Makhmud A. AKHMEDOV

ON COMPACTNESS OF IMBEDDING FOR ONE CLASS OF WEIGHT ANISOTROPIC PSEUDO-NORMALIZED SPACES

Abstract

The problem of compactness of imbedding for one class of weight nonlinear anisotropic spaces is considered. The theorem on compactness of imbedding of one class of weight pn -spaces in Lebesque weight space is proved.

The paper is devoted to the investigation of the problem of compactness of imbedding for one class of weight nonlinear anisotropic spaces. As a fact the paper is logic continuation of researches carried out by the author in [1,2], devoted to the theorems of imbedding of weight nonlinear pseudo-normalized spaces in Lebesque weight spaces. Many works have been devoted to the investigation of nonlinear pn-spaces. In weight case they were studied in [3,4]. The application of received results to the nonlinear differential equations is also possible to find in [5]. In weight case, as it was noted, the problems of imbedding for pn-spaces are considered by the author in [1,2].

Applying the results obtained in [2], and also using the methods of the paper [6] in the present paper the theorem on compactness of imbedding of one class of weight pn-spaces in Lebesque weight space is proved.

The basic concepts and notation used in the paper are cited in section 1. The second section contains some auxiliary results and the main theorem on compactness.

1. Preliminary data

Let $\mathbf{R}^n$ $(n \geq 1)$ be $n-$dimensional Euclidean space, Ω be a bounded set in $\mathbf{R}^n$.

The arbitrary measurable function ν on Ω such that $0 < \nu(x) < +\infty$ almost everywhere on Ω is said to be weight, and the set of all weights on Ω we'll denote by $W(\Omega)$.

For $\nu \in W(\Omega)$ we denote by $\nu(E) = |E|_\nu = \int\limits_E \nu(x)\,dx$ the ν weight measure of the measurable set $E \subset \Omega$. For $\nu(x) = 1$ we denote by $|E|$ the Lebesque measure of the set E.

Let $\nu \in W(\Omega)$, $1 \leq p \leq +\infty$. We determine the weight space $L_p(\Omega;\nu)$ as a set of all measurable on Ω functions $u(x)$ for which the norm

$$|u : L_p(\Omega;\nu)| = \begin{cases} \left(\int\limits_\Omega |u(x)|^p \nu(x)\,dx\right)^{1/p} & \text{at} \quad 1 \leq p < +\infty \\ esse\sup\limits_\Omega |u(x)|\,\nu(x) & \text{at} \quad p = +\infty \end{cases}$$

is finite.

Let $0 \leq p_i < +\infty$, $1 \leq q_i < +\infty$, $\nu_i \in W(\Omega)$, $i = \overline{0,n}$. Let's introduce the notation for the function $u \in C^1(\Omega)$

$$\left|u : S^1_{\bar{p},\bar{q}}(\Omega;\bar{\nu})\right| = |u : L_{p_0+q_0}(\Omega;\nu_0)| +$$

$$+\sum_{i=1}^{n}\left(\int_{\Omega}\nu_i(x)\,|u(x)|^{p_i}\left|\frac{\partial u(x)}{\partial x_i}\right|^{q_i}dx\right)^{1/(p_i+q_i)} \tag{1.1}$$

where $\bar{p}=(p_0,...,p_n)$, $\bar{q}=(q_0,...,q_n)$, $\bar{\nu}(x)=(\nu_0,...,\nu_n(x))$.

By $S^1_{\bar{p},\bar{q}}(\Omega;\bar{\nu})$ we denote the closure of the set of all $u\in C^1(\Omega)$ for which expression (1.1) is finite.

Expression (1.1) determines the pseudo-norm (pn) [3,4]. The properties of pn-spaces in unweighted case are sufficiently studied in [3-5] in which both questions of imbedding and applications to differential equations were considered.

Cite some notation and auxiliary assertions.

Consider the integral operators

$$Ku(x)=\int_{\Omega}k(x,y)\,u(y)\,dy,\qquad K^*u(x)=\int_{\Omega}k^*(x,y)\,u(y)\,dy,\quad x\in\Omega$$

where $k:\Omega\times\Omega\to\mathbf{R}$ is a non-negative measurable function $k^*(x,y)=k(y,x)$.

