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TWO-DIMENSIONAL EVOLUTIONALY EQUATION IN VISCOELASTIC SOILS THE PREPOTENT FREQUENCIES

Abstract

Two-dimensional evolutional equation, describing non-linear wave process in viscoelastic and linearly-hereditary soils has been introduced. It is shown, that the volume perturbation doesn't reduce to rise of non-linear waves are the product to cross waves of shear deformations. In particular case, an exact solutions describing the structures both of shock waves, and solutions are reduced. The conditions for the existence of prepotent frequences of harmonic oscillations are found.

Some one-dimenstional mathematic models of non-linear waves in elastic and viscoellastic soils [1-5] have been constructed to the present time. The one dimensional distribution of non-linear waves in porous media saturated by fluid, and also in multiphase media have been generalized in [1,6-9].

In the given paper, two-dimensional evolutionary equation, describing non-linear wave process in viscoelastic and linear ly-hereditary soils has been derived. In turns out, that these waves are the product of just cross waves of shear deformations. In particular case, the exact solutions, describing the structures both of show waves, and solutions are reduced. The conditions for the existence of prepotent frequences of hormonic oscillations are found.

1. By describing soil's motion we'll suppose, that the sizes of microcracks, pores and grains of rock are small with respect to (X_1, X_2) with macrocoordinates and length of the wave, and also $L >> X_1, X_2$.

On the base of above-stated assumption we'll take the system of equations of masses, impulses, tensor of speeds deformations on sense of Oldroyd's derivative and thermodinamic condition [3,9].

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_1}{\partial X_1} + \frac{\partial \rho v_2}{\partial X_2} = 0; \tag{1.1}$$

$$\frac{\partial \rho v_1}{\partial t} + \frac{\partial \rho v_1 v_1}{\partial X_1} + \frac{\partial \rho v_1 v_2}{\partial X_2} = \frac{\partial \sigma_{11}}{\partial X_1} + \frac{\partial \sigma_{12}}{\partial X_2}, \tag{1.2}$$

$$\frac{\partial \rho v_2}{\partial t} + \frac{\partial \rho v_1 v_2}{\partial X_1} + \frac{\partial \rho v_2 v_2}{\partial X_2} = \frac{\partial \sigma_{12}}{\partial X_1} + \frac{\partial \sigma_{22}}{\partial X_2};$$

$$\frac{De_{11}}{Dt}=\frac{\partial e_{11}}{\partial t}+\upsilon_1\frac{\partial e_{11}}{\partial X_1}+\upsilon_2\frac{\partial e_{11}}{\partial X_2}+2e_{11}\frac{\partial \upsilon_1}{\partial X_1}+2e_{12}\frac{\partial \upsilon_1}{\partial X_2}=\frac{\partial \upsilon_1}{\partial X_1}, \tag{1.3}$$

$$\frac{De_{22}}{Dt}=\frac{\partial e_{22}}{\partial t}+\upsilon_1\frac{\partial e_{22}}{\partial X_1}+\upsilon_2\frac{\partial e_{22}}{\partial X_2}+2e_{12}\frac{\partial \upsilon_2}{\partial X_1}+2e_{22}\frac{\partial \upsilon_2}{\partial X_2}=\frac{\partial \upsilon_2}{\partial X_2},$$

$$\frac{De_{12}}{Dt}=\frac{\partial e_{12}}{\partial t}+\upsilon_1\frac{\partial e_{12}}{\partial X_1}+\upsilon_2\frac{\partial e_{12}}{\partial X_2}+e_{12}\frac{\partial \upsilon_1}{\partial X_1}+e_{12}\frac{\partial \upsilon_2}{\partial X_2}+$$

$$+e_{11}\frac{\partial \upsilon_2}{\partial X_1}+e_{22}\frac{\partial \upsilon_1}{\partial X_2}=\frac{1}{2}\left(\frac{\partial \upsilon_1}{\partial X_2}+\frac{\partial \upsilon_2}{\partial X_1}\right); \tag{1.4}$$

Let's remark, that in experiments [3,10] some viscoelastic dynamic properties fragmentation rocks, which correspond to more complicated models, with including to the defining rhelogical correlations also higher time derivatives were detected.

