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HOMOTOPY RELATION ON THE GATEGORY OF INVERSE AND DIRECT SPECTRA OF TOPOLOGICAL SPACES

Abstract

In this article, a homotopy relation is introduced on the category of inverse and direct spectra of topological spaces. This relation is a generalizition of the usual homotopy relation in the class of topological spaces. It is proved that the given relation is an equivalence relation and the composition operation is invariant with respect to this relation. Later a link between the homotopy relation of inverse spectra and the homotopy relation of the limits of inverse spectra is stated.

1.Introduction. In algebraic topology many homology and cohomology theories have been built on subcategories of the category of topological spaces. For example, K-theory was built on the category of the finite CW-complexes [2]. It is natural to study the problem of expansion of the K-theory on a wider category. One of the approaches is to approximate a topological space by "good" spaces. Such an approximation has been introduced by P.S. Alexandrov by using an inverse spectrum. He proved that each compact space is a limit of the inverse spectrum of finite polyedra [1]. Further this result has been expanded by V.I. Zaytsev, V.I. Ponomaryov and others for a wider category of topological spaces [10],[12].

Inverse and direct spectra have many applications in algebra and topology. By using inverse spectra homology and cohomology theories are defined [7]. Thus the K-functor is firstly expanded on the category of locally finite CW-complexes, then on the category of all topological spaces [4],[5],[6],[7]. In recent years shape theory has been built on the basis of inverse spectra [3],[9].

In cited works inverse spectra are used as a tool to study different problems. However in order to expand the function of the tool it is natural to introduce a new equivalence relation involving also the usual equivalence relations. In this work a new homotopy relation is introduced on the category of inverse and direct spectra that is an expansion of the homotopy relation in the category of topological spaces. Further the relationship within homotopics of inverse spectra and limit spaces of these spectra is established.

2. Homotopy relation on the category of inverse and direct spectra

$Inv\,(Top)$ and $Dir\,(Top)$ are the appropriate categories of inverse and direct spectra of the topological spaces. Consider the inverse spectra :

$$\underline{X} = \left(\{X_\alpha\}_{\alpha\in A}, \left\{ p_\alpha^{\alpha'} : X_{\alpha'} \to X_\alpha \right\}_{\alpha\prec\alpha'} \right),$$

$$\underline{Y} = \left(\{Y_\beta\}_{\beta\in B}, \left\{ q_\beta^{\beta'} : Y_{\beta'} \to Y_\beta \right\}_{\beta\prec\beta'} \right).$$

For the morphisms of the above inverse spectra

$$\underline{f} = \left(\pi : B \to A, \left\{f_\beta : X_{\pi(\beta)} \to Y_\beta\right\}_{\beta\in B}\right),$$

$$\underline{g} = \left(\rho : B \to A, \left\{g_\beta : X_{\rho(\beta)} \to Y_\beta\right\}_{\beta\in B}\right)$$

we give the following definition [7],[11].

Definition 2.1. *If $\forall\beta \in B\ \exists\alpha \in A$ satisfying $\alpha \succ \pi(\beta), \alpha \succ \rho(\beta)$ and mappings $f_\beta \circ p^\alpha_{\pi(\beta)}$ and $g_\beta \circ p^\alpha_{\rho(\beta)}$ are homotopic ($f_\beta \circ p^\alpha_{\pi(\beta)} \sim g_\beta \circ p^\alpha_{\rho(\beta)}$), then the morphisms $\underline{f}, \underline{g} : \underline{X} \to \underline{Y}$ are said to be spectrally homotopic morphisms. If $f_\beta \circ p^\alpha_{\pi(\beta)} = g_\beta \circ p^\alpha_{\rho(\beta)}$, then the morphisms $\underline{f}$ and $\underline{g}$ are called canonically homotopic morphisms. We indicate that the morphisms $\underline{f}$ and $\underline{g}$ are spectrally homotopic by $\underline{f} \overset{s}{\sim} \underline{g}$.*

Similarly we define the concept of homotopy in a category of direct spectra of topological spaces.

Note that, if both inverse and direct spectra are formed from one space, the morphisms of the spectra (spectral homotopy) will give the continuous maps of topological spaces. Spectral homotopy will be transformed into an ordinary homotopy.