Let $\omega,\nu\in W(\Omega)$. For the operator K assume

$$[K]_{p\nu^{1-p'},q\omega}=$$

$$=\sup\left\{\left|K\nu^{1-p'}\chi_Q:L_q(\Omega;\omega)\right|\cdot\left|\chi_Q:L_p\left(\Omega;\nu^{1-p'}\right)\right|^{-1}:dyadic Q\subset\Omega\right\}. \tag{1.2}$$

For the kernel k of the operator K introduce the value

$$[k]_{p\nu^{1-p'},q\omega}=\sup_{x\in\Omega}\sup_{r>0}\left\{|\Omega\cap B(x,r)|_{\omega}^{1/q}\cdot\left|\chi_{\Omega\backslash B(x,r)}k(x,\cdot):L_p\left(\Omega;\nu^{1-p'}\right)\right|\right\}. \tag{1.3}$$

Denote by $k_{h,a}$ the kernel k of the operator K if it has the form

$$k_{h,a}(x,y)=\chi_{[h,a]}(|x-y|\,k(x,y)),\qquad 0\le h\le a<+\infty$$

Denote by $K_{h,a}\left(K^*_{h,a}\right)$ the operator K with the kernel $k_{h,a}\left(k^*_{h,a}\right)$.

Further in the paper we'll consider the operator K with the kernel

$$k(x,y)=\frac{1}{|x-y|^{n-1}}$$

The noted notation (1.2) and (1.3) are taken from the paper [7].

We need also the following imbedding theorem from the paper [2].

Theorem 1.1 *Let $\Omega\subset\mathbf{R}^n$ be an arbitrary bounded open set with non-empty interior $\alpha\ge 0$, $1\le p_0\le p<+\infty$, $1<p_i\le p<+\infty$, $\omega,\nu_0,\nu_i\in W(\Omega)$ and the following conditions be fulfilled*

1) $\tilde{C}_0(\Omega)=\sup\limits_{\substack{B(y,r)\\ y\in\Omega,\ r=\frac{g(y)}{12\sqrt{n}}}}C_0(B(y,r))<+\infty$

$$C_0(B(y,r))=\frac{1}{|B(y,r)|}\left(\int_{B(y,r)}\omega(z)\,dz\right)^{1/p}\cdot\left(\int_{B(y,r)}\nu_0^{1-p_0'}(z)\,dz\right)^{1/p_0'}$$

2) $\tilde{C}_i(\Omega) = \sup\limits_{\substack{B(y,r)\\ y\in\Omega,\ r=\frac{g(y)}{12\sqrt{n}}}} C_i(B(y,r)) < +\infty$

3) $\tilde{C}_i^*(\Omega) = \sup\limits_{\substack{B(y,r)\\ y\in\Omega,\ r=\frac{g(y)}{12\sqrt{n}}}} C_i^*(B(y,r)) < +\infty$

$$\left.\begin{aligned} C_i(B(y,r)) &= \left[K_{0,6\sqrt{n}r}\right]_{p_i\nu_i^{1-p_i'},p\omega} \\ C_i^*(B(y,r)) &= \left[K^*_{0,6\sqrt{n}r}\right]_{p'\omega p_i'\nu_i^{1-p_i'}} \end{aligned}\right\} \quad \text{at} \quad 1 < p_i \le p < +\infty$$

or

$$\left.\begin{aligned} C_i(B(y,r)) &= \left[k_{0,6\sqrt{n}r}\right]_{p_i\nu_i^{1-p_i'},p\omega} \\ C_i^*(B(y,r)) &= \left[k^*_{0,6\sqrt{n}r}\right]_{p'\omega p_i'\nu_i^{1-p_i'}} \end{aligned}\right\} \quad \text{at} \quad 1 < p_i < p < +\infty$$

$i = \overline{1,n}$.

Then the imbedding

$$S^1_{\alpha\bar{p},\bar{p}}(\Omega;\bar{\nu}) \subset L_{(\alpha+1)p}(\Omega;\omega)$$

holds.

For $\varepsilon > 0$ we assume that $\Omega_\varepsilon = \{y : dist(y,\Omega) < \varepsilon\}$. It is clear that if $x \in \Omega$ then the open ball $B(x,\varepsilon) \subset \Omega_\varepsilon$. Let $u \in L_1^{loc}(\Omega_\varepsilon)$. Determine the function

$$u_h(x) = \frac{1}{|B(x,h)|}\int\limits_{B(x,h)} u(y)\,dy,\ \ x \in \bar{\Omega},\ \ 0 < h < \varepsilon.$$

The following theorem holds.

