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DURABILITY OF THICK-WALLED PIPE FILLED WITH CORROSIVE MEDIUM SUBJECT TO DAMAGEABILITY OF MATERIAL OF PIPE AND RETAINED STRENGTH

Abstract

In the paper we investigate the process of diffused failure of thick-walled pipe with corrosive filler forming the uniform pressure on inner boundary of the pipe. It's accepted that the influence of corrosive medium manifests itself only on at the limit of instantaneous strength. Integral operator of hereditary type describes the damageability process. The problem is solved with respect to retained strength of material of the pipe beyond failure front. The numerical analysis is led and the curves of motion of failure front depending on the concentration of distribution of corrosive medium and measure of retained strength beyond failure front are constructed.

In the carried out investigations on definition of durability of thick-walled pipes with filler, the chemical activity of filler or some other its corrosiveness, as a rule, was ignored. One of the first attempts of account of this factor in the creeping conditions of material of the pipe was the paper [2]. In the paper [3] the similar investigation is carried out for elastically damaging material of the pipe following the hereditary model of hereditarily damaging solid of Suvorov-Akhundov. In that paper it was obtained the influence of corrosiveness of the material on origin and development of failure front provided complete loss of carrying capacity of the pipe beyond failure front.

In this paper this is solved provided the presence of retained strength beyond the failure front. As in the paper [3] we'll assume the corrosiveness of filler manifests itself on lowering of limit of short-term strength σ_{st} whose dependence on the concentration C of corrosive medium is defined by the formula [1]

$$\sigma_{st} = \sigma_{st0} \left(1 - \gamma C\right), \tag{1}$$

where γ_n is an empiric constant.

We also accept a linear distribution of concentration of corrosive medium by thickness of the pipe

$$C = \frac{b - r}{b - a}. \tag{2}$$

Here a and b are inner and outer radii of the pipe, r is current radius of the pipe, C is concentration of corrosive medium referred to the level of its concentration on the inner surface such that $C\left(a\right) = 1$ and $C\left(b\right) = 0$. As well as in [3] we accept that the influence of corrosive filler on rheological characteristics of the material of the pipe is insignificant and we can neglect them. Let the state of the material of the pipe be described by the physical relations given in the papers [5,6] provided the neglect of process of invertible creeping and assuming the volumetric deformation as elastic one

$$\ni_{ij}=\frac{1}{2\mu_0}\left(1+M^*\right)s_{ij};\quad \varepsilon=\frac{\sigma}{3K_0}, \tag{3}$$

where $\ni_{ij}, s_{ij}$ are deviators, ε and σ are sphere parts of the deformation and stress tensors, respectively. In turn M^* is an integral damageability operator of hereditary type. At monotone stress this operator is the operator of continuous action and then for determination of stresses we can apply the Volterra and Rabotnov correspondence principle.

We take the failure criterion in the form following the papers [5,6]

$$\left(1+M^*\right)\sigma_{in}=\sigma_{st}, \tag{4}$$

where σ_{in} is stress intensity which for thick pipe being under the condition of plane deformation has the form

$$\sigma_{in}=\sqrt{2}p\frac{a^2b^2}{b^2-a^2}\cdot\frac{1}{r^2}. \tag{5}$$

Here p is impression on inner surface of the pipe created by the filler.

Originally the pipe consists of single unsolved material. The maximum value of the stress intensity is reached on the inner surface of the pipe, where a more strong process of failure accumulation is. This process leads to origin of failure zone at time t_0 defined on the basis of failure criterion (4)

$$\left(1+M^*\right)\sigma_{in\max}=\sigma_{st0}\left(1-\gamma C\right). \tag{6}$$

Subject to the fact that $C\left(a\right)=1$ and representation

$$\sigma_{in\max}=\left.\sigma_{in}\right|_{r=a}=\sqrt{2}p\frac{b^2}{b^2-a^2} \tag{7}$$

denoting $a/b=\beta_0$ and $\dfrac{\sigma_{st0}}{\sqrt{2p}}=g$ and allowing for (7) in (6) we obtain

$$\int_0^{t_0}M\left(\tau\right)d\tau=g\left(1-\beta_0^2\right)\left(1-\gamma\right)-1. \tag{8}$$

