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ON SOLUTION OF A MIXED PROBLEM FOR AN EQUATION OF THE FOURTH ORDER WITH DISCONTINUOUS COEFFICIENT

Abstract

It is known that the solution of mixed problem for parabolic equation in sense of G.E.Shilov doesn't always exist. In this paper there was considered a mixed problem and found a solution in the form of contour integral for parabolic equation in the sense of Q.E.Shilov in one interval and in the sense of I.Q.Petrovsky in the other one.

We consider the mixed problem of the form

$$\frac{\partial u}{\partial t}=p\left(x\right)\frac{\partial^4 u}{\partial x^4}+q\left(x\right)\frac{\partial^2 u}{\partial x^2},\quad 0<x<1,\ t>0, \tag{1}$$

$$u\left(0,x\right)=\varphi_p\left(x\right),\ \alpha\left(p-1\right)<x<\alpha\left(p-2\right)+p-1,\ p=1,2. \tag{2}$$

$$L_{k+1}\left(u\right)\equiv\left.\frac{\partial^k u\left(x,t\right)}{\partial x^k}\right|_{x=0}=0,\quad k=0,1,$$

$$L_{k+3}\left(u\right)\equiv\left.\frac{\partial^k u\left(x,t\right)}{\partial x^k}\right|_{x=1}=0,\quad k=0,1,$$

$$L_k^1\left(u\right)+L_k^2\left(u\right)=0,\ k=\overline{5,8} \tag{3}$$

$$L_{k+5}^1\left(u\right)\equiv\gamma_{k+1}\left.\frac{\partial^k u\left(x,t\right)}{\partial x^k}\right|_{x=\alpha-0},\quad k=\overline{0,3},$$

$$L_{k+5}^2\left(u\right)\equiv\delta_{k+1}\left.\frac{\partial^k u\left(x,t\right)}{\partial x^k}\right|_{x=\alpha+0},\quad k=\overline{0,3},$$

where $p\left(x\right),q\left(x\right),\varphi_1\left(x\right),\varphi_2\left(x\right)$ are the known complex-valued functions, γ_k,δ_k are complex numbers $0<\alpha<1$

$$p\left(x\right)=\begin{cases} p_0\left(x\right)\left[\gamma+\sqrt{-1}\tau\right],\ x\in\left[0,\alpha\right), \\ \sqrt{-1}\tau p_0\left(x\right),\ x\in\left[0,1\right], \end{cases}$$

$$p_0\left(x\right)>0,\ x\in\left[0,1\right],\ \gamma<0,\ \tau>0,\ \operatorname{Re}q\left(x\right)>0,\ x\in\left[\alpha,1\right].$$

By $S_j^{(k)}$ $\left(j=\overline{1,8};\ k=1,2\right)$ we'll denote the following sectors of complex λ plane:

$$S_j^{(k)}=\left\{\lambda\left|(-1)^j\operatorname{Re}\left(1-\sqrt{-1}\right)V_k\left(x\right)\lambda>0;\ (-1)^j\operatorname{Re}\sqrt{-1}V_k\left(x\right)\lambda>0\right.\right\},$$

$$j=1,2;\ k=1,2;$$

$$S_j^{(k)}=\left\{\lambda\left|(-1)^j\operatorname{Re}\left(1-\sqrt{-1}\right)V_k\left(x\right)\lambda<0;\ (-1)^j\operatorname{Re}V_k\left(x\right)\lambda>0\right.\right\},$$

$$j=3,4;\ k=1,2;$$

$$S_j^{(k)} = \left\{ \lambda \left| (-1)^j \operatorname{Re} V_k(x)\lambda < 0;\ (-1)^j \operatorname{Re}\left(1+\sqrt{-1}\right) V_k(x)\lambda > 0 \right. \right\},$$

$$j = 5,6;\ k = 1,2;$$

$$S_j^{(k)} = \left\{ \lambda \left| (-1)^j \operatorname{Re}\left(1+\sqrt{-1}\right) V_k(x)\lambda < 0;\ (-1)^j \operatorname{Re}\sqrt{-1} V_k(x)\lambda > 0 \right. \right\},$$

$$j = 7,8;\ k = 1,2;$$

where $V_1(x) = \theta(x) = \left(p_0(x)\sqrt{\gamma^2+\tau^2}\right)^{-\frac{1}{4}} \exp\left[\sqrt{-1}\left(\frac{\pi}{4} - \frac{1}{4} arctg \frac{\tau}{\gamma}\right)\right]$, at $x \in [0,\alpha)$,