Theorem 1.2 [6]. *Let $1 \le p < +\infty$ and $\omega \in W(\Omega) \cap L_1(\Omega)$. Then if the set $E \subset L_p(\Omega;\omega)$ satisfies the following conditions*

1) $E \subset L_1^{loc}(\Omega_\varepsilon)$ for some $\varepsilon > 0$;

2) E is uniformly bounded in $L_p(\Omega;\omega)$;

3) $E_h = \{u_h : u \in E\}$ is uniformly bounded on $\bar{\Omega}$ at each $h :\ 0 < h < \varepsilon$

4) the functions of the set E_h are uniformly continuous on $\bar{\Omega}$;

5) $u_h \xrightarrow[h\to 0]{L_p(\Omega;\omega)} u$, then it is relatively compact in $L_p(\Omega;\omega)$.

2. Theorem on compactness.

The main result of the paper is the theorem on compactness of imbedding of weight pseudo-normalized space in Lebesque weight space. For proving of this result we need some auxiliary assertions which we'll show below:

Theorem 2.1 *Let $\Omega \subset \mathbf{R}^n$ be bounded measurable set and the counted system $\{G^m\}_{m=1}^\infty$ of measurable sets such that $G^m \subset G^{m+1} \subseteq \Omega$ and $\Omega = \bigcup\limits_{m=1}^{\infty} G^m$. Then the conditions:*

1. For each $m \in N$ it holds the compact imbedding

$$S^1_{\bar{p},\bar{q}}(\Omega;\bar{\nu}) \subset\subset L_r(G^m;\omega)\,; \tag{2.1}$$

holds

$$2. \lim_{m\to+\infty}\left(\sup_{|u:S_{\bar{p},\bar{q}}(\Omega;\bar{\nu})|\leq 1}|u:L_r(\Omega\backslash G^m;\omega)|\right)=0 \tag{2.2}$$

are necessary and sufficient for holding of the compact imbedding

$$S^1_{\bar{p},\bar{q}}(\Omega;\bar{\nu})\subset\subset L_r(\Omega;\omega) \tag{2.3}$$

The noted theorem is proved by the same way as analogously to the same result in case of Lebesque weight spaces ([6]).

Lemma 2.1 *Let* $1\leq p<+\infty,\ 0<h<+\infty$ *and* $\omega\in W(\Omega)$. *Then for any* $u\in C^0(\Omega_h)$ *it holds the inequality*

$$|u-u_h:L_p(\Omega;\omega)|\leq C\sum_{i=1}^n\left|\int\limits_{B(x,h)}\frac{|D_iu(y)|}{|x-y|^{n-1}}dy:L_p(\Omega;\omega)\right|. \tag{2.4}$$

The proof of the assertion follows from the integral representation for continuously differentiable functions [8] lemma 1, p.436.

Lemma 2.2 *Let* $h>0,\ \alpha\geq 0$ *be some numbers,* $\omega,\ \nu_i\in W(\Omega),\ \ i=\overline{1,n}$. *Then for any* $u\in C^1(\Omega_h)$ *the estimation*

$$|(|u|^\alpha u-(|u|^\alpha u)_h):L_p(\Omega;\omega)|\leq C\sum_{i=1}^n||u|^\alpha D_iu:L_{p_i}(\Omega;\nu_i)|\cdot\max_{i=\overline{1,n}}$$

$$\begin{array}{lll}\left[K_{0,3\sqrt{n}h}\right]_{p_i\nu_i^{1-p'},p\omega}+\left[K^*_{0,3\sqrt{n}h}\right]_{p'\omega,p'_i\nu_i^{1-p'_i}} & \text{at} & 1<p_i\leq p<+\infty\\ \left[k_{0,3\sqrt{n}h}\right]_{p_i\nu_i^{1-p'},p\omega}+\left[k_{0,3\sqrt{n}h}\right]_{p'\omega,p'_i\nu_i^{1-p'_i}} & \text{at} & 1<p_i\leq p<+\infty\end{array} \tag{2.5}$$

hold, where the relations connecting the weight functions are determined in section 1.

