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MATHEMATICAL SIMULATION AND OPTIMAL DISTRIBUTION OF RESOURCES IN INDUSTRIAL UNITS

Abstract

A mathematical model of optimal distribution of resources between the sub-divisions of the object is studied. The problem is reduced to the integral ,mathematical programming.

The profitability is taken as a quality criterion. To avoid the integrality the method of penalty functions are used. By solving the obtained nonlinear problem a conditional gradient method is used. The obtained result is analyzed on an example of repair of oil wells.

Introduction. In the paper we consider a mathematical problem on optimal distribution of resources (spares, repair and etc.) between sub-divisions (workship, plant and etc.) of a unit (industrial unit, corporation, industrial network and etc.) in industry.

At each unit there may be first of all chosen such functions of control that are directly connected with realization of production process.

The units engaged in production are subject to economical tasks: to put into operation funds and industrial resources by general centralized investigations at the expense of optimal distribution [21]. The production profitability is taken as a quality criteria.

The solution algorithm is made on the base on conditional gradient and penalty function methods.

The results are illustrated by a concrete example arising by repairing wells when optimal distribution of resources is required.

Optimal distribution of resources is one of main conditions for cutting of production costs. On its account a unit may defray some expenses and get high profit.

The main goal of object of many economical problems is increase of production, growth of production profitability and at last ensuring financial stability of enterprise.

The solution of this problem first of all depends on finding optimal distribution of existing resources.

At present some units with their sub-divisions are at their late development stage [5, 7]. They are unprofitable and make worse the financial state. Therefore economic control methods must be introduced. For this propose we must solve problems on development of optimal plan of production manufacture and attainment of production profitability to acceptable level in enterprise at minimal necessary expenditures.

2. Problem statement. Let there be n sub-divisions and N type resources. If in the i-th sub-division we allot the j-th resource then we denote the planned allotted resource by C_{ij}. On the other hand assume that after the j-th resource at the i-th sub-division the volume of produced production is known.

Let the planned allotted resources for all sub-divisions equal b, and maximally allotted resource for each sub-division equals b_i. By S_i we denote necessary forms of resources of the i-th sub-division.

Thus, it is required to solve the following problem. Which type resources does each sub-division need at the expense of allotted resource in order to achieve maximal production profitability at the whole of unit?

If we denote by x_{ij} the allotted j-th resources at the i-th sub-division, then

$$Q_i = Q_i\left(x_{i1},\ x_{i2}, ..., x_{in}\right)\ .$$

Here Q_i is the amount of produced production from the i-th sub-division. By the formula of dependence of profitability from the amount of produced production we get

$$\rho_i = \rho_i\left(x_{i1}, ..., x_{in}\right)$$

Thus, we get the following mathematical problem:

$$\sum_{i=1}^{n} \rho_i\left(x_{i1}, x_{i2}, ..., x_{in}\right) \to \max\ , \tag{1}$$

$$\sum_{j=1}^{N} x_{ij} \geq \sum_{j\in S_i} c_{ij}\ , \quad i = \overline{1,n}, \tag{2}$$

$$\sum_{j=1}^{N} x_{ij} \leq b_i\ , \quad i = \overline{1,n}, \tag{3}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{N} x_{ij} \leq b\ . \tag{4}$$

3. Reduction of (1)-(4) to integral programming. Note that each resource is completely allotted, i.e. the resource infact is allotted or is not allotted.

Then, it is known that

$$x_{ij} = \begin{cases} c_{ij}, & \text{if the } j\text{-th resource is allotted at the } j\text{-th sub-division} \\ 0, & \text{otherwise}\ . \end{cases}$$

After denotation

$$z_{ij} = \begin{cases} 1, & \text{if the } j\text{-th resource is allotted at the } j\text{-th sub-division} \\ 0, & \text{otherwise}\ . \end{cases}$$

Between x_{ij} and z_{ij} we have the following relation

$$x_{ij} = c_{ij}Z_{ij}. \tag{5}$$

Allowing it in (1)-(4) we get:

$$\sum_{i=1}^{n} \rho_i \left(Z_{i1}, Z_{i2}, ..., Z_{iN}\right) \rightarrow \max \ , \tag{6}$$

$$\sum_{j=1}^{N} c_{ij} Z_{ij} \geq \sum_{j \in S_i} c_{ij} \ , \quad i = \overline{1, n}, \tag{7}$$

$$\sum_{j=1}^{N} c_{ij} Z_{ij} \leq b_i \ , \quad i = \overline{1, n}, \tag{8}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{N} c_{ij} Z_{ij} \leq b \ . \tag{9}$$

Here z_{ij} is an integral variable that takes the value 0 or 1. We get integral problem (6)-(9).