$V_2(x) = \omega(x) = (\tau p_0(x))^{-\frac{1}{4}} \exp \frac{3\pi}{8}\sqrt{-1}$, at $x \in [\alpha, 1]$.

Problem (1)-(3) is associated to the following spectral problem:

$$p(x)\, y^{IV} + q(x)\, y^{II} - \lambda^4 y = -\varphi_p(x), \tag{4}$$

$$\alpha(p-1) < x < \alpha(p-2) + p - 1,\ p = 1,2,$$

$$L_{k+1}(y) \equiv y^{(k)}(0) = 0,\ k = 0,1,$$

$$L_{k+3}(y) \equiv y^{(k)}(1) = 0,\ k = 0,1,$$

$$L_k^1(y) + L_k^2 = 0,\ k = \overline{5,8}, \tag{5}$$

$$L_{k+5}^1(y) \equiv \gamma_{k+1} y^{(k)}(\alpha - 0),\ k = \overline{0,3},$$

$$L_{k+5}^2(y) \equiv \delta_{k+1} y^{(k)}(\alpha + 0),\ k = \overline{0,3},$$

The following theorem is proved:

Theorem 1. *Let the coefficients of equation (4) satisfy the conditions:*

$$p_0(x) \in C^2[0,\alpha),\ \left|p_0^{(k)}(x)\right| \le M,\ \left(k = \overline{0,2}\right),\ x \in [0,\alpha),$$

$$p_0(x) \in C^2[\alpha,1],\ q(x) \in C[0,1]$$

Then at $\lambda \in S_i^{(p)}$ $\left(p = 1,2;\ i = \overline{1,8}\right)$ in the period $(\alpha(p-1),\ \alpha(2-p) + p - 1)$ equation (4) has four linear-independent solution $y_{4(p-1)+k}(x,\lambda)$ $\left(p = 1,2;\ k = \overline{1,4}\right)$, allowing the asymptotic representations:

$$\frac{d^s y_{4(p-1)+k}(x,\lambda)}{dx^s} = \left[\left(\sqrt{-1}\right)^{k-1} V_p(x)\lambda\right]^s V_p^{-\frac{3}{2}}(x)\left[1 + \frac{E(x,\lambda)}{\lambda}\right] \times$$

$$\times \exp\left[\left(\sqrt{-1}\right)^{k-1} \lambda \int\limits_{\alpha(p-1)}^{x} V_p(\xi)\, d\xi\right], \tag{6}$$

$$\alpha(p-1) < x < \alpha(2-p) + p - 1;\ k = \overline{1,4};\ s = \overline{0,3};\ \lambda \in S_i^{(p)}\ \left(p = 1,2;\ i = \overline{1,8}\right),$$

where $E(x,\lambda)$ are bounded analytical functions for large $|\lambda|$.

