left(1+g_{1n-}(\lambda)\right)^{-1}\left(1+g_{11+}(\lambda)\right), \tag{23}$$

it follows from (22)

$$\begin{aligned} G^{n+1}_{11+}(\lambda)+&\left(G^{n+1}_{nn-}(\lambda)-g_{1n-}(\lambda)\right)\left(1+g_{1n-}(\lambda)\right)^{-1}\times\\ &\times\left(1+g_{11+}(\lambda)\right)=\bar{G}^{n}_{11-}(\lambda). \end{aligned} \tag{24}$$

Here $\bar{G}^{n}_{11-}(\lambda)$ is found from factorization (17) and $g_{1n-}(\lambda), g_{11+}(\lambda)$ from (23). From (24) we have:

$$\begin{aligned} G^{n+1}_{11+}(\lambda)\left(1+g_{11+}(\lambda)\right)^{-1}+\left(G^{n+1}_{nn-}(\lambda)-g_{1n-}(\lambda)\right)\left(1+g_{1n-}(\lambda)\right)^{-1}=\\ =\bar{G}^{n}_{11-}(\lambda)\left(1+g_{11+}(\lambda)\right)^{-1}. \end{aligned} \tag{25}$$

Solving this equation we find

$$\begin{aligned} G^{n+1}_{11+}(\lambda)&=\left[\bar{G}^{n}_{11-}(\lambda)\left(1+g_{11+}(\lambda)\right)\right]_+\left(1+g_{11+}(\lambda)\right),\\ G^{n+1}_{nn-}(\lambda)&=g_{1n-}(\lambda)+\left[\bar{G}^{n}_{11-}(\lambda)\left(1+g_{11+}(\lambda)\right)\right]_-\left(1+g_{1n-}(\lambda)\right). \end{aligned} \tag{26}$$

Similarly, from other equalities (21) for

$A_1 = ... = A_{k-1} = A_{k+1} = ... = A_{n-1} = 0$ we get

$$y_k^k(0,\lambda) = \left(1 + G_{kk+}^{n+1}(\lambda)\right) A_k + \left(G_{nn-}^{n+1}(\lambda) - g_{kn-}(\lambda)\right) B_k,$$

and from representations (11)

$$y_k^k(0,\lambda) = \left(1 + \bar{G}_{kk-}^{n}(\lambda)\right) A_k, \tag{27}$$

where

$$\bar{G}_{kk-}^{n}(\lambda) = \left(1 - C_{kn-}^{n}(\lambda)\right)^{-1} \left(1 + G_{kk-}^{n}(\lambda)\right).$$

Consequently,

$$\left(1 + G_{kk+}^{n+1}(\lambda)\right) A_k + \left(G_{nn-}^{n+1}(\lambda) - g_{kn-}(\lambda)\right) B_k = \left(1 + G_{kk-}^{n}(\lambda)\right) A_k$$

or

$$G_{kk+}^{n+1}(\lambda) + \left(G_{nn-}^{n+1}(\lambda) - g_{kn-}(\lambda)\right) S_{kk}(\lambda) = G_{k-}^{n}(\lambda).$$

Finally we get

$$G_{kk+}^{n+1}(\lambda) = -\left[\left(G_{nn-}^{n+1}(\lambda) - g_{kn-}(\lambda)\right) S_{kk}(\lambda)\right]_- \qquad (k = 2, 3, ..., n-1).$$

The theorem is proved.

The next theorem follows from this theorem.

Theorem 4. *Let the coefficients in system (1) be measurable functions and satisfy conditions (2) and the zeros of the problem are absent. Then by the known scattering matrix $S(\lambda)$ the coefficients are uniquely determined.*

References

[1]. Gasymov M.G., Levitan B.M. *Determination of Dirac's system on scattering phase.* Dokl. AN SSSR, 1966, 167, No6, pp.1219-1222. (Russian)

[2]. Gasymov M.G. *The inverse scattering problem for a system of Dirac equations of order $2n$.* Trudy Moskowskogo mat. obsh., 1968, 19, pp.41-112. (Russian)

[3]. Frolov I.G. *The inverse scattering problem for Dirac's system on the axis.* Dokl. AN SSSR, 1972, 207, No1, pp.44-47. (Russian)

[4]. Nizhnik L.P. Fam Loy Vu. *The inverse scattering problem on semi-axis with not self-adjoint potential matrix.* Ukr. mat. zhurn., 1974, v.26, No4, pp.469-486. (Russian)

[5]. Maksudov F.G., Veliev S.G. *The inverse scattering problem of self-adjoint Dirac operator on the axis.* DAN SSSR, 1975, v.225, No6, pp.1263-1266. (Russian)

[6]. Shabat A.B. *The inverse scattering problem for a system of differential equations.* Func. Analys. and its appl., 1975, v.9, No3, pp.75-79. (Russian)

[7]. Shabat A.B. *The inverse problem of scattering problem.* Diff. Uravn., 1979, v.15, No10, pp.1824-1834. (Russian)

[8]. Kaup D. *The three-wave interaction a non-dispersive phenomenon.* Stud. Appl. Math., 1976, v.55, pp.9-44.

[9]. Caudrey P.J. *The inverse spectral problem for a system of three coupled first order equations.* Proceedings of the Royal Irish Academy, 1983, v.83A, No1, pp.23-31.

[10]. Beals R., Coifman R.R. *Scattering and inverse scattering for first order system.* Comm. on pure and Appl. Mah., 1984, v.37, pp.39.

[11]. Iskenderov N.Sh. *The inverse scattering problem on a semi-axis for first order differential equations system.* Transactions of AS Azerb., 2000, v.XX, No4, pp.97-108.

[12]. Garayev A.P. *The inverse scattering problem for linear system on semi-axis.* Proceedings of Institute of Mathematics and Mechanics, 2002, v.XVI (XXIV), pp.29-37.

Fakhraddin A. Namazov
Institute of Mathematics and Mechanics of NAS of Azerbaijan
9, F.Agayev str., AZ1141, Baku, Azerbaijan
Tel.: (99412) 439 47 20 (off.)

Received January 06, 2006; Revised Aprel 11, 2006.
Translated by Nazirova S.H.