Note the another specific character of this problem. As it was said above, the formula of the dependence of ρ_i-profitability on the amount of produced production Q_i, is known [3, 4, 7, 14].

But on the other hand, the dependence of the form $Q_i = Q_i \left(x_{ij}, ..., x_{iN}\right)$ or $Q_i = \tilde{Q}_i \left(Z_{i1}, ..., Z_{iN}\right)$ function is given in the form of tables.

Then we get a problem on maximization (6)-(9).

The function given in the form of tables complicates the solution of problem (6)-(9). This is connected with non-smoothness and uncertainty of the function at the points with non-integral coordinates.

Solution algorithm of problem (6)-(9). If we assume that several forms of resources correspond to each sub-division we can define this function as linear.

If the amount of the production Q_{ij} at the i-th sub-division is known after the allotted j-th form of the resource, then the amount of the production Q_{ij} may be calculated as follows:

$$Q_i = Q_i^0 + Q_{i1} \cdot Z_{i1} + Q_{i2} \cdot Z_{i2} + ... + Q_{iN} \cdot Z_{iN} = Q_i^0 + \sum_{j=1}^{N} Q_{ij} \cdot Z_{ij}. \tag{10}$$

Here Q_i^0 is the amount of the produced product before the resource is allotted.

Then it is known that we can calculate the profitability as follows:

$$\rho_i = \rho_i^0 + \rho_{i1} Z_{i1} + \rho_{i2} Z_{i2} + ... + \rho_{iN} Z_{iN} = \rho_i^0 + \sum_{j=1}^{N} \rho_{ij} Z_{ij}. \tag{11}$$

In this case with conditions (7)-(9) we get the maximization of the function

$$J\left(Z\right) = \sum_{i=1}^{n} \sum_{j=1}^{N} \rho_{ij} \cdot Z_{ij}. \tag{12}$$

And now let's consider the solution of the stated problem.

At first, ignoring the integral property we maximize function (12) with conditions (7)-(9) and $0 \leq Z_{ij} \leq 1,\ i = \overline{1,n},\ \ j = \overline{1,N}$. Assume the solution of this problem

$$Z^* = \left(Z_{11}^*, ..., Z_{1N}^*,\ Z_{21}^*, ..., Z_{2N}^*, ..., z_{nN}^*\right)\ .$$

And then ignoring the integral property we maximize the function

$$I(Z) = \sum_{i=1}^{n}\sum_{j=1}^{N} \rho_{ij} Z_{ij} - C \prod_{i=1}^{n}\prod_{j=1}^{N} Z_{ij}\left(1 - Z_{ij}\right) \tag{13}$$

with the cited conditions. Here C is sufficiently large positive constant. Here we are to take $Z = Z^*$ as an initial point. So, having taken $Z = Z^*$ as an initial point we must maximize the linear function (13) with limitations (7)-(9) and $0 \leq Z_{ij} \leq 1,\ i = \overline{1,m},\ j = \overline{1,M}$.

4. Computing algorithm. Approximate method is used to solve this problem. To solve this problem with the help of conditional gradient is more effective. Because a linear function is maximized with linear limitations at each step of this method. In other words, we are to solve the linear programming problem at each iteration step. First we speak briefly about this method.

1. Having taken the initial point

$$Z_0 = Z^* = \left(Z_{11}^*, ..., Z_{1N}^*, Z_{21}^*, ..., Z_{2N}^*, .., Z_{nN}^*\right)$$

Maximizing the linear function

$$I(Z) = \sum_{i=1}^{n}\sum_{j=1}^{N} R_{ij} Z_{ij} \tag{14}$$

with conditions (7)-(9) and $0 \leq z_{ij} \leq 1,\ i = \overline{1,n},\ j = \overline{1,N}$ we find the subsidiary solutions

$$\bar{Z}_0 = \left(Z_{11}^0, Z_{12}^0 ..., Z_{1N}^0, Z_{21}^0, ..., Z_{2N}^0, .., Z_{nN}^0\right)\ .$$

Here

$$R_{km} = \frac{\partial I\left(Z_0\right)}{\partial Z_{km}} = \rho_{km} + C\left(1 - 2Z_{km}\right)\prod_{i=1}^{n}\prod_{j=1}^{N} Z^0\left(1 - Z_{ij}^0\right),\quad i \neq k,\quad j \neq m. \tag{15}$$

2. The next point.

$Z_1 = (1 - \alpha) Z_0, +\alpha \bar{Z}_0$ is found provided $0 \leq \alpha \leq 1$, α is selected by different methods [5, 9, 10, 11, 12, 18]

a) the number α may be found as a maximal point of the function $f(\alpha) = I\left(\alpha Z_0 + (1 - \alpha)\bar{Z}_0\right)$ on the interval $[0, 1]$

b) at $\alpha = 1$ the condition $I(Z_1) \geq I(Z_0)$ is verified. If this condition is not satisfied, we can bisect α, i.e. having taken $\alpha = \frac{1}{2}$ the monotonicity condition is verified. This process continues till the monotonicity condition is satisfied. As is shown in the paper [16, 17, 18] for some least estimation the indicated condition must be satisfied.

c) We can give at the first step $\alpha_1 = \frac{1}{2}$, at the next steps $\alpha_2 = \frac{1}{3}$, $\alpha_3 = \frac{1}{4}$ or as $\alpha_k = \frac{1}{k+1} Z_1$.