The assertion of the lemma follows from lemma 2.1 and the theorem on boundedness of integrals of potential type in weight spaces [7].

Lemma 2.3 *Let* $\varepsilon>0,\ \alpha\geq 0$ *be some numbers* $\omega,\ \nu_0,\ \nu_i\in W(\Omega_\varepsilon)$ *and the conditions*

at $1<p_i\leq p<+\infty$

$$\lim_{h\to 0}\left\{[K_{0,h}]_{p_i\nu_i^{1-p'_i},p\omega}+\left[K^*_{0,h}\right]_{p'\omega,p'_i\nu_i^{1-p'_i}}\right\}=0 \tag{2.6}$$

at $1<p_i<p<+\infty$

$$\lim_{h\to 0}\left\{[k_{0,h}]_{p_i\nu_i^{1-p'_i},p\omega}+[k_{0,h}]_{p'\omega,p'_i\nu_i^{1-p'_i}}\right\}=0 \tag{2.7}$$

where $i=\overline{1,n}$ *be fulfilled.*

Then for any $u\in S^1_{\alpha\bar{p},\bar{p}}(\Omega_\varepsilon;\bar{\nu})\cap C^1(\Omega_\varepsilon)$ the relation

$$\lim_{h\to 0}|(|u|^\alpha u-(|u|^\alpha u)_h):L_p(\Omega;\omega)|=0. \tag{2.8}$$

holds. The assertion of the lemma follows from inequality (2.5).

Lemma 2.4 *Let the conditions of theorem 1.1 be fulfilled. Then for any* $\varepsilon > 0$ and $u \in S^1_{\alpha\bar{p},\bar{p}}(\Omega;\bar{\nu})$ *there exists* $u^{(\varepsilon)} \in C^1(\Omega)$ *such that*

$$\left|\left(|u|^\alpha u - \left|u^{(\varepsilon)}\right|^\alpha u^{(\varepsilon)}\right) : L_p(\Omega;\omega)\right| < \frac{\varepsilon}{2} \qquad (2.9)$$

The proof of the assertion is based on continuity of imbedding (1.1) and determination of the weight pn-space.

Theorem 2.2 (local compactness). *Let* $\Omega \subset \mathbf{R}^n$ *be bounded open set, and* $G \subset \Omega$ *such that for some sufficiently small* $\varepsilon > 0,\ G_\varepsilon \subset \Omega,\ \omega,\ \nu_i \in W(\Omega)$, $\alpha \geq 0,\ 1 > p_i \leq p < +\infty$ *and*

1) $\omega, \nu_0^{1-p_0\xi'} \in L_1^{loc}(\Omega)$;

2) the conditions of theorem 1.1 are fulfilled;

3) relations (2.6) and (2.7) are fulfilled for the set G_ε.

Then the compact imbedding:

$$S^1_{\alpha\bar{p},\bar{p}}(\Omega;\bar{\nu}) \subset\subset L_{(\alpha+1)p}(G;\omega).$$

holds.

Proof. By virtue of Hausdorff theorem ([8], theorem 3, p.46) it is sufficient to prove that for any $\eta > 0$ and bounded set from $S^1_{\alpha\bar{p},\bar{p}}(\Omega;\bar{\nu})$ there exists the finite η -net in $L_{(\alpha+1)p}(G;\omega)$.

First of all note that the continuity of the mapping $g(t) = |t|^\alpha t :\ L_{(\alpha+1)p}(G;\omega) \to L_p(G;\omega)$ yields the following: if there exists the finite η -net for the image of the mapping $g(E)$ in $L_p(G;\omega)$, then there also exists the finite η -net for the set E in $L_{(\alpha+1)p}(G;\omega)$. Therefore it is sufficient to show the existence of finite η -net for the set

$$\{|u|^\alpha u \in L_p(G;\omega) :\ u \in L_p(G;\omega)\} \qquad (*)$$

Let's assume

$$E = \left\{u : \left|u : S^1_{\alpha\bar{p},\bar{p}}(\Omega;\bar{\nu})\right| \leq M < +\infty\right\}.$$

Let's take the sufficient small $\eta > 0$ and allowing for lemma 2.4 for each $u \in E$ we choose $u^{(\eta)} \in C^1(\Omega)$ such that inequality (2.9) be fulfilled. The set of such functions $u^{(\eta)}$ we denote by $E^{(\eta)}$.