Let us formulate the corresponding laws with sharing of sphere and deviator constituents for stress and strain tensors [3.9]

$$\left(b_\circ+\sum_{l=1}^{m}b_l\frac{D^l}{Dt^l}\right)\sigma=\left(a_0+\sum_{l=1}^{n}a_l\frac{D^l}{Dt^l}\right)e, \tag{1.5}$$

$$\left(b_0'+\sum b_l'\frac{D^l}{Dt^l}\right)\sigma_{12}=\left(a_0'+\sum_{l=1}^{n}a_l'\frac{D^l}{Dt^l}\right)e_{12}. \tag{1.6}$$

Similarly, for linearly-hereditary soils, we'll take

$$\sigma=3K\left(e-\int_\circ^t H_0\left(t-T\right)edT\right); \tag{1.7}$$

$$\sigma_{12}=2G\left(e_{12}-\int_0^t H\left(t-T\right)e_{12}dT\right). \tag{1.8}$$

Here $\sigma=\sigma_{11}+\sigma_{22},\ e=e_{11}+e_{22},\ H_0\left(t\right),\ H\left(t\right)$ are the memory functions, $K=E/3\left(1-3\nu\right),\ G=E/2\left(1+\nu\right),\ a_0,...,a_n',b_0,...,b_n'$ constant coefficients which are defined from concrete viscoelastic models.

In general case of equations (1.1)-(1.6) or (1.1)-(1.4), (1.7), (1.8) is closed with respect to unknown variables: $\sigma_{11},\sigma_{22},\sigma_{12},e_{11},e_{22},e_{12},\upsilon_1,\upsilon_2,\rho$. It is naturally to assume, that the forms of these functions slowly change with distance from entry, i.e we introduce new variables [1-6].

$$x=\eta X_1,y=\eta X_2,\tau=\left(ct-\bar{n}\cdot\bar{r}\right)/c=t-c_1^{-1}X_1-c_2^{-1}X_2. \tag{1.9}$$

Here $\bar{n}\left(n_1,n_2\right),\bar{r}\left(X_1,X_2\right),c_1=c/n_1,c_2=c/n_2,\eta<<1$. Substituting (1.9) into equations (1.1)-(1.8), we have

$$\frac{\partial\rho}{\partial\tau}+\eta\frac{\partial\rho\upsilon_1}{\partial x}+\eta\frac{\partial\rho\upsilon_2}{\partial y}-c_1^{-1}\frac{\partial\rho\upsilon_1}{\partial\tau}-c_2^{-1}\frac{\partial\rho\upsilon_2}{\partial\tau}=0; \tag{1.10}$$

$$\frac{\partial\rho\upsilon_1}{\partial\tau}+\eta\frac{\partial\rho\upsilon_1\upsilon_1}{\partial x}+\eta\frac{\partial\rho\upsilon_1\upsilon_2}{\partial y}-c_1^{-1}\frac{\partial\rho\upsilon_1\upsilon_1}{\partial\tau}-c_2^{-1}\frac{\partial\rho\upsilon_1\upsilon_2}{\partial\tau}= \tag{1.11}$$

$$=\eta\frac{\partial\sigma_{11}}{\partial x}+\eta\frac{\partial\sigma_{12}}{\partial y}-c_1^{-1}\frac{\partial\sigma_{11}}{\partial\tau}-c_2^{-1}\frac{\partial\sigma_{12}}{\partial\sigma},$$

$$\frac{\partial\rho\upsilon_2}{\partial\tau}+\eta\frac{\partial\rho\upsilon_1\upsilon_2}{\partial x}-c_1^{-1}\frac{\partial\rho\upsilon_1\upsilon_2}{\partial\tau}-c_2^{-1}\frac{\partial\rho\upsilon_2\upsilon_2}{\partial\tau}=$$

$$=\eta\frac{\partial\sigma_{12}}{\partial x}+\eta\frac{\partial\sigma_{22}}{\partial y}-c_1^{-1}\frac{\partial\sigma_{12}}{\partial\tau}-c_2^{-1}\frac{\partial\sigma_{22}}{\partial\tau};$$

$$\frac{\partial e_{11}}{\partial\tau}+\eta\upsilon_1\frac{\partial e_{11}}{\partial x}+\eta\upsilon_2\frac{\partial e_{11}}{\partial y}+2\eta e_{11}\frac{\partial\upsilon_1}{\partial x}+2\eta e_{12}\frac{\partial\upsilon_1}{\partial y}- \tag{1.12}$$

$$-\left(c_1^{-1}\upsilon_1+c_2^{-1}\upsilon_2\right)\frac{\partial e_{11}}{\partial\tau}-2\left(c_1^{-1}e_{11}+c_2^{-1}e_{12}\right)\frac{\partial\upsilon_1}{\partial\tau}=\eta\frac{\partial\upsilon_1}{\partial x}-c_1^{-1}\frac{\partial\upsilon_1}{\partial\tau},$$