Solution of spectral problem (4), (5) is found in the form of ([1], pp.162-167).

$$y(x,\lambda) = \int\limits_0^\alpha \frac{\Delta^{(p,1)}(x,\xi,\lambda)}{\Delta(\lambda)} f_1(\xi)\, d\xi + \int\limits_\alpha^1 \frac{\Delta^{(p,2)}(x,\xi,\lambda)}{\Delta(\lambda)} f_2(\xi)\, d\xi, \tag{7}$$

Here

$$\Delta^{(i,j)}(x,\xi,\lambda)=\begin{vmatrix} u_{11}^{(i,j)}(x,\xi,\lambda) & u_{12}^{(i,j)}(x,\lambda) & . & . & . & u_{19}^{(i,j)}(x,\lambda) \\ u_{21}^{(i,j)}(\xi,\lambda) & u_{11}(\lambda) & . & . & . & u_{18}(\lambda) \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ u_{91}^{(i,j)}(\xi,\lambda) & u_{81}(\lambda) & . & . & . & u_{88}(\lambda) \end{vmatrix}, \qquad (8)$$

$$\Delta(\lambda)=\begin{vmatrix} u_{11}(\lambda) & . & . & . & u_{18}(\lambda) \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ u_{81}(\lambda) & . & . & . & u_{88}(\lambda) \end{vmatrix}, \qquad (9)$$

where

$$u_{11}^{(i,j)}(x,\xi,\lambda)=\frac{1}{2}\left(1+(-1)^{i+j}\right)g_i(x,\xi,\lambda), \quad (i,j=1,2),$$

$$u_{1q}^{(1,j)}(x,\lambda)=y_{q-1}(x,\lambda), \quad \left(j=1,2;\ q=\overline{2,5}\right),$$

$$u_{1q}^{(1,j)}(x,\lambda)=0, \quad \left(j=1,2;\ q=\overline{6,9}\right),$$

$$u_{1q}^{(2,j)}(x,\lambda)=0, \quad \left(j=1,2;\ q=\overline{2,5}\right),$$

$$u_{1q}^{(2,j)}(x,\lambda)=y_{q-1}(x,\lambda), \quad \left(j=1,2;\ q=\overline{6,9}\right),$$

$$u_{21}^{,(j,2)}(\xi,\lambda)=u_{31}^{(j,2)}(\xi,\lambda)=u_{41}^{(j,1)}(\xi,\lambda)=u_{51}^{(j,1)}(\xi,\lambda)=0, \quad (j=1,2),$$

$$u_{p1}^{(j,1)}(\xi,\lambda)=L_{p-1}(g_2)_x, \quad (j=1,2;\ q=4,5),$$

$$u_{p1}^{(i,j)}(\xi,\lambda)=\frac{1}{2}\left(1+(-1)^{1+j}\right)L_{p-1}^1(g_1)_x+\frac{1}{2}\left(1+(-1)^{j}\right)L_{p-1}^2(g_2)_x,$$

$$\left(i,j=1,2;\ q=\overline{6,9}\right),$$

$$u_{ij}(\lambda)=\begin{cases} L_i(y_j(x,\lambda)) & \text{at } \left(i=1,2;\ j=\overline{1,4}\right), \\ 0 & \text{at } \left(i=1,2;\ j=\overline{5,8}\right), \end{cases} \qquad (10)$$

$$u_{ij}(\lambda)=\begin{cases} 0 & \text{at } \left(i=3,4;\ j=\overline{1,4}\right), \\ L_i(y_j(x,\lambda)) & \text{at } \left(i=3,4;\ j=\overline{5,8}\right), \end{cases}$$

$$u_{ij}(\lambda)=L_i^1(y_j(x,\lambda)), \ \left(i=\overline{5,8};\ j=\overline{1,4}\right),$$

$$u_{ij}(\lambda)=L_i^2(y_j(x,\lambda)), \ \left(i=\overline{5,8};\ j=\overline{5,8}\right),$$

$$f_1(x)=\frac{-\varphi_1(x)}{p_0(x)\left(\gamma+\sqrt{-1}\tau\right)}, \ f_2(x)=\frac{-\varphi_2(x)}{\sqrt{-1}\tau p_0(x)},$$

$$g_p(x,\xi,\lambda)=\pm\frac{1}{2}\sum_{k=4p-3}^{4p} z_k(\xi,\lambda)\,y_k(x,\lambda), \ (p=1,2),$$

$$\ll + \gg \ \text{at } \alpha(p-1)<\xi\le x<\alpha(2-p)+p-1,$$

$$\ll - \gg \text{ at } \alpha (p-1) < x \le \xi < \alpha (2-p) + p - 1, \ (p = 1, 2),$$

$$z_{k+4(p-1)} (\xi, \lambda) = \frac{W^p_{4k} (\xi, \lambda)}{W^p (\xi, \lambda)} \qquad \left(k = \overline{1,4};\ p = 1, 2\right) \tag{11}$$