Assume that the set $E^{(\eta,\alpha)} = \left\{|u|^\alpha u : u \in E^{(h)}\right\}$ is relatively compact in $L_p(G;\omega)$. For this we construct the fulfilment of the conditions of theorem 1.2 in conditions of theorem 2.2.

Take the sufficiently small $\eta > 0$ and assume $E_h^{(\eta,\alpha)} = \left\{u_h : u \in E^{(\eta,\alpha)}\right\}$. Since $E^{(\eta)} \in C^1(\Omega)$ then it is clear that $E^{(\eta,\alpha)} \subset L_q(G_\varepsilon;\varepsilon)$ (condition 1 of theorem 1.2).

The uniform boundedness $E^{(\eta,\alpha)}$ in $L_p(G;\omega)$ (condition 2 theorem 1.2) follows from the following inequalities

$$\left|\left|u^{(h)}\right|^\alpha u^{(h)} : L_p(G:\omega)\right| \leq \left|\left(\left|u^{(\eta)}\right|^\alpha u^{(\eta)} - |u|^\alpha u\right) : L_p(G;\omega)\right| +$$

$$+\left|\left|u^\alpha\right| u : L_p(G:\omega)\right| \leq \frac{\varepsilon}{2} + \tilde{C}(\Omega) M < +\infty$$

where $\tilde{C}(\Omega)$ is a constant determined in the imbedding theorem:

$$\tilde{C}(\Omega) = \max\left\{\tilde{C}_0(\Omega),\ \tilde{C}_i(\Omega), \tilde{C}_i(\Omega),\ \ i = \overline{1,n}\right\}, \qquad (**)$$

Let now $u_h \in E_h^{(\eta,\alpha)}$ be an arbitrary function. Then applying Holder inequality we'll obtain

$$|(|u|^{\alpha} u)_h (x)| \leq \frac{C}{h^n} \int\limits_{B(x,h)} |u(y)|^{\alpha+1} dy \leq$$

$$\leq \frac{C}{h^n} \left| u^{\alpha+1} : L_{p_0}(\Omega;\nu_0) \right| \leq \left(\int\limits_{B(x,h)} \nu_0^{1-p_0'}(y)\, dy \right)^{1/p_0'} \leq$$

$$\leq \frac{CM}{h^n} \left(\int\limits_{B(x,h)} \nu_0^{1-p_0'}(y)\, dy \right)^{1/p_0'} < +\infty \tag{2.10}$$

Since the right hand side of inequality (2.10) doesn't depend on $(|u|^{\alpha} u)_h$, and $|u|^{\alpha} u \in E_h^{(\eta)}$ is an arbitrary function then hence it follows the uniform boundedness $E_h^{(\nu,\alpha)}$ on $\overline{G}$ (condition 3 of theorem 1.2).

Take an arbitrary function $u \in E_h^{(\nu,\alpha)}$ and any two points $x_1, x_2 \in \overline{G}$. Applying Holder inequality we'll obtain

$$|u_h(x_1) - u_h(x_2)| \leq \frac{MC}{h^n} \left(\int\limits_{B(x_1,h)\Delta B(x_2,h)} \nu_0^{1-p_0'}(y)\, dy \right)^{1/p_0'}$$

The right hand side of this inequality doesn't depend on u and it is finite. Moreover, allowing for condition 1) and the absolute continuity of Lebesque integral for any beforehand given $\lambda > 0$ we can choose $\delta > 0$ such that as soon as $|x_1 - x_2| < \lambda$ the relation

$$|u_h(x_1) - u_h(x_2)| < \lambda$$

is fulfilled.

The last inequality by virtue of arbitrariness of $u \in E_h^{(\eta,\alpha)}$ and $x_1, x_2 \in \overline{G}$ gives equicontinuity functions from $E_h^{(\eta,\alpha)}$ on $\overline{G}$ (condition 4 of theorem 1.2).

Allowing for condition 3 and applying lemma 2.2 and 2.3 it is easy to see that

$$\lim_{h\to 0} |(|u|^{\alpha} u - (|u|^{\alpha} u)_h) : L_p(G;\omega)| = 0$$

uniformly on $E^{(\eta,\alpha)}$ (condition 5 of theorem 1.2).

