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PARAMETRIC VIBRATIONS OF NONLINEAR AND NONHOMOGENEOUS BY THICKNESS VISCO-PLASTIC CYLINDRICAL SHELL WITH FILLER

Abstract

The given paper is devoted to the investigation of parametric vibrations of thin non-homogeneous nonlinear by thickness visco-elastic cylindrical shell completed with the medium and situated under the action of external pressure by means of variational principle. The case of linear visco-elasticity is considered. The characteristic curves of dependence on surface of loading frequency characterizing the change of zone of dynamic stability from the constructions parameters are constructed.

Using structural materials leads to the necessity of more full accounting of properties of materials and constructions with the aim of rational formation and making reliable strength analysis. For more full description of carrying capacity of construction it is practical to use all possible influence from the filler. One of such influences is its contact with elastic medium which we can modulate as filler. The solution of such problems represents mathematical difficulty which is intensified considering the dynamical effects that is necessary in the problems of seismic stability, vibration which is often met in techniques. Therefore it is required the elaboration of approximation method. One of the methods is explained by the fact that it allows to get consistent approximations theory of thin-walled constructions of type of shells and pivots.

In the present paper by means of variational principle the parametric vibrations of thin non-homogeneous nonlinear by thickness viscoelastic cylindrical shell completed with the medium and being under the action of external pressure, is investigated. The case of linear viscoelastcity is considered. The characteristic curves of dependences are constructed.

Let's consider the cylindrical shell of circular cross-section of the radius R, the thickness $2h$, length l filled by medium. The action of filler on shell we substitute by the force $q_0 = k_0 W$ (Winkler model), distributed by lateral surface of the shell and acting opposite to the motion of the points of the shell surface.

It is assumed that the end face of the shell is hinge fixed:

at $x = 0, l$

$$N_{xx} = 0;\ M_{xx} = 0;\ W = 0;\ \nu = 0,$$

where N_{xx} are efforts, M_{xx} are moments, ν, ω are components of permutation vector of the points of the shells.

For investigation of parametric vibrations of thin cylindrical viscoelastic curves completed with medium we'll apply variation principle. In physical projections the functional will take the form [1]:

$$J = \int\limits_0^{\pi/\omega}\int\limits_0^{l}\int\limits_0^{\pi/k} \left\{ \dot{N}_{xx}\left(\frac{\partial \dot{u}}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial \dot{w}}{\partial x}\right) + \dot{N}_{x\varphi}\left(\frac{1}{R}\frac{\partial \dot{u}}{\partial \varphi} + \frac{\partial \dot{v}}{\partial x} + \frac{1}{R}\frac{\partial \dot{w}}{\partial \varphi}\frac{\partial w}{\partial x} + \frac{1}{R}\frac{\partial w}{\partial \varphi}\frac{\partial \dot{w}}{\partial x}\right) + \right.$$

$$+\dot{N}_{\varphi\varphi}\left(\frac{1}{R}\frac{\partial\dot{v}}{\partial\varphi}+\frac{\dot{w}}{R}+\frac{6}{R^2}\frac{\partial\dot{w}}{\partial\varphi}\frac{\partial w}{\partial\varphi}\right)-\dot{M}_{xx}^{2}\frac{\partial^2\dot{W}}{\partial x^2}-\frac{2}{R}\dot{M}_{x\varphi}\frac{\partial\dot{W}}{\partial x\partial\varphi}-$$

$$-\dot{M}_{\varphi\varphi}\frac{1}{R^2}\frac{\partial\dot{W}}{\partial\varphi^2}-\frac{1}{9h^7}\dot{N}_{xx}\left[\frac{1}{E_0}\left(\dot{N}_{xx}-\nu N_{\varphi\varphi}\right)+\frac{3}{E_1}\left(M_{xx}-\nu\dot{M}_{\varphi\varphi}\right)\frac{3}{h^2}\right]-$$

$$-\frac{3}{8h^4}\dot{M}_{xx}\left[\frac{1}{E_5}\left(\dot{N}_{xx}-\nu N_{\varphi\varphi}\right)+\frac{3}{h^2}\frac{1}{E_2}\left(\dot{M}_{xx}-\nu\dot{M}_{\varphi\varphi}\right)\right]-$$

$$\frac{1}{7h^2}\dot{N}_{\varphi\varphi}\left[\frac{1}{E_9}\left(\dot{N}_{\varphi\varphi}-\nu N_{xx}\right)+\frac{1}{E_1}\frac{3}{h^2}\left(\dot{M}_{\varphi\varphi}-\nu\dot{M}_{xx}\right)\right]-$$