Theorem 2.2. *The spectral homotopy relation of $Inv(Top)$ $[Dir(Top)]$ in the category of inverse [direct] spectra of topological spaces is an equivalence relation.*

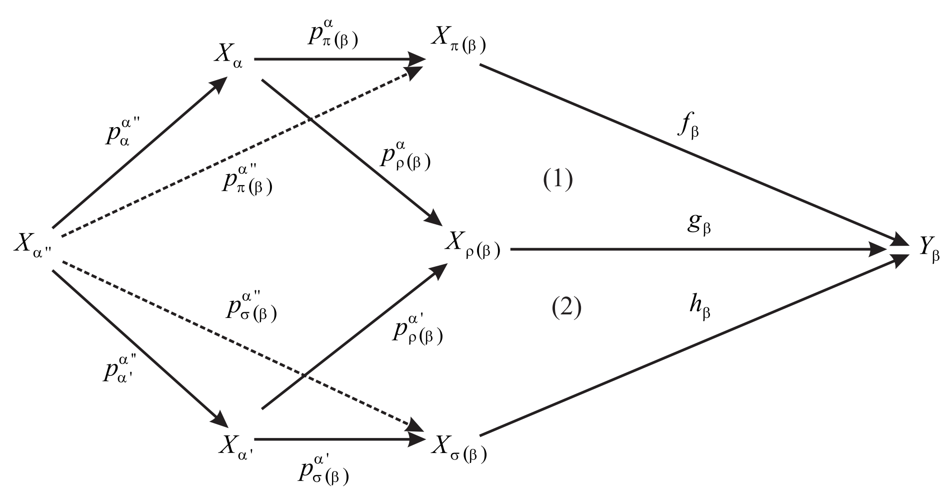
Proof. It is obvious that this relation is reflexive and symmetric. We show that spectral homotopy is transitive. Let $\underline{f} \overset{s}{\sim} \underline{g}$ and $\underline{g} \overset{s}{\sim} \underline{h}$.

Because $\underline{f} \overset{s}{\sim} \underline{g}, \forall\beta \in B\ \exists\alpha \in A$ such that $\alpha \succ \pi(\beta), \alpha \succ \rho(\beta)$, and $f_\beta \circ p^\alpha_{\pi(\beta)} \sim g_\beta \circ p^\alpha_{\rho(\beta)}$.

Because $\underline{g} \overset{s}{\sim} \underline{h}$, $\forall\beta \in B\ \exists\alpha' \in A$ such that $\alpha' \succ \rho(\beta), \alpha' \succ \sigma(\beta)$ and $g_\beta \circ p^{\alpha'}_{\rho(\beta)} \sim h_\beta \circ p^{\alpha'}_{\sigma(\beta)}$.

Because set A is a directed set, for α and α', there is $\alpha'' \in A$ such that $\alpha'' \succ \alpha$, $\alpha'' \succ \alpha'$.

Consider the following diagram:



Now (1) and (2) are homotopy commutative so by using the equations

$$p^{\alpha}_{\pi(\beta)} \circ p^{\alpha''}_{\alpha} = p^{\alpha''}_{\pi(\beta)}, \quad p^{\alpha'}_{\sigma(\beta)} \circ p^{\alpha''}_{\alpha'} = p^{\alpha''}_{\sigma(\beta)}, \quad p^{\alpha}_{\rho(\beta)} \circ p^{\alpha''}_{\alpha} = p^{\alpha'}_{\rho(\beta)} \circ p^{\alpha''}_{\alpha'}$$

we find,

$$f_{\beta} \circ p^{\alpha''}_{\pi(\beta)} \; h_{\beta} \circ p^{\alpha''}_{\sigma(\beta)}.$$

We can show that spectral homotopy in the category of $Dir(Top)$ is an equivalance relation in a similar way.

Theorem 2.3. *The composition operation in the categories $Inv(Top)$ and $Dir(Top)$ are invariant with respect to the spectral homotopy relation.*

Proof. Consider the following:

$$\underline{X} = \left(\{X_{\alpha}\}_{\alpha \in A}, \left\{p^{\alpha'}_{\alpha}\right\}_{\alpha \prec \alpha'}\right), \; \underline{Y} = \left(\{Y_{\beta}\}_{\beta \in B}, \left\{q^{\beta'}_{\beta}\right\}_{\beta \prec \beta'}\right),$$

$$\underline{Z} = \left(\{Z_{\gamma}\}_{\gamma \in C}, \left\{r^{\gamma'}_{\gamma}\right\}_{\gamma \prec \gamma'}\right)$$

and

$$\underline{f_0} = \left(\pi_0 : B \to A, \{f_{0\beta}\}_{\beta \in B}\right), \; \underline{f_1} = \left(\pi_1 : B \to A, \{f_{1\beta}\}_{\beta \in B}\right)$$