$$\frac{\partial e_{22}}{\partial\tau}+\eta\upsilon_1\frac{\partial e_{22}}{\partial x}+\eta\upsilon_2\frac{\partial e_{22}}{\partial y}+2e_{12}\frac{\partial\upsilon_2}{\partial x}+2\eta e_{22}\frac{\partial\upsilon_2}{\partial y}-$$

$$-\left(c_1^{-1}\upsilon_1+c_2^{-1}\upsilon_2\right)\frac{\partial e_{22}}{\partial\tau}-2\left(c_1^{-1}\upsilon_{12}+c_2^{-1}\upsilon_{22}\right)\frac{\partial\upsilon_2}{\partial\tau}=\eta\frac{\partial\upsilon_2}{\partial y}-c_2^{-1}\frac{\partial\upsilon_2}{\partial\tau},$$

$$\frac{\partial e_{12}}{\partial\tau}+\eta\upsilon_1\frac{\partial e_{12}}{\partial x}+\eta\upsilon_2\frac{\partial e_{12}}{\partial y}+\eta e_{11}\frac{\partial\upsilon_2}{\partial x}+\eta e_{11}\frac{\partial\upsilon_2}{\partial x}+\eta e_{22}\frac{\partial\upsilon_1}{\partial x}+\eta e_{12}\frac{\partial\upsilon_2}{\partial y}-$$

$$-\left(c_1^{-1}\upsilon_1+c_2^{-1}\upsilon_2\right)\frac{\partial e_{12}}{\partial\tau}-\left(c_2^{-1}e_{22}+c_1^{-1}e_{12}\right)\frac{\partial\upsilon_1}{\partial\tau}-\left(c_1^{-1}e_{11}+c_2^{-1}e_{12}\right)\frac{\partial\upsilon_2}{\partial\tau}=$$

$$=\frac{1}{2}\left(\eta\frac{\partial\upsilon_1}{\partial y}+\eta\frac{\partial\upsilon_2}{\partial x}-c_2^{-1}\frac{\partial\upsilon_1}{\partial\tau}-c_1^{-1}\frac{\partial\upsilon_2}{\partial\tau}\right);$$

$$\left[b_0+\sum_{l=1}^{m}b_l\prod_{q=1}^{l}\left(\frac{\partial}{\partial\tau}+\eta\upsilon_1\frac{\partial}{\partial x}+\eta\upsilon_2\frac{\partial}{\partial y}-c_1^{-1}\upsilon_1\frac{\partial}{\partial\tau}-c_2^{-1}\upsilon_2\frac{\partial}{\partial\tau}\right)^q\right]\sigma= \tag{1.13}$$

$$=\left[a_0+\sum_{l}^{n}a_l\prod_{q=1}^{l}\left(\frac{\partial}{\partial\tau}+\eta\upsilon_1\frac{\partial}{\partial x}+\eta\upsilon_2\frac{\partial}{\partial y}-c_1^{-1}\upsilon_1\frac{\partial}{\partial\tau}-c_2^{-1}\upsilon_2\frac{\partial}{\partial\tau}\right)^q\right]e;$$

$$\left[{}'b_0+\sum_{l}^{m}b_l\prod_{q=1}^{l}\left(\frac{\partial}{\partial\tau}+\eta\upsilon_1\frac{\partial}{\partial x}+\eta\upsilon_2\frac{\partial}{\partial y}-c_1^{-1}\upsilon_1\frac{\partial}{\partial\tau}-c_2^{-1}\upsilon_2\frac{\partial}{\partial\tau}\right)^q\right]\sigma_{12}= \tag{1.14}$$

$$=\left[a_0'+\sum_{l}^{n}a_l'\prod_{q=1}\left(\frac{\partial}{\partial\tau}+\eta\upsilon_1\frac{\partial}{\partial x}+\eta\upsilon_2\frac{\partial}{\partial y}-c_1^{-1}\frac{\partial}{\partial\tau}-c_2^{-1}\upsilon_2\frac{\partial}{\partial\tau}\right)^q\right]e_{12}.$$

The Rheological equations (1.13), (1.14) are written only for viscoelastic rocks, and for linearly-hereditary rocks the notation will be similar.