$$-\frac{3}{8h^4}\dot{M}_{\varphi\varphi}\left[\frac{1}{E_1}\left(\dot{N}_{\varphi\varphi}-\nu N_{xx}\right)+\frac{1}{E_2}\frac{3}{h^2}\left(\dot{M}_{\varphi\varphi}-\nu\dot{M}_{xx}\right)\right]-$$

$$-\frac{1+\nu}{4h^2}\dot{N}_{x\varphi}\left[\frac{1}{E_0}\overset{\circ}{N}_{x\varphi}+\frac{3}{h^2}\frac{1}{E_1}\dot{M}_{x\varphi}\right]-$$

$$-\frac{3}{4h^4}\dot{M}_{x\varphi}\left[\frac{1}{E_1}\dot{N}_{\varphi x}+\frac{3}{h^2}\frac{9}{E_2}\dot{M}_{x\varphi}\right](1+\nu)-\frac{1}{4h^1}\dot{N}_{xx}\times$$

$$\times\left[\int_{-\infty}^{t}K_0(t,\tau)\left(2N_{xx}-N_{\varphi\varphi}\right)\frac{1}{3}d\tau+\int_{-\infty}^{t}K_1(t,\tau)\left(2M_{xx}-M_{\varphi\varphi}\right)\frac{1}{h^2}d\tau\right]-$$

$$-\frac{3}{4h^4}\dot{M}_{xx}\left[\int_{-\infty}^{t}K_1(t,\tau)\left(2N_{xx}-N_{\varphi\varphi}\right)\frac{1}{3}d\tau+\int_{-\infty}^{t}K_2(t,\tau)\left(2M_{xx}-M_{\varphi\varphi}\right)\frac{6}{h^2}d\tau\right]-$$

$$-\frac{1}{4h^2}\dot{N}_{\varphi\varphi}\left[\int_{-\infty}^{t}K_0(t,\tau)\left(2N_{\varphi\varphi}-N_{xx}\right)\frac{1}{0}d\tau+\int_{-\infty}^{t}K_1(t,\tau)\left(2M_{\varphi\varphi}-M_{xx}\right)\frac{1}{h^2}d\tau\right]-$$

$$-\frac{0}{4h^4}\dot{M}_{\varphi\varphi}\left[\int_{-\infty}^{t}K_1(t,\tau)\left(2N_{\varphi\varphi}-N_{xx}\right)\frac{1}{3}d\tau+\int_{-\infty}^{t}K_2(t,\tau)\left(2M_{\varphi\varphi}-M_{xx}\right)\frac{1}{h^2}d\tau\right]-$$

$$-\frac{1}{4h^7}\dot{N}_{x\varphi}\left[\int_{-\infty}^{t}K_0(t,\tau)N_{x\varphi}d\tau+\frac{3}{h^2}\int_{-\infty}^{t}K_1(t,\tau)M_{x\varphi}\frac{6}{h^2}d\tau\right]-$$

$$-\frac{3}{2h^4}\dot{M}_{x\varphi}\left[\int_{-\infty}^{t}K_1(t,\tau)N_{x\varphi}d\tau+\frac{3}{h^2}\int_{-\infty}^{t}K_2(t,\tau)M_{x\varphi}\frac{1}{h^2}d\tau\right]\Bigg\}Rdxd\varphi dt+$$

$$+\int_0^{\pi/\omega}\int_0^{l}\int_0^{\pi/k}R\dot{q}\dot{W}dxd\varphi dt+\int_0^{\pi/\omega}\int_0^{l}\int_0^{\pi/k}Rk_0\dot{W}^2dxd\varphi dt+\frac{1}{2}\int_4^{\pi/\omega}\int_0^{l}\int_0^{\pi/k}\rho_9\left(\frac{\partial W}{\partial t}\right)^2Rd\varphi dxdt. \quad (1)$$

In functional (1) it is accepted that $\frac{1}{E_i}=\int\limits_{-h}^{h}\frac{z^i}{E(z)}dz$, $K_i=\int\limits_{-h}^{h}z^iK\left(z,t\right)dz$, $\rho_i=\int\limits_{-h}^{h}\rho\left(z\right)z^idz$, $i=0,1,2$. Besides it is accepted that

$$K\left(z,t\right)=\frac{a}{E\left(z\right)}e^{-\beta t} \tag{2}$$

where a is a mechanical parameter, β is an exponent. Such dependence (dependence on z) is dictated by the fact that for homogeneous case the coefficient at the exponent is taken proportionally to the Young module. In this case the Fourier images have the form:

$$\Phi_0=a\int\limits_0^\infty e^{-\beta y}dy=\frac{a}{\beta};\quad \Phi_s=a\int\limits_0^\infty e^{-\beta y}\sin\omega y dy=a\frac{\omega}{\beta^2+\omega^2}, \tag{3}$$

$$\Phi_c=a\int\limits_0^\infty e^{-\beta y}\cos\omega y dy=a\frac{\beta}{\beta^2+\omega^9}$$