$W^p (\xi, \lambda)$ is Wronskian determinant for fundamental system of partial solutions $y_{4p-3} (x, \lambda), y_{4p-2} (x, \lambda), y_{4p-1} (x, \lambda), y_{4p} (x, \lambda)$ of homogeneous equation, corresponding to equation (4) at $\alpha (p-1) < x < \alpha (2-p) + p - 1,\ p = 1, 2.$

$W^p_{4k} (\xi, \lambda)$ is algebraic complement of the element $(4, k)$ of the determinant $W^p (\xi, \lambda), \ \left(p = 1, 2;\ k = \overline{1,4}\right)$.

Denote by λ^j_n and μ^j_n $(j = 1, 2, 3, 4\ n = 1, 2, ...)$ zeros of the determinant $\Delta (\lambda)$ laying on the j-th quarter.

The following theorem is proved: ([2], pp.5-12).

Theorem 2. *Let*

$$p_0 (x) \in C^2 [0, \alpha), \ \left| p_0^{(k)} (x) \right| \le M, \ \left(k = \overline{0,2}\right), x \in [0, \alpha),$$

$$p_0 (x) \in C^2 [\alpha, 1], \ q (x) \in C [0, 1],$$

$$\varphi_1 (x) \in C [0, \alpha), \ |\varphi_1 (x)| \le M, \ x \in [0, \alpha), \ \varphi_2 (x) \in [\alpha, 1] \ \text{and } B_k \ne 0, \ k = 1, 2$$

Then solution of problem (4), (5) is out of δ neighborhood of the points $\lambda = 0$, λ^j_n, μ^j_n whose asymptotic distributions are determined by the formulae

$$\lambda^j_n = e^{\sqrt{-1}\frac{\pi}{2}(j-1)} \left\{ \left[2 \int_0^\alpha \theta (\xi) \, d\xi \right]^{-1} \left[\ln_0 \left(-\frac{B_2}{B_1} \right) + 2\pi k \sqrt{-1} \right] \right\} + O\left(\frac{1}{k}\right),$$

$$k \to +\infty,$$

$$\mu^j_n = e^{\sqrt{-1}\frac{\pi}{2}(j-1)} \left\{ \left[2 \int_0^\alpha \omega (\xi) \, d\xi \right]^{-1} \left[2\pi n \sqrt{-1} - \ln_0 \left(-\frac{B_2}{B_1} \right) \right] \right\} + O\left(\frac{1}{n}\right),$$

$$n \to +\infty, \tag{12}$$

analytically and it holds the estimation

$$y = O\left(\frac{1}{\lambda^3}\right), \ |\lambda| \to +\infty, \ \lambda \in S^{(p)}_k \quad \left(p = 1, 2;\ k = \overline{1,8}\right), \tag{13}$$

where

$$B_1 = a_1 + \sqrt{-1} a_2 - a_3 - a_4 - \sqrt{-1} a_5 + a_6, \ B_6 = \sum_{k=1}^{6} a_k.$$

$$a_1 = \gamma_1 \gamma_2 \delta_3 \delta_4 \omega^4 (\alpha), \ a_2 = 2\gamma_1 \gamma_3 \delta_2 \delta_4 \theta (\alpha) \omega^3 (\alpha) \quad , a_3 = \gamma_1 \gamma_4 \delta_2 \delta_3 \omega^2 (\alpha) \theta^2 (\alpha),$$

$$a_4 = \gamma_2 \gamma_3 \delta_1 \delta_4 \theta^2 (\alpha) \omega^2 (\alpha), \ a_5 = 2\gamma_2 \gamma_4 \delta_1 \delta_3 \theta^3 (\alpha) \omega (\alpha) \quad , a_6 = \gamma_3 \gamma_4 \delta_1 \delta_2 \theta^4 (\alpha).$$