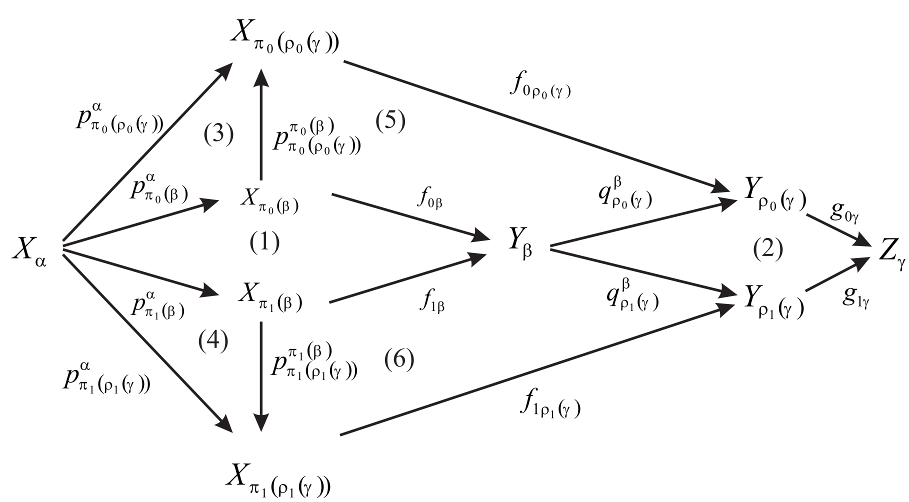
$$\underline{g_0} = \left(\rho_0 : C \to B, \{g_{0\gamma}\}_{\gamma \in C}\right), \; \underline{g_1} = \left(\rho_1 : C \to B, \{g_{1\gamma}\}_{\gamma \in C}\right)$$

and assume that $\underline{f_0} \overset{s}{\sim} \underline{f_1}$ and $\underline{g_0} \overset{s}{\sim} \underline{g_1}$.

Because $\underline{f_0} \overset{s}{\sim} \underline{f_1}$ then $\forall \beta \in B \exists \alpha \in A$ such that $\alpha \succ \pi_0(\beta), \alpha \succ \pi_1(\beta)$ and $f_{0\beta} \circ p^{\alpha}_{\pi_0(\beta)} \sim f_{1\beta} \circ p^{\alpha}_{\pi_{1(\beta)}}$.

Because $\underline{g_0} \overset{s}{\sim} \underline{g_1}$, hen $\forall \gamma \in C \; \exists \beta \in B$ such that $\beta \succ \rho_0(\gamma), \beta \succ \rho_1(\gamma)$ and $g_{0\gamma} \circ q^{\beta}_{\rho_0(\gamma)} \sim g_{1\gamma} \circ q^{\beta}_{\rho_1(\gamma)}$.

Consider the following diagram



In this diagram (1) and (2) are homotopy commutative, (3) and (4) are commutative from the definition of inverse spectrum, and (5), (6) are commutative from the definition of the morphism of inverse spectra. Using these, we obtain:

$$g_{0\gamma} \circ f_{0\rho_0(\gamma)} \circ p^{\alpha}_{\pi_0(\rho_0(\gamma))} \sim g_{1\gamma} \circ f_{1\rho_1(\gamma)} \circ p^{\alpha}_{\pi_1\rho_1(\gamma)}$$

i. e.

$$\underline{g_0} \circ \underline{f}_0 \overset{s}{\sim} \underline{g}_1 \circ \underline{f_1}.$$

Analogously one can prove the theorem in the category $Dir(Top)$.

3. Relation between spectral homotopy and usual homotopy

Let C be a category for which we can describe the categories $Inv(C)$ and $Dir(C)$, and the limit definition of the inverse and direct spectra in these categories.

It is obvious that, we can define canonical homotopy in the categories $Inv(C)$ and $Dir(C)$.