Thus all conditions of theorem 1.2 are fulfilled and so $E^{(\eta,\alpha)}$ is relatively compact in $L_p(G;\omega)$. Consequently, for $E^{(\eta,\alpha)}$ in $L_p(G;\omega)$ there exists the finite $\frac{\eta}{2}-$ net. Then from inequality (2.9) it follows the existence of finite η -net for the set $(*)$ in $L_p(G;\omega)$. The theorem is proved.

Now we introduce the basic assertion on compactness of imbedding of weight spaces.

Theorem 2.3 *Let* $\Omega \subset \mathbf{R}^n,\ \omega, \nu_i \in W(\Omega),\ 1 < p_i \leq p < +\infty,\ i = \overline{0,n},\ \alpha \geq 0$ *and the following conditions be fulfilled:*

1) Ω *is a bounded and* $\Omega = \underset{m\geq 1}{U} G^m$ *where* $G^m \subset G^{+1} \subsetneqq \Omega$ *are open and for each* $m \in N$ *will be find the number* $\varepsilon_m > 0$ *such that* $G^m_{\varepsilon_m} \subset \Omega$*;*

2) $\omega, \nu_0^{1-p_0'} \in L_1^{loc}(\Omega)$;

3) the conditions of theorem 1.2 are fulfilled;

4) $\lim\limits_{m\to+\infty} \tilde{C}(\Omega\backslash G^m) = 0$ *where* $\tilde{C}(\Omega)$ *is determined in (**);*

5) for $G^m_{\varepsilon_m}$, $m \in N$ *one of relations (2.6) or (2.7) is fulfilled.*

Then the compact imbedding

$$S^1_{\alpha\bar{p},\bar{p}}(\Omega;\bar{\nu}) \subset\subset L_{(\alpha+1)p}(\Omega;\omega) \tag{2.11}$$

holds.

Proof. It is sufficient to show that in conditions of this theorem the assertion of theorem 2.1 is true.

Allowing for condition 1)-3) and 5) by theorem 2.2 we'll obtain the compact imbedding

$$S^1_{\alpha\bar{p}_i,\bar{p}}(\Omega;\bar{0}) \subset\subset L_{(\alpha+1)p}(G^m;\omega)$$

for all $m \in N$. Further, applying theorem 1.1 and taking into account condition 4) we'll obtain the fulfilment of relation (2.2).

Thus all conditions of theorem 2.1 are fulfilled and consequently the compact imbedding (2.11) holds. The theorem is proved.

Condition of theorem 2.2 are rather difficult from the point of view of application. Therefore cite the assertion which a little bit more weaker than the previous one, but with more visible conditions.

Theorem 2.4 *Let* $\Omega \subset \mathbf{R}^n$, $\omega, \nu_0, \nu_i \in W(\Omega)$, $1 < p_0 \le p < +\infty$, $1 < p_i < p < +\infty$, $i = \overline{1,n}$, $\alpha \ge 0$ *and the following conditions be fulfilled:*

1) Ω *is open,* $\Omega = \bigcup\limits_{m\ge1} G^m$ *where* $G^m \subset G^{m+1} \subsetneqq \Omega$ *are open and for each* $m \in N$ *it will be found the number* $\varepsilon_m > 0$ *such that* $G^m_{\varepsilon_m} \subset \Omega$*;*

2) $\omega, \nu_0^{1-p_0'} \in L_1^{loc}(\Omega)$;

3) $\tilde{C}(\Omega) = \max\left\{\tilde{C}_0(\Omega),\ \tilde{C}_i(\Omega),\ \tilde{C}_i^*(\Omega),\ i = \overline{1,n}\right\} < +\infty$ *where*

$$\tilde{C}_0(\Omega) = \sup_{\substack{B(y,r)\\ y\in\Omega,\ r=\frac{g(y)}{12\sqrt{n}}}} \frac{1}{|B(y,r)|}\left(\int\limits_{B(y,r)} \omega(x)\,dx\right)^{1/p}\left(\int\limits_{B(y,r)} \nu_0^{1-p_0'}(x)\,dx\right)^{1/p_0'}$$