Theorem 3. *Let the functions* $p_0^{(k)} (x)$ *and* $\varphi_1^{(k)} (x)$ $\left(k = \overline{0,2}\right)$ *be continuous and bounded on* $[0, \alpha)$ *and conditions*

$$p_0 (x), \varphi_2 (x) \in C^2 [\alpha, 1], \ q (x) \in C [0, 1], \ \varphi_1 (0) = \varphi_1 (\alpha) = \varphi_2 (\alpha) = \varphi_2 (1) = 0.$$

be fulfilled. Assume that $|B_2| > |B_1| > 0$*.*

Then problem (1)-(3) has solution which is represented by the formula

$$u(x,\lambda) = \frac{2}{\pi\sqrt{-1}} \int\limits_{L_1} \lambda^3 e^{\lambda^4 t} y(x,\lambda)\, d\lambda. \tag{14}$$

Here L_1 *is unbounded open circuit contour determined by the relation*

$$L_1 = \left\{ \lambda \left| \lambda = -l \cot\frac{\pi}{8} + r e^{\sqrt{-1}\varphi},\ -\frac{\pi}{8} \le \varphi \le \frac{\pi}{8} \right.\right\} \cup$$

$$\cup \left\{ \lambda \left| \lambda = -l \cot\frac{\pi}{8} + \mathrm{Re}^{\pm\frac{\pi}{8}\sqrt{-1}},\ R \ge r \right.\right\}.$$

Here $r > 0$ *is sufficiently big number, and* l *satisfied the inequality*

$$0 < l < \frac{\ln|B_2| - \ln|B_1|}{2\cos\frac{\pi}{8} \int\limits_{\alpha}^{1} (\tau p_0(\xi))^{-\frac{1}{4}} d\xi}.$$

Proof. Let r_n be a monotonically increasing and positive numerical sequence satisfying the condition $\lim\limits_{n\to\infty} r_n = +\infty$ on λ complex surface. Consider such O_n circumferences of the radiuses r_n which do not intersect with the circumferences $\left|\lambda - \lambda_n^j\right| = \delta$ and $\left|\mu - \mu_n^j\right| = \delta,\ \left(j = \overline{1,4}\right)$. It is always possible.

Really, from formula (12) we have:

$$\left|\lambda_{n+1}^j - \lambda_n^j\right| = \pi \left[\int\limits_0^{\alpha} \theta(\xi)\, d\xi \right]^{-1} + O\left(\frac{1}{n}\right),\ n \to +\infty,$$

$$\left|\mu_{n+1}^j - \mu_n^j\right| = \pi \left[\int\limits_{\alpha}^{1} \omega(\xi)\, d\xi \right]^{-1} + O\left(\frac{1}{n}\right),\ n \to +\infty.$$

On positive direction of the axis $O\lambda_2$ we'll choose such points H and K for which conditions $OH = l$ and $OK = r$ are fulfilled.

Here $0 < l < \dfrac{\ln|B_2| - \ln|B_1|}{2\cos\frac{\pi}{8} \int\limits_{\alpha}^{1} (\tau p_0(\xi))^{-\frac{1}{4}} d\xi}$,and r is sufficiently large number.