Let's represent the desired variables as series small parameter η

$$\sigma_{ij}=\sigma_{ij}^0+\eta\sigma_{ij}^{(1)}+\eta^2\sigma_{ij}^{(2)}+...,r_{ij}=\eta e_{ij}^{(1)}+\eta^2\upsilon_{ij}^{(2)}+...,\upsilon_i=\eta\upsilon_i^{(1)}+\eta^2\upsilon_i^{(2)}+...,$$

$$\rho=\rho_0+\eta D_1\left(\sigma_{11}^{(!)}+\sigma_{22}^{(1)}\right)+\eta^2[D_1\left(\sigma_{11}^{(2)}+\sigma_{22}^{(2)}\right)+D_2\left(\sigma_{11}^{(1)}+\sigma_{22}^{(1)}\right)^2]+... \tag{1.15}$$

Here $i,j=1,2, D_1=(\partial\rho/\partial\sigma)_{\sigma_0}<0,\ D_2=(1/2)\left(\partial^2\gamma r/\partial\sigma^2\right)_{\sigma_0},\ \sigma_{12}^0,\sigma_{11}^0,\sigma_{12}^0,\sigma_0$ are values of stress in stationary state of rock.

Substituting expansion (1.15) to the system of equations (1.10)-(1.14) and equate the coefficients of members with the same degrees η in the first approximation we have

$$D_1\left(\sigma_{11}^{(1)}+\sigma_{22}^{(1)}\right)=\rho_0\left(c_1^{-1}v_1^{(1)}+c_2^{-1}v_2^{(1)}\right), \tag{1.16}$$

$$\rho_0 v_1^{(1)}=-c_1^{-1}\sigma_{11}^{(1)}-c_2^{-1}\sigma_{12}^{(1)},\ \rho_0 v_2^{(1)}=-c_1^{-1}\sigma_{12}^{(1)}-c_2^{-1}\sigma_{22}^{(1)}; \tag{1.17}$$

$$e_{11}^{(1)}=-c_1^{-1}v_1^{(1)},\ e_{22}^{(1)}=-c_2^{-1}v_2^{(1)},\ e_{12}^{(1)}=-\left(c_2^{-1}v_1^{(1)}+c_1^{-1}v_2^{(1)}\right)/2; \tag{1.18}$$

$$b_0\left(\sigma_{11}^{(1)}+\sigma_{22}^{(1)}\right)=a_0\left(e_{11}^{(1)}+e_{22}^{(1)}\right), \tag{1.19}$$

$$b_0'\sigma_{12}^{(1)}=a_0'e_{12}^{(1)}. \tag{1.20}$$

The closed system of linear homogeneous equations (1.16)-(1.20) has a non-zero solution if and only if the determinant of this system equals zero:

$$\left(D_1a_0/b_0\rho_0+1\right)^2\left[1-a_0'\left(c_1^{-2}+c_2^{-2}\right)/\left(2b_0'\rho\right)\right]=0.$$

From the first two equations (1.18) and (1.19), (1.16) we'll obtain:

$$\left(D_1\frac{a_0}{b_0\rho_0}+1\right)\left(c_1^{-1}v_1^{(1)}+c_2^{-1}v_2^{(1)}\right)=0. \tag{1.21}$$

Similarly, from the third equation (1.18) and (1.20), (1.17) we find

$$\sigma_{11}^{(1)}+\sigma_{22}^{(1)}=\rho_0c_1c_2\left(\frac{a_0'\left(c_1^{-2}+c_2^{-2}\right)}{2b_0'\rho_0}-1\right)\left(c_2^{-1}v_1^{(1)}+c_1^{-1}v_2^{(1)}\right). \tag{1.22}$$

It is known, that volumetric and cross waves extend independently from each other.

On the other hand, at distribution of the volumetric waves $e^{(1)}=c_1^{-1}v_1^{(1)}+c_2^{-1}v_2^{(1)}\neq 0$, therefore from (1.21) follows

$$c_p=\left(-1/D_1\right)^{1/2}=\left(\frac{a_0}{b_0\rho_0}\right)^{1/2}. \tag{1.23}$$

On the other hand, at distribution of the cross waves $e_{12}^{(1)}=c_2^{-1}v_1^{(1)}+c_1^{-1}v_2^{(1)}\neq 0$, therefore from (1.22) and (1.19) it follows: $\sigma^{(1)}=0\Rightarrow e^{(1)}=0$

$$\frac{a_0'}{2b_0'\rho_0}=\frac{c_1^2c_2^2}{c_1^2+c_2^2}=c_s^2,\ v_2^{(1)}=-\left(c_2/c_1\right)v_1^{(1)}. \tag{1.24}$$

Let us note, that the speeds of volumetric c_p and cross s_s waves are connected with c_1,c_2 correlations $c_1=c_{p,s}/\sin\alpha,\ c_2=--c_{p,s}/\cos\alpha$, where $\alpha\ (0<\alpha<\pi/2)$

is an angle of irridence and Snellews correlation $\sin\alpha/c_{p,s}=const$ connects α with speed of waves .

