MECHANICS

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FLEXURAL GRAVITATIONAL CAPILLARY WAVES IN FLOATING PLATE CAUSED BY RANDOM LOAD

Abstract

In the paper the formula describing the propagation of plane nonstationary flexural gravity capillary waves in plate caused by normal random load is obtained. In the bend equation the presence of pressure jump is taken into account by both sides of contact between viscoelastic plate and noncompressible fluid stipulated by forces of surface tension. Analysis of flexure of elastic plate under the action of random load moving with constant velocity, is carried out.

In this paper first the problem on propagation of unsteady plane flexural gravity capillary waves in plate caused by normal random load is considered. The formula taking into account the presence of pressure jump by both sides of contact between viscoelastic plate and noncompressible fluid stipulated by forces of surface tension is obtained. Analysis of flexure of elastic plate under the action of random load moving with constant velocity is carried out.

Nonstationary oscillations of elastic plate floating on surface of fluid of total depth have been investigated in [1,2]. Stationary waves caused by displacement of load has been studied earlier in [3], nonstationary plane gravity elastic waves with periodic by time pressure in [1,4] with moving pressure in [1,3].

1. Let the nonstationary random load $P(\xi_1, \xi_2, t)$ act on the surface of the plate, where t is time, (ξ_1, ξ_2) is a random point. The mathematical expectation (mean value) of the function $P(\xi_1, \xi_2, t)$ of two random variables ξ_1, ξ_2 is determined by their joint distribution by formula [5]

$$MP(\xi_1, \xi_2, t) = \iint\limits_{-\infty}^{\infty} \Phi(\xi_1, \xi_2)\, P(\xi_1, \xi_2, t)\, d\xi_1 d\xi_2, \tag{1.1}$$

if this expression exists in the sense of absolute convergence. Here $\Phi(\xi_1, \xi_2)$ is density of two-dimensional distribution of the vector (ξ_1, ξ_2).

Formula (1.1) is more practical, when by Gaussian distribution the continuous random variables ξ_1, ξ_2 are distributed normally with mathematical expectation (center) x_1, x_2 and dispersion $d^2/2$

$$\Phi\left(\xi_1,\xi_2\right)=\Phi\left(\frac{x_i-\xi_i}{d}\right)=\frac{1}{\pi d^2}\exp\left(-\sum_{i=1}^{2}\frac{\left(x_i-\xi_i\right)^2}{d^2}\right). \quad (1.2)$$

Assuming independence of P of the polar angle θ, we write condition (1.1) with the help of (1.2) in polar coordinates that is characteristic for a problem with axial symmetry

$$MP\left(\xi_1,\xi_2,t\right)=\frac{2}{d^2}\int_0^\infty I_0\left(2\frac{r\rho}{d^2}\right)\exp\left(-\frac{r^2+\rho^2}{d^2}\right)P\left(\rho,t\right)\rho d\rho. \quad (1.3)$$

Here we use the integral [6]

$$\int_0^{2\pi}\exp\left(2\frac{r\rho}{d^2}\cos\theta\right)d\theta=2\pi I_0\left(2\frac{r\rho}{d^2}\right),$$

where I_0 is the Bessel function of imaginary argument and as $d\to 0$ the mean value of (1.3) passes to $P\left(r,t\right)$.

In the case of potential flow of noncompressible viscous fluid the continuity equation and third motion equation are reduced to the form of [7]

$$\frac{\partial^2\varphi}{\partial x_1^2}+\frac{\partial^2\varphi}{\partial x_2^2}+\frac{\partial^2\varphi}{\partial z^2}=\left(\Delta_-+\frac{\partial^2}{\partial z^2}\right)\varphi, \quad (1.4)$$

$$-P_2=\rho\frac{\partial\varphi}{\partial t}+\rho_2 gz. \quad (1.5)$$

The dynamic linearized boundary condition on unperturbed contact $z=0$ is determined by the known Laplace formula: $P_2^0\left(w\right)+\left.P_2\right|_{z=w}-P_{pl}=-\sigma R^{-1}$

$$-P_{pl}=\rho_2\frac{\partial\varphi}{\partial t}+\rho_2 gw-\sigma\Delta_- w. \quad (1.6)$$