On λ-complex surface we'll construct unbounded open contour $L_k\ \left(k = \overline{1,4}\right)$ determined by the equalities

$$L_k \left\{ \lambda \left| \lambda = e^{\sqrt{-1}\frac{\pi}{2}(k-1)} \left(-l\cot\frac{\pi}{8} + r e^{\sqrt{-1}\varphi} \right),\ -\frac{\pi}{8} \le \varphi \le \frac{\pi}{8} \right.\right\} \cup$$

$$\cup \left\{ \lambda \left| \lambda = e^{\sqrt{-1}\frac{\pi}{2}(k-1)} \left(-l\cot\frac{\pi}{8} + \mathrm{Re}^{\pm\frac{\pi}{8}\sqrt{-1}} \right),\ R \ge r \right.\right\},\ k = \overline{1,4}. \tag{15}$$

Intersection points of the contour $L_k\ \left(k = \overline{1,4}\right)$ with the circumference O_n we'll denote by A_{kn} and $B_{kn}\ \left(k = \overline{1,4}\right)$.

Fig.1.

We'll show that the integrals determined by formulae (14) uniformly converges at $t>0$. For this we'll take into account (13) in (14):

$$\left|\lambda^3 y(x,\lambda)\, e^{\lambda^4 t}\right| \le Me^{t|\lambda|^4 \cos 4\arg\lambda} = Me^{t|\lambda|^4\cos 4\left(\frac{\pi}{8}+\delta_n\right)} = Me^{-t|\lambda|^4 \sin 4\delta_n} \le$$

$$\le Me^{-t|\lambda|^4 \frac{8}{\pi}\delta_n} = Me^{-t\frac{8}{\pi}\arcsin\left(\frac{1}{|\lambda|}l\sin\frac{3\pi}{8}\right)} \le Me^{-t\frac{8}{\pi}l\sin\frac{3\pi}{8}|\lambda|^3}.$$

Here $\delta_n = \arcsin\left(\dfrac{1}{|\lambda|}l\sin\dfrac{3\pi}{8}\right)$.

Allowing for solution (14) in equation (1) we'll get:

$$\frac{\partial u}{\partial t} - p(x)\frac{\partial^4 u}{\partial x^4} - q(x)\frac{\partial^2 u}{\partial x^2} =$$

$$= \frac{2}{\pi\sqrt{-1}}\int_{L_1}\left[\lambda^4 y(x,\lambda) - p(x)\, y^{IV}(x,\lambda) - q(x)\, y^{II}(x,\lambda)\right]\lambda^3 e^{\lambda^4 t} d\lambda =$$

$$= \frac{2}{\pi\sqrt{-1}}\int_{L_1}\lambda^3\varphi(x)\, e^{\lambda^4 t} d\lambda. \tag{16}$$

By the Cauchy integral formula we have:

$$\sum_{k=1}^{4}\int_{A_{kn}E_kF_kB_{kn}}\lambda^3 e^{\lambda^4 t} d\lambda + \sum_{k=1}^{4}\int_{\mu_{kn}}\lambda^3 e^{\lambda^4 t} d\lambda = 0, \tag{17}$$

where $\mu_{1n} = B_{1n}A_{2n},\ \mu_{2n} = B_{2n}A_{3n},\ \mu_{3n} = B_{3n}A_{4n},\ \mu_{4n} = B_{4n}A_{1n}$.

We'll show that the truth of the following equality at $t > 0$:

$$\lim_{n\to\infty}\int\limits_{\mu_{kn}} \lambda^3 e^{\lambda^4 t} d\lambda = 0,\ k = \overline{1,4}. \tag{18}$$

Really for $k = 1$ we have:

$$\left|\int\limits_{B_{1n}A_{2n}} \lambda^3 e^{\lambda^4 t} d\lambda\right| = \frac{1}{4} r_n^4 \int\limits_{4\delta_n}^{\pi-4\delta_n} e^{-tr_n^4 \sin\varphi} d\varphi \le \frac{r_n^4}{4}\int\limits_{4\delta n}^{\pi/2} e^{-tr_n^4 \frac{2}{\pi}\varphi} d\varphi +$$

$$+\frac{r_n^4}{4}\int\limits_{\pi/2}^{\pi-4\delta_n} e^{-tr_n^4\left(2-\frac{2}{\pi}\varphi\right)} d\varphi = \frac{\pi}{4t}\left[e^{-tr_n^4 \frac{8}{\pi}\delta_n} - e^{tr_n^4}\right] \le$$

$$\le \frac{\pi}{4t}\left[e^{-tr_n^3 \frac{8}{\pi} l \sin\frac{3\pi}{8}} - e^{-tr_n^4}\right] \xrightarrow[n\to\infty]{} 0.$$