Similarly to the first, in the second approximation we have

$$D_1\frac{\partial\left(\sigma_{11}^{(2)}+\sigma_{22}^{(2)}\right)}{\partial\tau}-\rho_0\frac{\partial}{\partial\tau}\left(c_1^{-1}v_1^{(2)}+c_2^{-1}v_2^{(2)}\right)=-D_2\frac{\partial}{\partial\tau}\left(\sigma_{11}^{(1)}+\sigma_{22}^{(1)}\right)^2+ \tag{1.25}$$

$$+\rho_0\frac{\partial}{\partial\tau}\left(c_1^{-1}v_1^{(1)}+c_2^{-1}v_2^{(1)}\right)^2-\rho_0\left(\frac{\partial v_1^{(1)}}{\partial x}+\frac{\partial v_2^{(1)}}{\partial y}\right);$$

$$\rho_0\frac{\partial v_1^{(2)}}{\partial\tau}+c_1^{-1}\frac{\partial\sigma_{11}^{(2)}}{\partial\tau}+c_2^{-1}\frac{\partial\sigma_{12}^{(2)}}{\partial\tau}=\frac{\partial\sigma_{11}^{(1)}}{\partial x}+\frac{\partial\sigma_{12}^{(1)}}{\partial y}, \tag{1.26}$$

$$\rho_0\frac{\partial v_2^{(2)}}{\partial\tau}+c_1^{-1}\frac{\partial\sigma_{12}^{(2)}}{\partial\tau}+c_2^{-1}\frac{\partial\sigma_{22}^{(2)}}{\partial\tau}=\frac{\partial\sigma_{12}^{(1)}}{\partial x}+\frac{\partial\sigma_{22}^{(1)}}{\partial y},$$

$$\frac{\partial e_{11}^{(2)}}{\partial\tau}+c_1^{-1}\frac{\partial\sigma_1^{(2)}}{\partial\tau}=\frac{\partial v_1^{(1)}}{\partial x}+\left(c_1^{-1}v_1^{(1)}+c_2^{-1}v_2^{(1)}\right)\frac{\partial e_{11}^{(1)}}{\partial\tau}+$$

$$+2\left(c_1^{-1}e_{11}^{(1)}+c_2^{-1}e_{12}^{(1)}\right)\frac{\partial v_1^{(1)}}{\partial\tau}, \tag{1.27}$$

$$\frac{\partial e_{22}^{(2)}}{\partial\tau}+c_2^{-1}\frac{\partial\sigma_2^{(2)}}{\partial\tau}=\frac{\partial v_2^{(1)}}{\partial y}+\left(c_1^{-1}v_1^{(1)}+c_2^{-1}v_2^{(1)}\right)\frac{\partial e_{22}^{(1)}}{\partial\tau}+$$

$$+2\left(c_1^{-1}e_{12}^{(1)}+c_2^{-1}e_{22}^{(1)}\right)\frac{\partial v_2^{(1)}}{\partial\tau};$$

$$\frac{\partial e_{12}^{(2)}}{\partial\tau}+\frac{1}{2}\frac{\partial}{\partial\tau}\left(c_2^{-1}v_1^{(2)}+c_1^{-1}v_2^{(2)}\right)=\frac{1}{2}\left(\frac{\partial v_1^{(1)}}{\partial y}+\frac{\partial v_2^{(1)}}{\partial x}\right)+ \tag{1.28}$$

$$\left(c_1^{-1}v_1^{(1)}+c_2^{-1}v_2^{(1)}\right)\frac{\partial e_{12}^{(1)}}{\partial\tau}+\left(c_1^{-1}e_{12}^{(2)}+c_2^{-1}e_{22}^{(1)}\right)\frac{\partial v_1^{(1)}}{\partial\tau}+$$

$$+\left(c_1^{-1}e_{11}^{(1)}+c_2^{-1}e_{12}^{(1)}\right)\frac{\partial v_2^{(1)}}{\partial\tau};$$