Hence for weakly singular Abel kernel $M\left(t\right)=\lambda t^{-\alpha};\ 0<\alpha<1;$

$$t_0=\left\{\frac{1-\alpha}{m}\left[g\left(1-\beta_0^2\right)\left(1-\gamma\right)-1\right]\right\}^{\frac{1}{1-\alpha}}. \tag{9}$$

For the constant damageability kernel $M\left(t\right)=m=const$

$$mt_0 = g\left(1-\beta_0^2\right)(1-\gamma)-1. \tag{10}$$

Further the boundary of the failure zone – failure front will move towards the outer surface of the pipe. Failure zone itself is an annular zone. The material of this failure zone remains the load-carrying ability, but significantly least than original material in front of failure front. We'll assume that on the failure front the material of the pipe sharply changes its instantaneous rheological characteristics, in this paper it changes the value of shear modulus μ. We take as μ_1 the shear modulus of the material of the pipe in front of failure front, and μ_0 beyond failure front. Introduce the notation

$$\chi = \frac{\mu_0}{\mu_1}. \tag{11}$$

It's obvious that $\chi < 1$.

The stressed state in arbitrary time is determined as for two-layer pipe with different elastic characteristics.

For intensity of stresses in the domain of the pipe in front of failure front that will supply with index 1 according to the known formulae [1] we obtain accepting the material as incompressible

$$\sigma_{in}^{(1)} = \sqrt{2}q\frac{k^2b^2}{b^2-k^2}\cdot\frac{1}{r^2}. \tag{12}$$

where q is pressure on failure front, k is radius of failure front.

Then using this expression for intensity of stresses in outer annular zone in front of failure front in strength criterion (4) we obtain the following equation

$$\frac{q(t)}{b^2-k^2(t)} + \frac{1}{k^2(t)}\int_0^t M(t-\tau)\frac{k^2(\tau)\,q(\tau)}{b^2-k^2(\tau)}d\tau = \frac{\sigma_{st0}}{\sqrt{2b^2}}\left(1-\gamma\frac{b-k(t)}{b-a}\right). \tag{13}$$

For the radial displacements of points of the domain beyond and in front of failure front provided incompressibility of the material again according to [3] we have

$$W^{(1)} = \frac{1}{2\mu_1}\left(1+M^*\right)\frac{k^2b^2q}{b^2-k^2}\cdot\frac{1}{r}; \quad W^{(0)} = \frac{a^2k^2(p-q)}{2\mu_0\left(k^2-a^2\right)}\cdot\frac{1}{r}. \tag{14}$$

From the continuity condition of displacements on failure front

$$W^{(1)}\Big|_{r=k} = W^{(0)}\Big|_{r=k} \tag{15}$$

we obtain

$$\frac{b^2k(t)\,q(t)}{b^2-k^2(t)} + \int_0^t M(t-\tau)\frac{b^2k^2(\tau)\,q(\tau)}{b^2-k^2(\tau)}\cdot\frac{1}{k(t)}d\tau = \frac{1}{\chi}\frac{a^2k(t)}{k^2(\tau)-a^2}\left(p-q(t)\right). \tag{16}$$

Thus we have a system of two nonlinear integral equations (13),(16) with respect to the radial coordinate $k(t)$ of failure front and pressure $q(t)$ on it. Note that if

t_0 is time of initial failure i.e. failure of inner surface of the pipe $r=a$ defined according to formula (8) then in system (13),(16) at $\tau \le t_0$ it should be supposed $k(\tau)=a;\ q(\tau)=p$. Integral equation (16) makes sense only at $t>t_0$.