Using the formulae for determination of contour L_k we'll obtain:

$$\int\limits_{L_1} \lambda^3 e^{\lambda^4 t} d\lambda = \lim_{n\to\infty} \int\limits_{A_{1n}E_1F_1B_{1n}} \lambda^3 e^{\lambda^4 t} d\lambda =$$

$$= \lim_{n\to\infty} \int\limits_{A_{kn}E_kF_kB_{kn}} \lambda^3 e^{\lambda^4 t} d\lambda = \int\limits_{L_k} \lambda^3 e^{\lambda^4 t} d\lambda,\ k = \overline{2,4}. \tag{19}$$

Passing to limit as $n \to \infty$ in equality (17) and using equalities (18), (19) we'll obtain:

$$\int\limits_{L_1} \lambda^3 e^{\lambda^4 t} d\lambda = 0.$$

Allowing for this equality in (16) we see that function (14) satisfies equation (1).

It is easy to check that integral determined by formula (14) satisfies boundary conditions (3).

Now check the fulfillment of initial condition (2).

Allowing for the conditions of the theorem we can show the truth of the equality

$$y(x,\lambda) = \frac{-2\varphi_p(x)}{\lambda^4} + \frac{W_p(x,\lambda)}{\lambda^5},\ \alpha(p-1) < x < \alpha(2-p) + p - 1,\ p = 1,2,$$

$$\lambda \in S_j^k\ \left(j = \overline{1,8},\ k = 1,2\right), \tag{20}$$

Here $W_p(x,\lambda)$ $(p = 1,2)$ are bounded and analytical functions out of δ neighborhood of poles of Green function.

Using the found formulae (12) for pole of Green function we can show that the function $y(x,\lambda)$ is analytical inside of closed contour $A_{1n}E_1F_1B_{1n}A_{1n}$ at $|B_2| > |B_1|$.

By the Cauchy integral formula we have:

$$\int\limits_{L_1} \lambda^3 y(x,\lambda) d\lambda = \lim_{n\to\infty} \int\limits_{A_{1n}E_1F_1B_{1n}} \lambda^3 y(x,\lambda) d\lambda = \lim_{n\to\infty} \int\limits_{B_{kn}A_{kn}} \lambda^3 y(x,\lambda) d\lambda. \tag{21}$$

Allowing for (20) and (21) we find:

$$\lim_{n\to\infty}\int\limits_{B_{1n}A_{1n}}\lambda^3 y\left(x,\lambda\right)d\lambda=-2\varphi\left(x\right)\sqrt{-1}\lim_{n\to\infty}\int\limits_{\frac{\pi}{8}+\delta_n}^{-\frac{\pi}{8}-\delta_n}d\lambda+$$

$$+\sqrt{-1}\lim_{n\to\infty}\int\limits_{\frac{\pi}{8}+\delta_n}^{-\frac{\pi}{8}-\delta_n}\frac{W_p\left(x,r_n e^{\sqrt{-1}\varphi}\right)}{r_n e^{\sqrt{-1}\varphi}}d\varphi=\frac{\pi\sqrt{-1}}{2}\varphi\left(x\right).\tag{22}$$

On the other hand

$$u\left(x,0\right)=\frac{2}{\pi\sqrt{-1}}\int\limits_{L_1}\lambda^3 y\left(x,\lambda\right)d\lambda.$$

Allowing for (22) in (21) we'll get the truth of the following equality:

$$u\left(x,0\right)=\varphi\left(x\right).$$

The theorem is proved.

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Received February 14, 2006; Revised Aprel 7, 2006.
Translated by Mamedova V.A.