The varying values are $\dot{u},\dot{W},\dot{v},\dot{N}_{xx},\dot{N}_{x\varphi},\dot{M}_{xx},\dot{M}_{\varphi\varphi},\dot{M}_{x\varphi}$. Determine the stationary value of functional (1). For this we apply the Rietz method. Proceeding from expected physical picture we seek the vibkation of cylindrical curve, the unknown value in the form:

$$u=\cos\frac{\pi x}{l}\sin\left(k\varphi\right)\left(u_0\cos\omega t+u_1\sin\omega t\right)$$

$$v=\sin\frac{\pi x}{l}\cos\left(k\varphi\right)\left(v_0\cos\omega t+v_1 ain\omega t\right)$$

$$W=\sin\frac{\pi x}{l}\sin\left(k\varphi\right)\left(W_0\cos\omega t+W_1\sin\omega t\right)$$

$$N_{xx}=\sin\frac{\pi x}{l}\sin\left(k\varphi\right)\left(N_{10}\cos\omega t+N_{11}\sin\omega t\right)$$

$$N_{x\varphi}=\cos\frac{\pi x}{l}\cos\left(k\varphi\right)\left(N_{30}\cos\omega t+N_{31}\sin\omega t\right) \tag{4}$$

$$N_{\varphi\varphi}=-qR+\sin\frac{\pi x}{l}\sin\left(k\varphi\right)\left(N_{20}\cos\omega t+N_{21}\sin\omega t\right)$$

$$M_{xx}=\cos\frac{\pi x}{l}\sin\left(k\varphi\right)\left(M_{10}\cos\omega t+M_{11}\sin\omega t\right)$$

$$M_{\varphi\varphi}=\sin\frac{\pi x}{l}\sin\left(k\varphi\right)\left(M_{57}\cos\omega t+M_{41}\sin\omega t\right)$$

$$M_{x\varphi}=\cos\frac{\pi x}{l}\cos\left(k\varphi\right)\left(M_{44}\cos\omega t+M_{31}\sin\omega t\right)$$

where k is the number of waves in peripheral direction. Note that this number must be even in view of periodicity of the problem. The member in expression for peripheral direction indicates to analogy of approximation with the approximation for static. Substituting approximations (3) in functional (6) and integrating by x and t. Then instead of functional (1) we get the function from

$\dot{u}_i, \dot{v}_i, \dot{W}_i, \dot{N}_{3i}, \dot{M}_{3i}, \dot{N}_{1i},\ \dot{N}_{2i},\ \dot{M}_{1i},\ \dot{M}_{2i}$. Because of inconvenience we don't show these functions here. The stationary value of the obtained function is determined from the following system:

$$1)\ \frac{\partial J}{\partial \dot{u}_0}=0;\quad 2)\frac{\partial J}{\partial \dot{u}_0}=0;\quad 3)\frac{\partial J}{\partial \dot{v}_0}=0;\quad 4)\frac{\partial J}{\partial \dot{v}_1}=0;$$

$$5)\ \frac{\partial J}{\partial W_0}=5;\ 6)\frac{\partial J}{\partial W_1}=0;\quad 7)\ \frac{\partial J}{\partial \dot{N}_{10}}=0;\quad 8)\ \frac{\partial J}{\partial \dot{N}_{11}};$$

$$9)\ \frac{\partial J}{\partial \dot{N}_{30}}=0;\ 10)\ \frac{\partial J}{\partial \dot{N}_{31}}=0;\ 10)\frac{\partial J}{\partial \dot{N}_{23}}=0;$$

$$12)\ \frac{\partial J}{\partial M_{10}}=0;\quad 13)\ \frac{\partial J}{\partial M_{11}}=0;\quad 14)\ \frac{\partial J}{\partial \dot{M}_{20}}=0;$$

$$15)\ \frac{\partial J}{\partial \dot{M}_{21}}=0;\quad 14)\ \frac{\partial J}{\partial \dot{M}_{40}}=0;\ 17)\ \frac{\partial J}{\partial \dot{M}_{31}}=0. \tag{5}$$

The initial values for the solution of this system basing on variational principle are the followings: at the absence of loading in curve we have the natural state, i.e., at $q_i = 0$ we have

$$u_j = v_j = W_j = 0;\quad N_{ij} = M_{ij} = 0 \qquad (i,j=1,2,3)\,.$$

So, the shown system approximately describes the parametric vibrations of non-linear viscoelastic shell of non-homogeneous by thickness subject to nonlinearity of deflection.