The kinematic condition on the contact $z=0$

$$\frac{\partial w}{\partial t}=\frac{\partial\varphi}{\partial z}, \quad (1.7)$$

where φ is potential of velocities, ρ_2 is density of fluid, w is flexure of a plate. σ is a coefficient of the surface, P_2 is pressure in fluid, P_{pl} is pressure in lower surface of plate.

Let the viscoelastic properties of the material of a plate be described by linear theory of heredity [8]

$$Lw=w-\int_{-\infty}^{t}H\left(t-\tau\right)w\left(\tau\right)d\tau. \quad (1.8)$$

Substituting (1.1) and (1.6) in the motion equation of viscoelastic plate we obtain

$$D\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)^2 Lw+\rho_1 h\frac{\partial^2 w}{\partial t^2}+\rho_2 gw-\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)w+\rho_2\left.\frac{\partial\varphi}{\partial t}\right|_{z=0}=$$

$$=\iint\limits_{-\infty}^{\infty}\Phi\left(\frac{x_i-\xi_i}{d}\right)P\left(\xi_i,t\right)d\xi_i, \tag{1.9}$$

where $D=Eh^3/\left(1-\nu^2\right)$ is cylindrical rigidity, ρ_1, h are density and thickness of plate.

Equations (1.4) and (1.9) describe motions of viscoelastic plate under the action of normal random load applied to free surface of plate and floating on surface of incompressible fluid.

2. Consider the problem on excitation of flexural gravity capillary waves in viscoelastic plate, assuming on the level $z=-H$ the presence of boundary of fluid with absolutely rigid bottom $(\partial\varphi/\partial z)=0$.

For the solution of problem (1.4),(1.9) and (1.7) we use the space-time Fourier transformation defined by the formula $\left(\bar{x}=\{x_1,x_2\}\,,\quad \bar{k}=\{k_1,k_2\}\right)$ [9]

$$\bar{g}\left(\bar{k},\omega\right)=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}g\left(\bar{x},t\right)\ e^{-i\left(\bar{k}\bar{x}+\omega t\right)}d\bar{x}dt. \tag{2.1}$$

After integral transformation of (2.1) equations (1.4),(1.9) and boundary conditions get the form

$$\frac{\partial^2\bar{\varphi}}{\partial z^2}-k^2\bar{\varphi}=0,\ \ k=\sqrt{k_1^2+k_2^2}, \tag{2.2}$$

$$\left\{Dk^4\left[1-\bar{H}\left(\omega\right)\right]-\rho_1 h\omega^2+\rho_2 g+\sigma k^2\right\}+\rho_2\ i\omega\bar{\varphi}|_{z=0}=$$

$$=\exp\left(-\frac{d^2}{4}k^2\right)\bar{P}\left(k,\omega\right), \tag{2.3}$$

$$i\omega\bar{w}=\left.\frac{\partial\bar{\varphi}}{\partial z}\right|_{z=0},\quad \frac{\partial\bar{\varphi}}{\partial z}=0. \tag{2.4}$$

The solution of equation (2.2) with the help of the second condition of (2.4) is reduced to the form

$$\bar{\varphi}=2C_2\left(\omega\right)e^{kH}ch\left[k\left(z+h\right)\right]. \tag{2.5}$$