We can lead equations (13),(16) to the solution of one nonlinear integral equation. For this using the identity of structure of integral members of equations (13) and (16), excepting them we obtain the following explicit representation of dependence of pressure on failure front on its radial coordinate

$$q(t)=p-\frac{\sigma_{st0}\chi}{\sqrt{2}}\left(\frac{k^2(t)}{a^2}-1\right)\left(1-\gamma\frac{b-k(t)}{b-a}\right). \tag{17}$$

Allowing for this representation in integral equation (13) and introducing the dimensionless quantity $\beta(t)=k(t)/b$ with respect to this dimensionless radial coordinate of failure front we obtain the following nonlinear integral equations

$$\frac{1}{1-\beta^2(t)}\left\{1+\chi g\left(1-\frac{\beta^2(t)}{\beta_0^2}\right)\left(1-\gamma\frac{1-\beta(t)}{1-\beta_0}\right)\right\}+\frac{1}{\beta^2(t)}\times$$
$$\times\int_0^t M(t-\tau)\frac{\beta^2(\tau)}{1-\beta^2(\tau)}\left\{1+\chi g\left(1-\frac{\beta^2(\tau)}{\beta_0^2}\right)\left(1-\gamma\frac{1-\beta(\tau)}{1-\beta_0}\right)\right\}d\tau=$$
$$=g\left(1-\gamma\frac{1-\beta(\tau)}{1-\beta_0}\right);\quad g=\frac{\sigma_{st0}}{\sqrt{2}p}. \tag{18}$$

The solution of equation (18) defines the character of extension of annular failure zone $\beta=\beta(t)$. Further by formula (17) the pressure on failure front is defined. It should be noted that the solution of an integral equation is valid until the pressure $q(t)$ calculated by formula (17) is positive. Its equality to zero and its negativity means violation of entirety of the material by formation of arc cracks along failure front.

To clarify the quality picture of failure process we take failure process as the damageability operator $M(t-\tau)=m=const$, then introducing the dimensionless time $\theta=mt$ and $\eta=m\tau$, equation (18) gets the form

$$\frac{1}{1-\beta^2(\theta)}\left\{1+\chi g\left(1-\frac{\beta^2(\theta)}{\beta_0^2}\right)\left(1-\gamma\frac{1-\beta(\theta)}{1-\beta_0}\right)\right\}+\frac{1}{\beta^2(\theta)}\times$$
$$\times\int_0^\theta\frac{\beta^2(\eta)}{1-\beta^2(\eta)}\left\{1+\chi g\left(1-\frac{\beta^2(\eta)}{\beta_0^2}\right)\left(1-\gamma\frac{1-\beta(\eta)}{1-\beta_0}\right)\right\}d\eta=$$
$$=g\left(1-\gamma\frac{1-\beta(\theta)}{1-\beta_0}\right). \tag{19}$$

Multiplying the obtained one by $\beta^2(\theta)$ and differentiating with respect to dimensionless time θ we obtain the following differential equation

$$\frac{d\beta}{d\theta}=\frac{A(\beta)}{B(\beta)} \tag{20}$$

$$
\left\{
\begin{array}{l}
A(\beta) = \dfrac{\beta^2}{1-\beta^2}\left\{1+\chi g\left(1-\dfrac{\beta^2}{\beta_0^2}\right)\left(1-\gamma\dfrac{1-\beta}{1-\beta_0}\right)\right\} \\
B(\beta) = 2\beta\left(1-\gamma\dfrac{1-\beta}{1-\beta_0}\right)+\gamma\dfrac{\beta^2}{1-\beta_0}- \\
-\dfrac{2\beta}{\left(1-\beta^2\right)^2}\left\{1+\chi g\left(1-\dfrac{\beta^2}{\beta_0^2}\right)\left(1-\gamma\dfrac{1-\beta}{1-\beta_0}\right)\right\}- \\
-\chi g\dfrac{\beta^2}{1-\beta^2}\left\{-\dfrac{2\beta^2}{\beta_0^2}\left(1-\gamma\dfrac{1-\beta}{1-\beta_0}\right)+\dfrac{\gamma}{1-\beta_0}\left(1-\dfrac{\beta^2}{\beta_0^2}\right)\right\}.
\end{array}
\right. \tag{21}
$$

The initial condition for it will be condition (10)

$$
\beta|_{\theta=\theta_0} = \beta_0;\ \theta_0 = g\left(1-\beta_0^2\right)(1-\gamma)-1. \tag{22}
$$

The obtained Cauchy problem (20),(22) was solved numerically by the Runge-Kutta method for the values of incoming parameters $\beta_0 = 0,5;\ g = 4;\ \ \gamma = 0;\ 0,01;\ 0,025;\ 0,1$ and $\chi = 0;\ 0,01$ and 0,1. In fig.1-3 the curve of motions of the failure front for three values of parameter of the retained strength χ depending on the concentration parameter γ of corrosive medium are led. The led graphs argue that the increase of concentration of corrosive medium in the material of the pipe leads to sufficiently tangible identification of process of diffused failure. Moreover for respectively large retained strength the failure process qualitatively varies, it occurs with decreasing velocity.