Let $F : Top \to C$ be any covariant (contravariant) functor. It is obvious that the functor F in the categories $Inv(Top)$ and $Dir(Top)$ transforms the covariant (contravariant) functors;

$$F_* : Inv(Top) \to Inv(C) \qquad (F^* : Inv(Top) \to Dir(C))$$

$$F_* : Dir(Top) \to Dir(C) \qquad (F^* : Dir(Top) \to Inv(C))$$

If $F : Top \to C$ is a homotopy invariant functor, then the indicated functor $F_*(F^*)$ transforms the homotopy mappings $f_\beta \circ p^{\alpha}_{\pi(\beta)} \sim g_\beta \circ p^{\alpha}_{\rho(\beta)}$ (Definition 2.1) into equal mappings:

$$F_*(f_\beta) \circ F_*\left(p^{\alpha}_{\pi(\beta)}\right) = F_*(g_\beta) \circ F_*\left(p^{\alpha}_{\rho(\beta)}\right)$$

$$\left[F^*\left(p^{\alpha}_{\pi(\beta)}\right) \circ F^*(f_\beta) = F^*\left(p^{\alpha}_{\rho(\beta)}\right) \circ F^*(g_\beta)\right]$$

That is ,the images of spectrally homotopic morphisms are ctually canonically homotopic.

Thus we have the following proposition.

Proposition 3.1. *If the functor $F : Top \to C$ is homotopy invariant, then the indicated functor $F_*(F^*)$ transforms the spectrally homotopic morphisms into canonically homotopic morphisms. Let*

$$\underline{f} = \left(\pi : B \to A, \left\{f_\beta : X_{\pi(\beta)} \to Y_\beta\right\}_{\beta\in B}\right), \ \ \underline{g} = \left(\rho : B \to A, \left\{g_\beta : X_{\rho(\beta)} \to Y_\beta\right\}_{\beta\in B}\right)$$

$(\overline{f}, \overline{g} : \overline{X} \to \overline{Y})$ be canonically homotopic in the category of $Inv(C)$ $(Dir(C))$. Consider the limits of these morphisms and

$$\lim_{\leftarrow} \underline{f}, \lim_{\leftarrow} \underline{g} : \lim_{\leftarrow} \underline{X} \to \lim_{\leftarrow} \underline{Y} \qquad (\lim_{\rightarrow} \overline{f}, \lim_{\rightarrow} \overline{g} : \lim_{\rightarrow} \overline{X} \to \lim_{\rightarrow} \overline{Y})$$

Proposition 3.2. *If $\underline{f}, \underline{g} : \underline{X} \to \underline{Y}$ $(\overline{f}, \overline{g} : \overline{X} \to \overline{Y})$ are canonically homotopic, then the limit morphisms are equal:*

$$\lim_{\leftarrow} \underline{f} = \lim_{\leftarrow} \underline{g} \qquad (\lim_{\rightarrow} \overline{f} = \lim_{\rightarrow} \overline{g}).$$

Proof. If $\left\{x_{\pi(\beta)}\right\}$ is any element of $\lim\limits_{\leftarrow} \underline{X}$, then

$$\lim_{\leftarrow} \underline{f}\left(\left\{x_{\pi(\beta)}\right\}\right) = \left\{f_\beta\left(x_{\pi(\beta)}\right)\right\}.$$

Because the morphisms $\underline{f}, \underline{g}$ are canonically homotopic, for each β, $\exists\alpha$ such that

$$\alpha \succ \pi(\beta), \alpha \succ \rho(\beta)$$

and

$$f_\beta\left(p^\alpha_{\pi(\beta)}(x_\alpha)\right) = g_\beta\left(p^\alpha_{\rho(\beta)}(x_\alpha)\right)$$

Therefore

$$f_\beta\left(x_{\pi(\beta)}\right) = f_\beta\left(p^\alpha_{\pi(\beta)}(x_\alpha)\right) = g_\beta\left(p^\alpha_{\rho(\beta)}(x_\alpha)\right) = g_\beta\left(x_{\rho(\beta)}\right).$$

Thus

$$\lim_{\leftarrow} \underline{f} = \lim_{\leftarrow} \underline{g}.$$

Now we look at

$$\lim_{\rightarrow} \overline{f}, \quad \lim_{\rightarrow} \overline{g} : \lim_{\rightarrow} \overline{X} \to \lim_{\rightarrow} \overline{Y}.$$

Assume $[x_\alpha] \in \lim_{\rightarrow} \overline{X}$ and $x_\alpha \in X_\alpha$. We need to show the following

$$[f_\alpha(x_\alpha)] = [g_\alpha(x_\alpha)]$$

Because $\overline{f}$ and $\overline{g}$ are canonically homotopic, for each $\alpha \in A$, $\exists\beta$ such that $\beta \succ \pi(\alpha), \beta \succ \rho(\alpha)$, and $q^\beta_{\pi(\alpha)} \circ f_\alpha = q^\beta_{\rho(\alpha)} \circ g_\alpha$ is obtained. Thus

$$q^\beta_{\pi(\alpha)}(f_\alpha(x_\alpha)) = q^\beta_{\rho(\alpha)}(g_\alpha(x_\alpha)).$$