3. After finding the point Z_1 we are to verify some accuracy conditions. The accuracy conditions may be given as:

a) $\|Z_1 - Z_0\| \leq \varepsilon$

b) $|I(Z_1) - I(Z_0)| \leq \varepsilon$

d) as combination of items a) and b)

$$\|Z_1 - Z_0\| + |I(Z_1) - I(Z_0)| < \varepsilon .$$

Here $\varepsilon > 0$ is the given number. The process is assumed to be complete if the accuracy condition is satisfied. And with the found point

$$Z_1 = \left(Z_{11}^1, Z_{12}^1 ..., Z_{1N}^1, Z_{21}^1, ..., Z_{nN}^1\right)$$

we shall solve problem (13), (7)-(9) ignoring the integral property condition.

It $C > 0$, then it suffices to have a large number, then each coordinate of the found one will be close to 0 or 1 [20]. Considering this, rounding off all coordinates to integral estimation we get the approximate solution of the problem (6)-(10).

When the accuracy condition is not fulfilled, taking $Z = Z_1$ as the first step we come back and the process continues till the accuracy condition is fulfilled.

5. Example. Recourse distribution of wells in oilgasrecovery.

Assume that in oil field there are n wells where N forms of repair is possible. If the j-th repair is performed at the i-th well, we'll denote by c_{ij} the allotted resource. On the other hand, assume that after the j-th repair at the i-th well the volume of oil recovery is known.

Simultaneously, adopt that oil recovery corresponds to several types of repair.

Let the allotted recourse for repairing in all the wells equal b, and maximal resource for each well equals b_i. By S_i we denote necessary forms of repairs at the i-th well. Thus, we are to solve the following problem

$$Q_i = Q_i^0 + Q_{i1} \cdot Z_{i1} + Q_{i2} \cdot Z_{i2} + ... + Q_{iN} \cdot Z_{iN} = Q_i^0 + \sum_{j=1}^{N} Q_{ij} \cdot Z_{ij}$$

Q_i is the amount of oil recovered from the i-th well. By the formula of dependence of profitability on the amount of recovered oil we'll get [1, 2, 6, 15].

Here Q_i^0 is the amount of oil recovery before repair. If non repair will be performed at the i-th well, then $z_{ij} = 0,\ j = \overline{1, N}$ and the value $Q_i = Q_i^0$.

By the formula on the dependence of profitability on the amount of recovered oil

$$\rho_i = \frac{(G - C_i - N)\, Q_i}{H_i} \qquad (16)$$

Here G is the price of 1t oil.
C_i is the oil production cost at the i-th well
N is transport expenses
H_i is exploitational expenses in wells

$$p_i^0 = \frac{(G - C_1 - N)\, Q_1^0}{H_i}$$

$$p_{ij} = \frac{(G - C_i - N)\, Q_{ij}}{H_i}$$

Case 1.

c_{11}	c_{12}	c_{13}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}
15	18	23	10	15	20	10	14	18

ρ_{11}	ρ_{12}	ρ_{13}	ρ_{21}	ρ_{22}	ρ_{23}	ρ_{31}	ρ_{32}	ρ_{33}
30	40	36	20	36	42	21	30	35

The following result is obtained with these data

z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}
0.8	1	1	0.15	1	1	1	1	0

As is seen, for a well where expenditure is greater than profitability more than two times, the value $z - a$ will be 1, and in other cases it will be 0.

Case 2.

c_{11}	c_{12}	c_{13}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}
15	18	23	10	15	20	10	14	18

ρ_{11}	ρ_{12}	ρ_{13}	ρ_{21}	ρ_{22}	ρ_{23}	ρ_{31}	ρ_{32}	ρ_{33}
30	40	36	20	36	42	21	30	35

In this example all data are as in example 2, but only in this case the value for ρ_{13} is taken not 48, but 36. Then, the results will be as follows

z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}
1	1	0	0.15	1	1	1	1	0

As is seen the decrease of profitability of the value z_{13} falls from 1 to 0.

Iterations	3	4	5	6
Values of the function	304, 0637	304, 0620	304, 0617	304, 0617
difference of values by iteration	0, 0312	0.0016	0, 0031	0, 0001

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