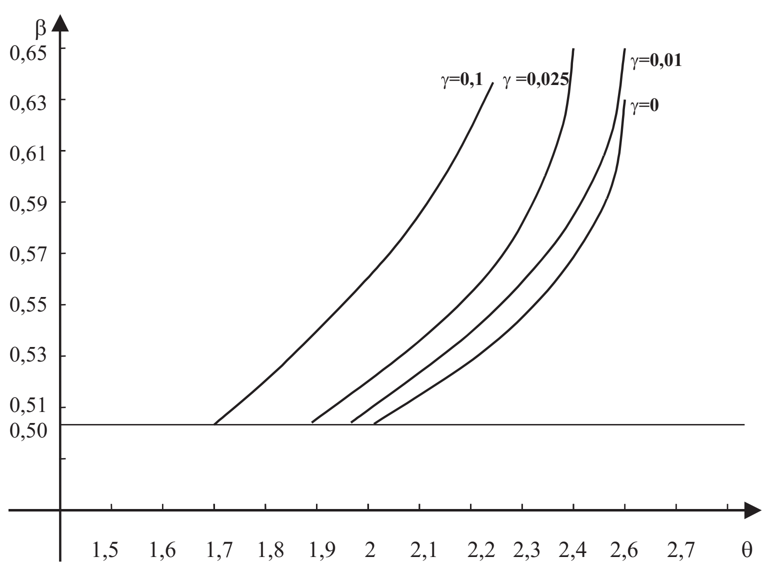

Fig. 1. $\chi = 0$.