This means that the elements $f_\alpha(x_\alpha)$ and $g_\alpha(x_\alpha)$ are equivalent and hence

$$[f_\alpha(x_\alpha)] = [g_\alpha(x_\alpha)].$$

From Proposition 3.1. and Proposition 3.2. the theorem below is derived

Theorem 3.3 *Let $F : Top \to C$ be a homotopy invariant functor. Then, the composition of the induced functor $F_*(F^*)$ and one of the functors $\lim\limits_{\leftarrow}$ or $\lim\limits_{\rightarrow}$ will be a spectral homotopy invariant functor.*

From Theorem 3.3, It is proved that Chech extension is homotopy invariant. Therefore we are able to extend the homotopy invariant functor to a wider category without changing its invariant characteristic since the topological spaces could be given as the limit of inverse spectrum of the "good" spaces. Here "good" spaces are the CW-complexes [1],[3],[9],[12].

Now, we show the relationship between homotopy of limit spaces and the spectral homotopy of inverse spectra. Let

$$\underline{X} = \left(\{X_\alpha\}_{\alpha\in A}, \left\{p_\alpha^{\alpha'}\right\}_{\alpha\prec\alpha'}\right), \quad \underline{Y} = \left(\{Y_\beta\}_{\beta\in B}, \left\{q_\beta^{\beta'}\right\}_{\beta\prec\beta'}\right)$$

be the inverse spectra of topological spaces $\underline{f}, \underline{g} : \underline{X} \to \underline{Y}$ be morphisms, and

$$f = \lim_{\leftarrow} \underline{f}, g = \lim_{\leftarrow} \underline{g} : X = \lim_{\leftarrow} \underline{X} \to Y = \lim_{\leftarrow} \underline{Y}$$

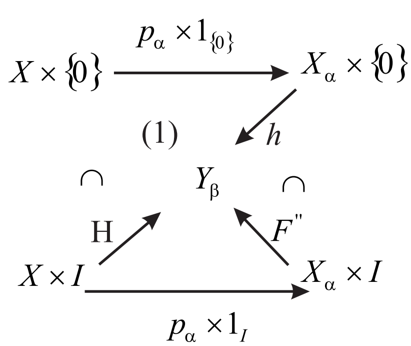
be the limit morphisms of the limit spaces. Then we can prove following.

Theorem 3.4. *If $f, g : X \to Y$ are homotopic maps and for $\forall\alpha \in A$, $p_\alpha : X \to X_\alpha$ is a cofibration and map onto X_α [8],[11], then the morphisms $\underline{f}, \underline{g} : \underline{X} \to \underline{Y}$ of the inverse spectra are spectrally homotopic.*

Proof. Let $\beta \in B$. Because $\pi(\beta), \rho(\beta) \in A$ and A is a directed set, there is $\alpha \in A$ such that $\alpha \succ \pi(\beta)$ and $\alpha \succ \rho(\beta)$. If $F : X \times I \to Y$ is a homotopy between f and g that is $F(x,0) = f(x), F(x,1) = g(x)$ and $q_\beta : Y \to Y_\beta$ is the projection, then H and h could be defined as follows:

$$H = q_\beta \circ F : X \times I \to Y_\beta \quad \textit{and} \quad h = f_\beta \circ p^\alpha_{\pi(\beta)}$$

Consider at the diagram below;



Note that (1) commutes. Therefore, the condition of cofibration is provided. Because of this, a homotopy $F'' : X_\alpha \times I \to Y_\beta$ having the following properties is obtained

$$F''(x_\alpha, 0) = h(x_\alpha) = f_\beta\left(p^\alpha_{\pi(\beta)}(x_\alpha)\right) \qquad H(x,t) = F''(p_\alpha(x), t) = F''(x_\alpha, t)$$

Then

$$F''(x_\alpha, 1) = H(x,1) = (q_\beta \circ F)(x,1) = q_\beta(F(x,1)) = q_\beta(g(x)) = g_\beta\left(p^\alpha_{\rho(\beta)}(x_\alpha)\right)$$

Therefore

$$F''(x_\alpha, 0) = f_\beta\left(p^\alpha_{\pi(\beta)}(x_\alpha)\right), F''(x_\alpha, 1) = g_\beta\left(p^\alpha_{\rho(\beta)}(x_\alpha)\right),$$

that is $\forall\beta \in B$ there is $\alpha \in A$ such that $f_\beta \circ p^\alpha_{\pi(\beta)} \sim g_\beta \circ p^\alpha_{\rho(\beta)}$. Thus the morphisms $\underline{f}, \underline{g} : \underline{X} \to \underline{Y}$ are spectrally homotopic. Hence, the theorem is proved.