$$b_0\left(\sigma_{11}^{(2)}+\sigma_{22}^{(2)}\right)-a_0\left(e_{11}^{(2)}+e_{22}^{(2)}\right)=\frac{1}{\eta}T, \tag{1.29}$$

$$b_0'\sigma_{12}^{(2)}-a_0'e_{12}^{(2)}=\frac{1}{\eta}T_{12}. \tag{1.30}$$

The expression T and T_{12}, which is present at equations (1.29) and (1.30) are simplified under the conditions $c_1^{-1}v_1\partial/\partial\tau\sim c_2^{-1}v_2\partial/\partial\tau$, since amplitudes of the speeds of displacements greatly less, than the speeds of the waves c_p or c_s

$$T=\sum_{l=1}^{n}a_l\frac{\partial^l\left(e_{11}^{(1)}+e_{22}^{(1)}\right)}{\partial\tau^l}-\sum_{l=1}^{m}b_l\frac{\partial^l\left(e_{11}^{(1)}+\sigma_{22}^{(1)}\right)}{\partial\tau^l}, \tag{1.31}$$

$$T_{12}=\sum_{l=1}^{n}a_l'\frac{\partial^l e_{22}^{(1)}}{\partial\tau^l}-\sum_{l=1}^{m}b_l'\frac{\partial^l\sigma_{12}^{(1)}}{\partial\tau^l}. \qquad (1.32)$$

In the second approximation the volume perturbation doesn't reduce to rise of two-dimensional waves. Really, equation with unknown $e^{(2)}=c_1^{-1}v_1^{(2)}+c_2^{-1}v_2^{(2)}\neq 0$ obtained from the system of equation (1.16), (1.18), (1.31) and (1.25), (1.27), (1.29) identically vanishes.

In this connection in the second approximation we deduce evolution equation of shear deformation of perturbations. Using expression (1.24), we'll express from equations (1.18), (1.20) and (1.17) $e_{11}^{(1)},...,\sigma_{22}^{(1)}$ though $v_1^{(1)}$

$$e_{11}^{(1)}=-c_1^{-1}v_1^{(1)},e_{22}^{(1)}=c_1^{-1}v_1^{(1)},e_{12}^{(1)}=-(1/2)\,c_2\left(c_2^{-2}-c_1^{-2}\right)v_1^{(1)}, \qquad (1.33)$$

$$\sigma_{12}^{(1)}=-\frac{\alpha_0'c_2}{2b_0'}\left(c_2^{-2}-c_1^{-2}\right)v_1^{(1)},\sigma_{11}^{(1)}=\left[\frac{\alpha_0'c_1}{2b_0'}\left(c_2^{-2}-c_1^{-2}\right)-\rho_0c_1\right]v_1^{(1)},$$

$$\sigma_{22}^{(1)}=\left[\frac{\alpha_0'c_1^{-1}}{\partial b_0'c_1}\left(c_2^{-2}-c_1^{-2}\right)+\rho_0c_1^2c_1^{-1}\right]v_1^{(1)},\sigma_{11}^{(1)}+\sigma_{22}^{(1)}=0.$$

Then, with help of relations (1.32) and (1.33) from the system of equations (1.30), (1.26), (1.28) we'll obtain the equation with the unknown $c_2^{-1}v_1^{(2)}+c_1^{-1}v_2^{(2)}\neq 0$, whose coefficient is equal to expression (1.24) $\left(\alpha_0'/\left(2b_0'\rho_0\right)-c_s^2\equiv 0\right)$. The right-hand side of this equation is a two-dimensional equation with Kortweg-de Vries (Kdv) non-linearlity, containing higher derivatives with respect to ordinary Kdv equation.

$$\frac{\partial v_1^{(1)}}{\partial x}-\frac{c_1}{c_2}\frac{\partial v_1^{(1)}}{\partial y}+R_1v_1^{(1)}\frac{\partial v_1^{(1)}}{\partial\tau}+\frac{R_3}{\eta}\sum_{l=1}^{n}S_{l+1}\frac{\partial^{l+1}v_1^{(1)}}{\partial\tau^{l+1}}=0, \qquad (1.34)$$

where

$$R_1=\frac{c_1^2+c_2^2}{c_s^2\left(c_1^2-c_2^2\right)},R_3=\frac{c_1}{4\rho_0b_0'c_s^4},S_{l+1}=\Gamma_{(n-l)}b_l'\frac{\alpha_0'}{b_0'}-\Gamma_{(n-l)}\alpha_l'. \qquad (1.35)$$