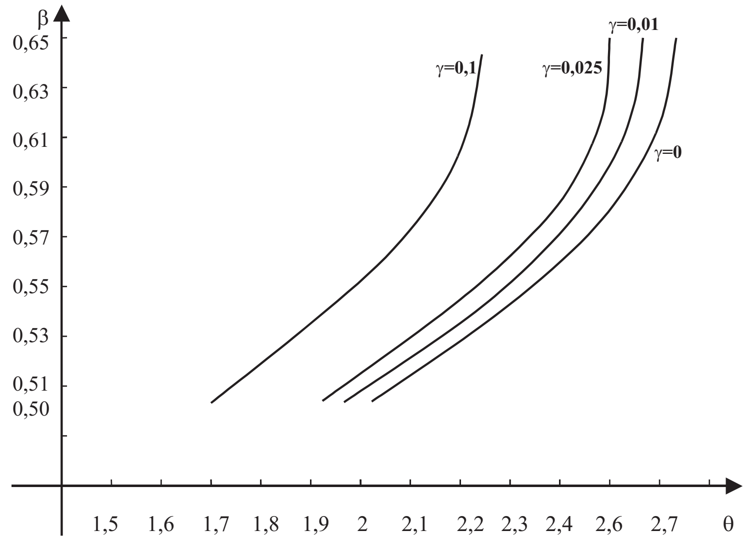

Fig. 2. $\chi = 0,01.$

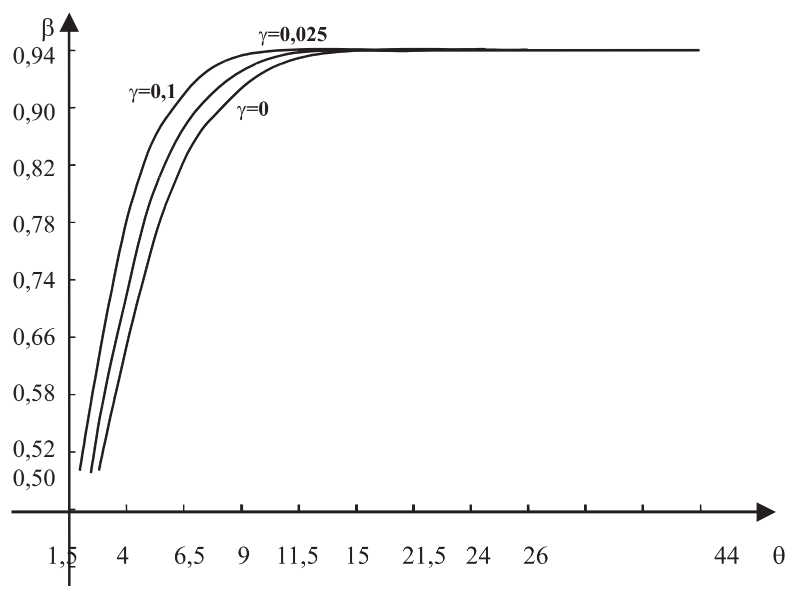

Fig. 3. $\chi = 0,1.$

In fig.4-5 the curves of motion of failure front are given for different values of concentration parameter of corrosive medium depending on parameter of retained strength.

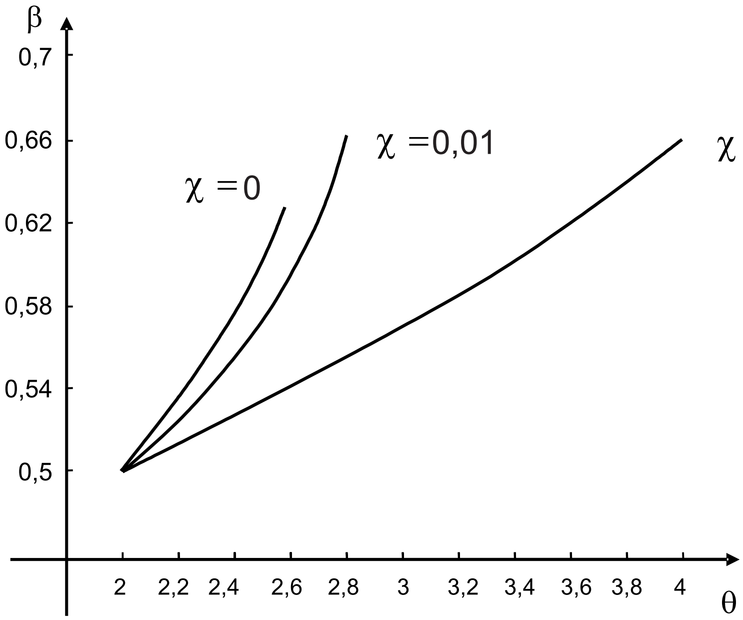

Fig. 4. $\gamma = 0$.

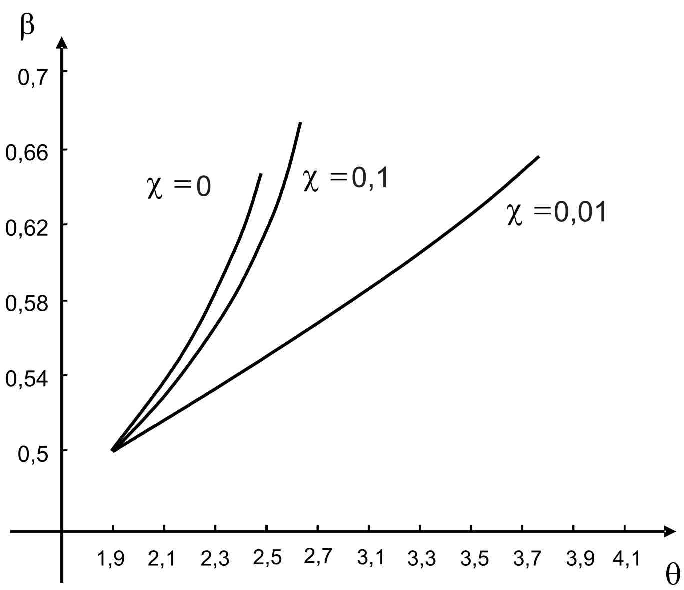

Fig. 5. $\gamma = 0,025$.

They argue that by increasing the retained strength beyond the failure front uuof the failure process of itself is stretched.

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