Theorem 3.4 is proved under the cofibration condition of the maps $p_\alpha : X \to X_\alpha$. Can we omit this condition?

Consider the class of homotopy types of mappings of topological spaces. It means that the homotopic mappings are equal.

Theorem 3.5. *If $f, g : X \to Y$ are homotopic mappings in the class of homotopy types then the morphisms of the inverse spectra $\underline{f}, \underline{g} : \underline{X} \to \underline{Y}$ are spectrally homotopic.*

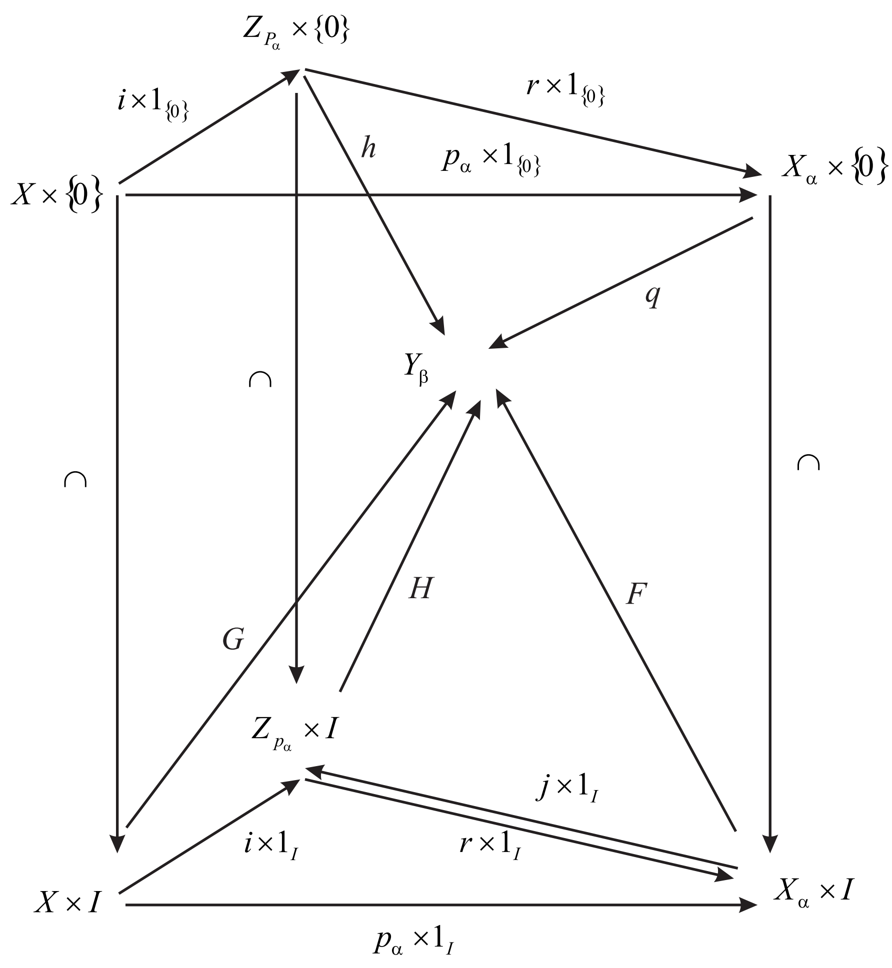
Proof. Let Z_{p_α} be the cylinder of the mapping $p_\alpha, i : X \to Z_{p_\alpha} j : X_\alpha \to Z_{p_\alpha}$ the injection maps and $r : Z_{p_\alpha} \to X_\alpha$ be the retract map. It is known that $i : X \to Z_{p_\alpha}$ is a cofibration.