$$\tilde{C}_i(\Omega) = \sup_{\substack{B(y,r)\\ y\in\Omega,\ r=\frac{g(y)}{12\sqrt{n}}}}\ \sup_{x\in B(y,r)}\ \sup_{0<h\le2r} |B(x,h)|_\omega^{1/p} \times$$

$$\times\left(\int\limits_{B(x,2r)\backslash B(x,h)} |x-z|^{(1-n)p_i'}\,\nu_i^{1-p_i'}(z)\,dz\right)^{1/p_i'}$$

$$\tilde{C}_i^*(\Omega) = \sup_{\substack{B(y,r)\\ y\in\Omega,\ r=\frac{g(y)}{12\sqrt{n}}}}\ \sup_{x\in B(y,r)}\ \sup_{0<h\le2r} |B(x,h)|_{\nu_i^{1-p_i'}}^{1/p_i'} \times$$

$$\times\left(\int\limits_{B(x,2r)\backslash B(x,h)} |x-z|^{(1-n)p}\,\omega(z)\,dz\right)^{1/p}$$

4) $\lim\limits_{m\to+\infty}\tilde{C}\left(\Omega\backslash G^m\right)=0;$

5) $$\lim_{r\to 0}\sup_{x\in G^m_{\varepsilon_m}}\sup_{0<h\le r}\left|B\left(x,h\right)\right|^{1/p}_{\omega}\left(\int\limits_{B(x,r)\backslash B(x,h)}\left|x-y\right|^{(1-n)p'_i}\nu_i^{1-p'_i}\left(y\right)dy\right)^{1/p'_i}=0$$

$$\lim_{r\to 0}\sup_{x\in G^m_{\varepsilon_m}}\sup_{0<h\le r}\left|B\left(x,h\right)\right|^{1/p'_i}_{\nu_i^{1-p'_i}}\left(\int\limits_{B(x,r)\backslash B(x,h)}\left|x-y\right|^{(1-n)p}\omega\left(y\right)dy\right)^{1/p}=0$$

$i=\overline{1,n}$.

Then the compact imbedding

$$S^1_{\alpha\bar{p},\bar{p}}\left(\Omega;\bar{\nu}\right)\subset\subset L_{(\alpha+1)p}\left(\Omega;\omega\right).$$

holds.

Theorem 2.4 follows from theorem 2.3 in case of $1<p_i<p<+\infty,\ i=\overline{1,n}$. For this it is sufficient to note that in conditions 5) of theorem 2.4 conditions 5) of theorem 2.3 are fulfilled.

References

[1]. Akhmedov M.A. *Imbedding theorems for one class of weight spaces.* Izv. AN Azerb., series of phys.-math. and tech. scien. 1996, v.XVII, No1-3, pp.37-48. (Russian)

[2]. Akhmedov M.A. *Imbedding theorem for one class of weight pseudo-parabolic spaces.* Izv. AN Azerb., (to appear) (Russian)

[3]. Soltanov K.N. *Imbedding theorem for nonlinear spaces and solvability of some nonlinear coercive equations.* Dep VINITI, 1991, No3697-B-91, 72p.

[4]. Soltanov K.N. *Imbedding theorem for nonlinear spaces and solvability of some nonlinear noncoercive equations.* Trudy IMM AM Azerb., 1996, v.V(XIII), pp.72-103. (Russian)

[5]. Soltanov K.N. *Some applications of nonlinear analysis to differential equations.* Baku, "Elm", 2002, 296p. (Russian)

[6]. Akhmedov M.A. *Some weight spaces, weight boundary values and their applications.* Dissertation work for scientific degree of Ph.D., Baku, BSU, 2000, 118p. (Russian)

[7]. Besov O.V. *Imbedding spaces of differential functions of variable smoothness.* Trudy V.A. Steklov, Mathematical Institute. 1997, v.214, pp.25-58. (Russian)

[8]. Kontorovich M.V., Akhilov G.P. *Functional analysis.* M.: "Nauka", 1984, 751p. (Russian)

Makhmud A. Akhmedov
Institute of Mathematics and Mechanics of NAS of Azerbaijan.
9, F.Agayev str., AZ1141, Baku, Azerbaijan.
Tel.: (99412) 439 02 21(off.)

Received October 10, 2003; Revised February 09, 2004.
Translated by Mamedova V.A.