To get rid of η and the distortion of length's scale we'll substitute in (1.34) $v=\eta R_1v_1^{(1)}=R_1v_1$ and $X_1=x/\eta,X_2=y/\eta$

$$\frac{\partial v}{\partial X_1}-\frac{c_1}{c_2}\frac{\partial v}{\partial X_2}+v\frac{\partial v}{\partial\tau}+R_3\sum_{l=1}^{n}S_{l+1}\frac{\partial^{l+1}v}{\partial\tau^{l+1}}=0. \qquad (1.36)$$

The obtained equation (1.36) describes evolution of non-linear two-dimensional waves in soils. After solution of this equation under the given boundary conditions the other parameters of the problem are found. If we substitute the differentiable operator (1.6) by intepral operator (1.8), then we'll get the evolution of non-linear waves in the linearly-hereditary mediua

$$\frac{\partial v}{\partial X_1}-\frac{c_1}{c_2}\frac{\partial v}{\partial X_2}+v\frac{\partial v}{\partial \tau}+R_3\frac{\partial}{\partial \tau}\int\limits_0^{\tau} H\left(\tau-T\right)vdT=0, \tag{1.37}$$

where

$$R_3=\frac{c_1}{2c_s^2},\ \frac{G}{\rho_0}=\frac{c_1^2c_2^2}{c_1^2+c_2^2}=c_s^2,\ v=\eta R_1 v_1^{(1)}. \tag{1.38}$$

In particular case the viscoelastic models [3.9] are used.

$$S_2=-E_1\theta_1-E_*\theta_*,\ S_3=-\left(E_1-E_*\right)\theta_1\theta_*-\bar{M}_2$$

$$S_4=\frac{E_2}{E_1}\theta_1\bar{M}_1-\left(\bar{M}_1+\bar{M}_2\right)\theta_1\left(\bar{M}_1+\bar{M}_2\frac{E_2+E_*}{E_1}\right)\theta_*,$$

$$S_5=-\bar{M}_1\left(\theta_1\theta_*+\frac{\bar{M}_2}{E_1}\right),\ S_6=-\frac{\bar{M}_1\bar{M}_2}{E_1}\theta_*.$$

2. Take in (1.36) $l=2$ $\left(\theta_1=\theta_*=0,\ \bar{M}=0\right)$ and substitute $v\to -v, \tau\to -\tau$

$$\frac{\partial v}{\partial X_1}-\frac{c_1}{c_2}\frac{\partial v}{\partial X_2}+v\frac{\partial v}{\partial \tau}+\beta\frac{\partial^3 v}{\partial \tau^3}=0,\ \beta=R_3\bar{M}_2. \tag{2.1}$$

Obtained equation (2.1) is Kdv equation in two variables. Search its solution in the form of stationary travelling wave $\xi=\tau-c_0^{-1}[X_1-(c_2/c_1)\,X_2]$

$$-2c_0^{-1}v_\xi+vv_\xi+\beta v_{\xi\xi\xi}=0, \tag{2.2}$$

$$3\beta v_\xi^2=-v^3+6c_0^{-1}v^2+6A_0v+6B_0=f\left(v\right). \tag{2.3}$$

It is known [4.5], that the sum of roots of equation (2.3) gives us the inverse value of speed of wave propagation. If two roots of this equation are equal to $v_1=v_2\neq v_3$, then solution (2.1) as $\xi\to\pm\infty,\ v\to 0$ has the form [11]

$$v=6c_0^{-1}\sec h^2\left\{\left(\frac{1}{2c_0\beta}\right)^{1/2}\left[\left(c_1^{-}-c_0^{-1}\right)X_1+\left(c_2^{-1}+c_0^{-1}c_1^{-1}c_2\right)X_2-t\right]\right\}. \tag{2.4}$$

In two-dimensional solution (2.4) An amplitude of the wave is twice greater than in one-dimensional case, and it's period (width) in $\sqrt{2}$ times decreasing equals to $2\tau\left(2c_0\beta\right)^{1/2}$. If equation (2.3) has three different roots $v_1\neq v_2\neq v_3$, then solution (2.1) takes the form:

$$v\left(\xi\right)=v_2+\left(v_3-v_2\right)cn^2\left[\xi\sqrt{\frac{v_3-v_1}{12\beta}},s\right]. \tag{2.5}$$

Hence, by coordinate ξ we'll define the period of the wave P

$$P=4\left(\frac{2\beta}{\upsilon_3-\upsilon_1}\right)^{1/2}K(s). \qquad (2.6)$$

Here $cn(x)$ is elliptical Jacobian's function, $K(s)$ is a first type of complete elliptical function.