Define the maps $q : X_\alpha \to Y_\beta, h : Z_{p_\alpha} \to Y_\beta$ and $G : X \times I \to Y_\beta$ as follows:

Let $q = f_\beta \circ p^\alpha_{\pi(\beta)}$ and $h = q \circ r$, and G is any homotopy satisfying the condition $G(x, 0) = h(i(x))$. Then, because $i : X \to Z_{p_\alpha}$ is a cofibration, there is $H : Z_{p_\alpha} \times I \to Y_\beta$ which satisfies

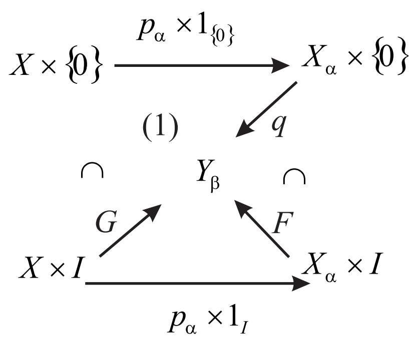
$$H(z, 0) = h(z) \quad \text{and} \quad G(x, t) = H \circ (i \times 1_I)(x, t).$$

Consider the following diagram :



For the homotopy

$G, G(x,0) = h(i(x)) = q \circ r \circ i(x) = q(p_\alpha(x))$ is assumed. There $F : X_\alpha \times I \to Y_\beta$ is defined to be $H \circ (j \times 1_I)$. Consider the diagram:



We check whether the conditions are verified with the $p_\alpha : X \to X_\alpha$ mapping of F homotopy with respect to the cofibration rules.

$F(x_\alpha, 0) = q(x_\alpha)$. Now, we check only the condition $F \circ (p_\alpha \times 1_I) = G$. Hence $j \circ r \sim 1_{Z_{p_\alpha}}$ and $F = H \circ (j \times 1_I)$, so $F \circ (r \times 1_I) \sim H$ On the other hand, there is also $F \circ (p_\alpha \times 1_I) \sim G$

Therefore, the second condition of cofibration is provided with homotopy equivalence exactness. Consequently this theorem has been proved.

If the morphisms of inverse spectra are homotopy the question follows the limit morphisms are homotopy?

We pass to homotopy classes in the category of inverse spectra instead of maps, denoting the new category by $Inv\,[Top]$. We can also define homotopy in the category $Inv\,[Top]$.

Consider the following special situation in the category $Inv\,[Top]$. Let the inverse spectrum $\underline{X}$ derive from a single space X,

$$\underline{Y} = \left(\{Y_\beta\}_{\beta \in B}, \left\{ q_\beta^{\beta'} \right\}_{\beta \prec \beta'} \right)$$

be any inverse spectrum and the morphisms

$$\underline{f} = \left(c : B \to \{*\}, \{f_\beta : X \to Y_\beta\}_{\beta \in B} \right), \underline{g} = \left(c : B \to \{*\}, \{g_\beta : X \to Y_\beta\}_{\beta \in B} \right)$$

are spectrally homotopic. Then $\forall \beta \in B$, $f_\beta, g_\beta : X \to Y_\beta$ are homotopic. Here, $q_\beta^\beta, f_\beta, g_\beta$ maps are homotopy classes of their own.

Lemma 3.6. *If the morphisms* $\underline{f}, \underline{g} : X \to \{Y_\beta\}_{\beta \in B}$ *are spectrally homotopic in the category* $Inv\,[Top]$*, then their limits* $\lim_{\leftarrow} \underline{f}, \lim_{\leftarrow} \underline{g}$ *are homotopic.*