Substituting (2.5) in (2.3) and in the first condition of (2.4) we obtain

$$\bar{w}=\frac{\exp\left(-d^2k^2/4\right)}{F\left(k,\omega\right)}\bar{P}\left(k,\omega\right), \tag{2.6}$$

$$\bar{\varphi}=\frac{i\omega ch\left[k\left(z+h\right)\right]\exp\left(-d^2k^2/4\right)}{ksh\left(kH\right)F\left(k,\omega\right)}P\left(k,\omega\right), \tag{2.7}$$

where

$$F(k,\omega) = Dk^4 + \sigma k^2 + \rho_2 g - \left(\rho_1 h + \frac{\rho_2}{kth(kH)}\right)\omega^2 - Dk^4 \bar{H}(\omega), \tag{2.8}$$

$$\bar{H}(\omega) = \int_0^\infty e^{-i\omega\tau} H(\tau)\, d\tau.$$

Using the theorem on the inverse Fourier transformation we have

$$w(x_1,x_2,t) = \frac{1}{(2\pi)^3} \iiint_{-\infty}^{\infty} \frac{\exp\left(-d^2k^2/4\right)\bar{P}(k,\omega)}{F(k,\omega)} e^{i(k_1x_1+k_2x_2+\omega t)} dk_1 dk_2 dt. \tag{2.9}$$

Noting that $\bar{k}\cdot\bar{x} = kr\cos\theta$, where θ is an angle between $\bar{k}$ and $\bar{x}$, and for the given case element of "area" in $\bar{k}$ -space is written down in the form of $dk_1 dk_2 = (kd\theta)\, dk$. Then it follows from (2.9)

$$w(r,t) = \frac{1}{(2\pi)^3} \int_0^{2\pi}\int_0^{\infty}\int_{-\infty}^{\infty} \frac{\exp\left(-d^2k^2/4\right)\bar{P}(k,\omega)}{F(k,\omega)} e^{i(kr\cos\theta t+\omega t)} d\theta dk d\omega. \tag{2.10}$$

Taking into account the integral representation of the Bessel functions [6,9]

$$J_0(kr) = \frac{1}{2\pi}\int_0^{2\pi} e^{ikr\cos\theta} d\theta.$$

in axisymmetric case we'll have

$$w(r,t) = \frac{1}{(2\pi)^2} \int_0^{\infty}\int_{-\infty}^{\infty} \frac{\exp\left(-d^2k^2/4\right)\bar{P}(k,\omega)}{F(k,\omega)} kJ_0(kr)\, e^{i\omega t} dk d\omega. \tag{2.11}$$

Applying the convolution theorem to (2.11) we obtain [9]

$$w(r,t) = \int_0^{\infty}\int_{-\infty}^{\infty} f(r-\rho, t-\tau)\, P(\rho,t)\, d\rho d\tau, \tag{2.12}$$

where

$$f(r,t) = \frac{1}{(2\pi)^2} \int_0^{\infty}\int_{-\infty}^{\infty} \frac{\exp\left(-\frac{d^2}{4}k^2\right)}{F(k,\omega)} kJ_0(kr)\, e^{i\omega t} dk d\omega. \tag{2.13}$$

In (2.12) the Green function $f(r-\rho, t-\tau)$ as a matter of fact is a solution of the boundary-value problem with homogeneous boundary conditions for random instantaneous unit load $P(\xi_1,\xi_2,t) = \delta(\xi_1)\,\delta(\xi_2)\,\delta(t)$ and $f = 0$ at $t < \tau$, δ is a Dirac delta function [5]. For elastic plate $H(t) \equiv 0$ expressions (2.8),(2.11),(2.13) are significantly simplified and gain simpler form

$$w(r,t) = \frac{P_0 kth(kH)}{2\pi\left[\rho_1 hkth(kH) + \rho_2\right]} \int_0^{\infty} \frac{\exp\left(-d^2k^2/4\right)\sin\left[\omega(k)\, t\right] kJ_0(kr)}{\omega(k)} dk, \tag{2.14}$$

where

$$\omega(k)=\sqrt{\frac{Dk^4+\sigma k^2+\rho_2 g}{\rho_1 hkth\,(kH)+\rho_2}kth\,(kH)},\quad \bar{P}\,(k,\omega)=P_0.$$

In integrand (2.14) has no singular points on the real axis k. Therefore improper integral (2.14) exists and to integrate it by the numerical method is not difficult. From here it is obvious that the dispersion decreases flexure of plate and as $r\to\infty,\ \omega\,(r,t)\sim r^{-1/2}$.