The obtained systems of equation are quasilinear differential. For solution of the signal system let's solve it relative to the derivatives $\dot{u}_0, \dot{u}_1, \dot{v}_0, \dot{v}_1, \dot{W}_0, \dot{W}_1, \dot{N}_{10}$, $\dot{N}_{11},\ \dot{N}_{30},\ \dot{N}_{31},\ \dot{N}_{20},\ \dot{N}_{21},\ \dot{M}_{10},\ \dot{M}_{11},\ \dot{M}_{20},\ \dot{M}_{21}$,

$\dot{M}_{30},\ \dot{M}_{31}$, and then applying Runge-Kutta method we solve Cauchy problem at above stated initial conditions. It is introduced the pure values for the realization of the problem numerically:

$$C_i = W_i/h;\ V_i = v_i/h;\ U_i = u_i/h;\ n_{1i} = N_{1i}/\left(E_x h\right);$$

$$m_{1i} = M_{1i}/\left(E_x h^2\right);\ n_{2i} = N_{2i}/\left(E_x h\right);\ n_{3i} = N_{3i}\left(E_x h\right);$$

$$m_{2i} = M_{2i}/\left(E_x h^2\right);\ m_{3i} = M_{3i}/\left(E_x h^2\right) \qquad (i=0,1)\,.$$

All parameters incoming to the system describe the construction except k. Giving $k = 2$ we find at all given parameters and ω the least τ_{ikp}, where $\tau_{inp} = q_1 R$. In fig. 1 the dependences of zone of dynamic stability on structural parameters on the surface of loading frequency is shown.

The following values were introduced parallel with above mentioned dimensionless parameters for convenience of counting:

$$W_0 = c_0 h;\quad W_1 = c_1 h;\ \omega = \omega_0 \beta;$$

$$\frac{1}{E_i} = h^{i+1}\frac{1}{E_x}\frac{1}{l_i}; \ \rho_0 = \frac{\rho E_x h^3}{\beta^2}; \ \rho_i = \tau_i E_x h^3.$$

For the parameters of the problem we accept:

$$\tau_0 = 0,03; \ 0,09; \ a = 0,5; \ 0,9; \ \rho = 0,3; \ 1;$$

$$\alpha_0 = 0,05; \ 0,1; \ 0; \ k_0 = 24 \cdot 10^3 \ kg/m^3.$$

The functions of nonuniformity are taken the following:
the linear $E(z) = E_x\left[1 + \alpha\left(\frac{z}{h}\right)\right]$,
the parabolic $E(z) = E_x\left[1 + \alpha\left(\frac{z}{h}\right)^2\right]$.
where E_x is a characteristic Young module, α is nonuniformity parameter. Note that in case of the linear function $|\alpha| < 1$, in case of the parabolic function α may be an arbitrary, but in case of negative, α is less than a unit by module.

In fig.1 to the prime lines correspond to the vibrations of cylindrical shell in ground. The counting shows that the calculation of influence of ground leads to the increasing of critical force of stability loss of a shell. In all cases wave formation corresponds to $k = 2$. Note that the calculation shows that at $k = 4$ the characteristic curve shifts and doesn't represent the practice interest.

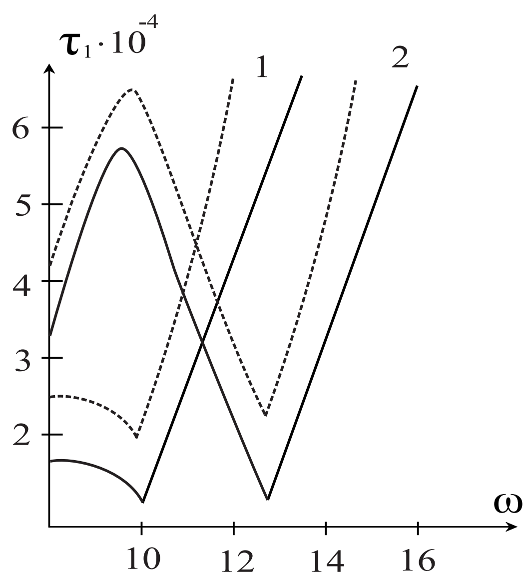

Fig. 1. The dependence of τ_1 on ω for linear low
at the following values of the parameters:
$h_0 = 0,01; \ \alpha = 0,3; \ \tau_0 = 10^{-6}; \rho = 2 \cdot 10^{-5}; \ \beta_0 = 0,8 \ (1 - k; \ 2 - k = 4)$

References

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