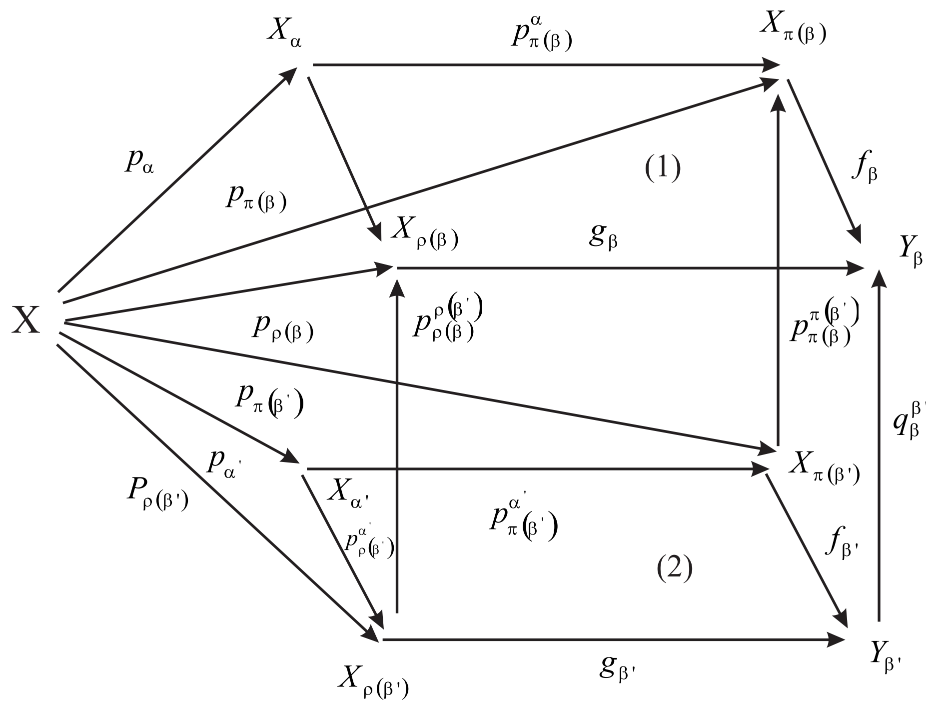
Proof. Let $f = \lim_{\leftarrow} \underline{f}$ and $g = \lim_{\leftarrow} \underline{g}$. Then it is enough to show that $[f] = [g]$.

$$[f](x) = \{[f_\beta](x)\} = \{[g_\beta](x)\} = [g](x) .$$

This equation could be derived by using $[f_\beta] = [g_\beta]$ as a result of f_β and g_β being homotopic.

Theorem 3.7. *In the category $Inv\,[Top]$ the limits of spectrally homotopic morphisms are homotopic.*

Proof. Assume that the morphisms $\underline{f}, \underline{g} : \underline{X} \to \underline{Y}$ are spectrally homotopic in the category $Inv\,[Top]$. For any $\beta' \succ \beta \in B$ thus the following diagram:



In this diagram (1) and (2) are homotopy commutative and the others are commutative. From this diagram, we will note

$$q_\beta^{\beta'} \circ f_{\beta'} \circ p_{\pi(\beta')} = f_\beta \circ p_{\pi(\beta)}^{\pi(\beta')} \circ p_{\pi(\beta')} = f_\beta \circ p_{\pi(\beta)}$$

$$q_\beta^{\beta'} \circ g_{\beta'} \circ p_{\rho(\beta')} = g_\beta \circ p_{\rho(\beta)}^{\rho(\beta')} \circ p_{\rho(\beta')} = g_\beta \circ p_{\rho(\beta)}$$

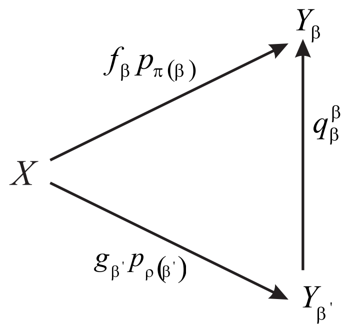
$$f_\beta \circ p_{\pi(\beta)} \sim g_\beta \circ p_{\rho(\beta)},$$

$$f_{\beta'} \circ p_{\pi(\beta')} \sim g_{\beta'} \circ p_{\rho\left(\beta'\right)}$$

and when we pass to homotopy classes,

$$\left[f_\beta \circ p_{\pi(\beta)}\right] = \left[g_\beta \circ p_{\rho(\beta)}\right], \quad \left[f_{\beta'} \circ p_{\pi(\beta')}\right] = \left[g_{\beta'} \circ p_{\rho(\beta')}\right]$$

follow. Thus the diagram



is homotopy commutative.

From Lemma 3.6., their limits are equal to each other. Because

$$\left(\lim_{\leftarrow} f \circ p_{\pi(\beta)}\right)\{x_\alpha\} = \left\{f_\beta\left(p_{\pi(\beta)}\{x_\alpha\}\right)\right\} = \left\{f_\beta\left(x_{\pi(\beta)}\right)\right\}$$

and $(\lim\limits_{\leftarrow} f_\beta)\{x_\alpha\} = \left\{f_\beta\left(x_{\pi(\beta)}\right)\right\}$ it follows that $\lim\limits_{\leftarrow} f_\beta \circ p_{\pi(\beta)} = \lim\limits_{\leftarrow} f_\beta$ and the theorem has been proved.

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