3. Let the normal random load move in the line of ξ_1 with the constant velocity υ and $P\,(\xi-\upsilon t)=P_0\delta\,(\xi-\upsilon t)$, $\xi-\xi_1$, $P_0=const$, $H\,(t)\equiv 0$. Then mathematical expectation (1.1) of this function by Gauss distribution (1.2) is determined by the following formula

$$MP\,(\xi,x,t)=\frac{P_0}{d\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\exp\left[-\frac{(x-\xi)^2}{d^2}\right]\delta\,(\xi-\upsilon t)\,d\xi=$$
$$=\frac{P_0}{d\sqrt{\pi}}\exp\left[-\frac{(x-\upsilon t)^2}{d^2}\right].$$

In two-dimensional statement equations (1.1), (1.9) and boundary conditions (1.7) $\left(\varphi_z'\right)_{z=H}=0$ in moving coordinates $\eta=x-\upsilon t$ get the form

$$\frac{\partial^2\varphi}{\partial z^2}+\frac{\partial^2\varphi}{\partial\eta^2}=0,\quad \eta\in(-\infty,\infty) \tag{3.1}$$

$$D\frac{\partial^4 w}{\partial\eta^4}+\rho_1 h\upsilon^2\frac{\partial^2 w}{\partial\eta^2}+\rho_2 g\omega-\sigma\frac{\partial^2 w}{\partial\eta^2}-\rho_2\upsilon\left.\frac{\partial\varphi}{\partial\eta}\right|_{z=0}=\frac{P_0}{d\sqrt{\pi}}\exp\left(-\frac{\eta^2}{d^2}\right), \tag{3.2}$$

$$-\upsilon\left.\frac{\partial w}{\partial\eta}\right|_{z=0}=\left.\frac{\partial\varphi}{\partial z}\right|_{z=0},\quad \left.\frac{\partial\varphi}{\partial z}\right|_{z=-H}=0. \tag{3.3}$$

Problem (3.1)-(3.3) is solved with the help of Fourier integral transformation (2.1) $\bar{w}\,(k)=\int\limits_0^{2\pi}w\,(\eta)\,e^{-ik\eta}d\eta$. After application of the inverse transformation we obtain

$$w=\frac{P_0}{2\pi}\int\limits_{-\infty}^{\infty}\frac{\exp\left(-d^2k^2/4\right)e^{ik(x-\upsilon t)}}{F_1\,(k,\omega)}dk, \tag{3.4}$$

$$\varphi=-\frac{i\upsilon P_0}{2\pi}\int\limits_{-\infty}^{\infty}\exp\left(-\frac{d^2}{4}k^2\right)\frac{kch\,[|k|\,(z+H)]\,e^{ik(x-\upsilon t)}}{|k|\,F_1\,(k,\upsilon)\,sh\,(|k|\,H)}dk,$$

where

$$F_1\,(k,\upsilon)=Dk^4-\left(\rho_1 h\upsilon^2-\sigma\right)k^2+\rho_2 g\left(1-\frac{k\upsilon^2}{gth\,(kH)}\right). \tag{3.5}$$

If the random load is concentrated on the area $\eta \in [l/2, -l/2]$, $P \neq 0$; $\eta \notin [l/2, -l/2]$, $P \equiv 0$, then solution (3.4) is easily generalized $P = q(\xi - \upsilon t)$

$$w = \frac{1}{2\pi} \int\limits_{-l/2}^{l/2} \int\limits_{-\infty}^{\infty} \frac{\exp\left(-d^2k^2/4\right) q(\eta)\, e^{ik(x-\eta-\upsilon t)}}{F_1(k,\omega)} dk d\eta. \tag{3.6}$$