$\upsilon_2\to\upsilon_1,\ s\to 1$ correspond to large values of amplitude of oscillations, at which the limit value $K(s)$ is simplified

$$K(s)\approx\frac{1}{2}\ln\frac{16}{1-s^2}=\frac{1}{2}\ln\frac{\upsilon_3-\upsilon_1}{\upsilon_2-\upsilon_1}. \qquad (2.7)$$

In this case, the sequence of wave families diverges with distance and closer to each wave profile at $s=1$ will take form (2.5)

If take in (1.36) $\bar{M}_1=\bar{M}_2=0,\ \theta_*=0,\ \theta_1E_1=\mu_1$, then we'll obtain the Burges equation

$$\frac{\partial\upsilon}{\partial X_1}-\frac{c_1}{c_2}\frac{\partial\upsilon}{\partial X_2}+\upsilon\frac{\partial\upsilon}{\partial\tau}-\mu\frac{\partial^2\upsilon}{\partial\tau^2}=0,\ \mu=R_3\mu_1. \qquad (2.8)$$

In literature [?] the solution of this equation and structure of the wave are well studied. The conditions for the existence of shock waves in solution (2.8) are shown.

The analytical solutions of equation (1.36) in other values $l=3,4$ and etc., with the help of Becklund's transformation are obtained in [12-14].

In the linear approximation equation (1.36) allowing for (1.39) takes the form

$$\frac{\partial\upsilon}{\partial X_1}-\frac{c_1}{c_2}\frac{\partial\upsilon}{\partial X_2}+A_2\frac{\partial^2\upsilon}{\partial\tau^2}+A_3\frac{\partial^3\upsilon}{\partial\tau^3}+A_4\frac{\partial^4\upsilon}{\partial\tau^4}+A_5\frac{\partial^5\upsilon}{\partial\tau^5}+A\frac{\partial^6u}{\partial\tau^6}, \qquad (2.9)$$

where $A_2=R_3S_2, A_3=R_3S_2, A_4=R_3S_4, A_5=R_3S_5, A_6=R_3S_6$.

The harmonic perturbations

$$\upsilon=\upsilon_0\exp i[\omega\tau-(k_1X_1+k_2X_2)] \qquad (2.10)$$

reduce (2.9) to the dispersing correlation

$$-i\left(k_1-k_2\frac{c_1}{c_2}\right)=i\omega\upsilon_*+\omega^2\left(A_2-A_4\omega^2+A_6\omega^4\right)+i\omega^3\left(A_3-A_5\omega^2\right), \qquad (2.11)$$

where υ_* is a value speed, in circumference of which the linearization is realized.

For the existence of preportent frequences it is necessary, that

$$A_2-A_4\omega^2+A_6\omega^4\geq 0. \qquad (2.12)$$

The given calculations show, that seismic signal with spectrum white noise, within the propagation of waves passes to the oscillations of preportent frequency

ω_d, corresponding to the wave number k_d. However, that grows of amplitude can be limited by nonlinear addends of Kdv, where generate the oscillation of higher frequences out of the considered interval, and hence, provide dissipation of waves.

As a kernal of integrodifferential equation (1.37) we take damping exponential function $H(t) = \theta_* \exp(-\theta' t)$ which is equivalent to use of model of viscoelastic Kelvin's soil [3]. Putting this in (1.3.7), often some transformations and linearizations we'll get

$$\frac{\partial v}{\partial X_1} - \frac{c_1}{c_2}\frac{\partial v}{\partial X_2} + \frac{1}{\theta'}\left(\frac{\partial^4 v}{\partial \tau \partial X_1} - \frac{c_1}{c_2}\frac{\partial^2 v}{\partial \tau \partial X_2}\right) + \frac{R_3 \theta_*}{\theta'}\frac{\partial v}{\partial \tau} = 0. \qquad (2.13)$$

Dispersional correlation, corresponding to (2.13) has the form

$$-i\left(k_1 - k_2 \frac{c_1}{c_2}\right) = -\frac{R_3 \theta_* \omega^2}{\omega^2 + \theta'^2} - i\frac{R_3 \theta_* \theta' \omega}{\omega^2 + \theta'^2}. \qquad (2.14)$$

From here, it is easy to see, that in rheological medium the preportent frequences don't exist, since the real part of complex number (2.14) is always less than zero. So, linearly-hereditary soils with exponentially damping memory reduce to the harmonic perturbations.

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