For the uniform distributed force $q(\eta) = q_0$ integrating expression (3.6) with respect to η and allowing for evenness of integrand functions with respect to k we obtain

$$w = \frac{2q}{\pi} \int\limits_{-\infty}^{\infty} \frac{\sin\frac{kl}{2} \cos k(x - \upsilon t) \exp\left(-d^2k^2/4\right)}{kF_1(k,\omega)} dk, \tag{3.7}$$

Usually in theory flexure-gravity-capillary waves two extreme cases differentiated by the relation H and length of wave are qualitatively investigated. At $kH << 1$ in (4.5) $thkH \approx kH$ and $kh >> 1$, $thkH \approx 1$. For fine water when depth of fluid is small [2,7]

$$F_1(k,\upsilon) = Dk^4 - \left(\rho_1 h\upsilon^2 - \sigma\right) k^2 + \rho_2 g \left(1 - \frac{\upsilon^2}{gH}\right). \tag{3.8}$$

Introduce the dimensionless quantities [2,3]

$$\begin{gathered} \alpha = (\rho_2 g/D)^{1/4}, \quad k = \alpha\xi, \quad m = \alpha(x - \upsilon t), \\ \upsilon = \xi (gH)^{1/2}, \quad \beta^2 = \rho_1 ghH/\sqrt{\rho_2 gD}, \quad \gamma = \sigma/\sqrt{\rho_2 gD}. \end{gathered} \tag{3.9}$$

Substituting (3.9) in (3.7) we rewrite this integral in the form of

$$w = \frac{2q_0}{\rho_2 g\pi} \int\limits_0^{\infty} \frac{\sin\frac{\alpha\xi l}{2} \cos\xi m \exp\left(-\frac{d^2}{4}\alpha^2\xi^2\right) d\xi}{\xi\left[\xi^4 - \left(\beta^2\rho^2 - \gamma^2\right)\xi^2 + 1 - \xi^2\right]}. \tag{3.10}$$

The flexure of the plate depends on the root of polynomial being in denominator of integrand expression (3.10). Equating the polynomial to zero

$$\xi^4 - \left(\beta^2\zeta^2 - \gamma^2\right)\xi^2 + 1 - \zeta^2 = 0, \tag{3.11}$$

we find its roots

$$\xi_i = \pm\sqrt{\frac{\beta^2\zeta^2 - \gamma^2}{2}} \left(1 \pm \sqrt{1 - \frac{4\left(1 - \zeta^2\right)}{\left(\beta^2\zeta^2 - \gamma^2\right)}}\right)^{1/2}. \tag{3.12}$$

In particular, at $\gamma = 0$ the analysis of impact of the roots of polynomial on flexure is explicitly studied [2,3] and in case of $\beta^2\zeta^2 - \gamma^2 \geq 0$ essence of analysis doesn't change.

If all roots of (3.12) are complex, then integral (3.10) converges. In this case the value of flexure depends on the form of complex number, i.e. at

$$\beta^2\zeta^2-\gamma^2<0,\ \Delta=\left(1\pm\sqrt{1-\frac{4\left(1-\zeta^2\right)}{\left(\beta^2\zeta^2-\gamma^2\right)^2}}\right)^{1/2}>0.$$

the dispersion intensifies amplitude of flexures due to multipliers $\exp\left(-d^2\alpha^2\xi_i^2/4\right)$.

At real roots (3.11) integral (3.10) diverges and at simple roots it exists only in the sense of principal value. As the velocity of movement of load approaches to the velocity of wave propagation $\left(\zeta\to 1,\ \ \upsilon\to\sqrt{gH}\right)$, amplitude of flexure at $\beta^2-\gamma^2>0$ increases and at fine water resonance arises. However, at $\beta^2-\gamma^2<0$, $\zeta\to 1$ integral (3.10) converges and flexural-gravity-capillary wave is propagated. Finally, integral (3.10) is investigated in the case when polynomial (3.11) has two real and two imaginary roots.

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