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A THIN ROUND VISCOELASTIC DISK IN THE FIELD OF NONSTATIONARY AND NON-HOMOGENOUS TEMPERATURE

Abstract

The components of the displacements vectors of deformations and stress tensors, which arise in a thin viscoelastic continuous disk and disk with central hole under the action of the field of non stationary and non homogenous temperature are defined. The essential influence of non stationary and non homogenous filed of temperature on mechanical properties of disks material has been taken into account. The given problem of thermoviscoelasticity for disk with essentially temperature dependent material properties has been reduced to the problem of thermoviscoelasticity for the disk, the material properties of which don't depend on temperature. An exact analytical solution of the last problem, which is of separate autonomus interest has been obtained.

Let a round disk made of viscoelastic material be physically linearly deformed in the field of non-stationary and non-homogenous temperature $T(r,t)$, where r is a current radius of disk, t is a time. We assume, that the temperature T isn't changed by thickness of the disk. At that, one can assume, that radial displacement and the radial σ_r, hoopential stress σ_φ aren't also changed by thickness. In the considered disk, two-dimensional stress state is realized. The material of the disk we take as mechanically incompressible and we'll assume, that its mechanical properties are described by the following correlations of hereditary type [1]:

$$\frac{S_{ij}}{2G_0} = \omega_1(T)\, e_{ij} - \int_0^t R(t-\tau)\,\omega_2(T)\, e_{ij}\, d\tau, \tag{1}$$

$$\theta = 3\alpha\Delta T. \tag{2}$$

Here $S_{ij} = \sigma_{ij} - \sigma\delta_{ij}$, $e_{ij} = \varepsilon_{ij} - \varepsilon\delta_{ij}$ is stress-deviator σ_{ij} and deformation deviator ε_{ij}; $\sigma = \sigma_{ij}\delta_{ij}/3$ is an average stress, $\varepsilon = \varepsilon_{ij}\delta_{ij}/3$ is an average deformation, δ_{ij} are Kroneker's symbols; $G_0 = const$ is a momentary shear modulus at some standard temperature. T_s, $\omega_1(T)$ and $\omega_2(T)$ are functions, which characterize the influence of the temperature on material properties at momentary loading and reonomial properties of materials, respectively; $R(t)$ is a hereditary function at standard temperature T_s; $\theta = 3\varepsilon$ is a volume deformation; $\alpha = const$ is a coefficient of linear heat extension; $\Delta T = T - T_0$; T_0 is an initial temperature, at which the initial stress and deformations are absent; T is a current disk temperature. The method of experimental definition of the functions $\omega_1(T), \omega_2(T), k(T)$ and constant G_0 is given in [1].

From (1), we see, that properties of disk material essentially depend on temperature T, which is non-homogenous and non-stationary, i.e. dependent on coordinates r and time t. Account of this dependent is realized by two functions $\omega_1(T)$ and $\omega_2(T)$, which are contained in correlations (1). We'll consider disconnected problem of thermoviscoelasticity; the temperature field $T = T(r,t)$ of disk we consider as known from the solution of corresponding heat problem.

With reference to our problem we have:

$$\sigma = \frac{1}{3}(\sigma_r + \sigma_\varphi),\ S_r = \frac{1}{3}(2\sigma_r - \sigma_\varphi);\ S_\varphi = \frac{1}{3}(2\sigma_\varphi - \sigma_r),$$

$$S_z = -\frac{1}{3}(\sigma_r + \sigma_\varphi); \ \varepsilon = \alpha_\Delta T; \ e_r = \varepsilon_r - \alpha_\Delta T;$$

$$e_\varphi = \varepsilon_\varphi - \alpha_\Delta T; \ e_z = \varepsilon_z - \alpha_\Delta T; \ \varepsilon_z = 3\alpha_\Delta T - (\varepsilon_r + \varepsilon_\varphi).$$

Taking into account these formulae determinative equations (1), (2) are transformed to the correlations:

$$\frac{\sigma_r}{2G_0} = \omega_1(T)(2\varepsilon_r + \varepsilon_\varphi - 3\alpha_\Delta T) - \int_0^t R(t-\tau)\,\omega_2(T)(2\varepsilon_r + \varepsilon_\varphi - 3\alpha_\Delta T)\,d\tau, \quad (3)$$

$$\frac{\sigma_\varphi}{2G_0} = \omega_1(T)(2\varepsilon_\varphi + \varepsilon_r - 3\alpha_\Delta T) - \int_0^t R(t-\tau)\,\omega_2(T)(2\varepsilon_\varphi + \varepsilon_r - 3\alpha_\Delta T)\,d\tau. \quad (4)$$

For the statement of the problem it is necessary to join to equations (3) and (4) the equilibrium equation:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\varphi}{r} = 0. \quad (5)$$

Connection between displacement u and deformation components $\varepsilon_r, \varepsilon_\varphi$,

$$\varepsilon_r = \frac{\partial u}{\partial r}, \ \varepsilon_\varphi = \frac{u}{r} \quad (6)$$

and boundary conditions, which for continuous disk of radius b have the form:

$$\sigma_r/_{r=b} = 0, \quad u/_{r=0} = 0, \quad (7)$$

for disk with circular central hole of radius a

$$\sigma_r/_{r=a} = 0, \quad \sigma_r/_{r=b} = 0. \quad (8)$$

The represented work by classification of the authors of the work [2] relates to the third type problems, since it is taken account the essential influence of temperature on mechanical properties of materials, at the same time, temperature field in the disk is nonstationary and non-homogenous. Let's solve the considered problem by the successive approximations method, developed in [3].

Previously we'll solve problem (3)-(7) (or (8)) under the condition $\omega_1(T) = \omega_2(T) = 1$, that is of autonomus interest.

The problem in this case is the problem of linear theory of thermoviscoelasticity of first type [2], in which it is disregarded the mechanical properties of the material, due to change of temperatures; non stationary and non-homogenous field of temperature includes just in correlation (2). Let's rewrite equations (3) and (4) at $\omega_1(T) = \omega_2(T) = 1$;

$$\sigma_r = 2G_0\left[2\varepsilon_r + \varepsilon_\varphi - 3\alpha_\Delta T - \int_0^t R(t-\tau)(2\varepsilon_r + \varepsilon_r - 3\alpha_\Delta T)\,d\tau\right], \quad (9)$$

$$\sigma_r = 2G_0\left[2\varepsilon_\varphi + \varepsilon_r - 3\alpha_\Delta T - \int_0^t R(t-\tau)(2\varepsilon_\varphi + \varepsilon_r - 3\alpha_\Delta T)\,d\tau\right]. \quad (10)$$

Take into account (6) in correlations (9) and (10) with the following substitution of results to the equilibrium equations (5). At that for definition of radial displacement u we'll obtain the equations:

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}=\frac{3}{2}\alpha\frac{\partial(\Delta T)}{\partial r}. \tag{11}$$

It is possible to rewrite equation (11) in the form:

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (ru)}{\partial r}\right]=\frac{3}{2}\alpha\frac{(\Delta T)}{\partial r}. \tag{12}$$

As a result of integrating this equation we'll define u:

$$u=\frac{3\alpha}{2r}\int\limits_{r_0}^{r} r_\Delta T(r,t)\,dr+C_1(t)\,r+\frac{C_2(t)}{2}. \tag{13}$$

Here $r_0=0$ for continuous disk, $r_0=a$ for disk with hole of radius a; $c_1(t)$, $c_2(t)$ are unknown time functions to be defined from the boundary conditions.

Using expression (13) for u we'll define the deformation components ε_r and ε_φ:

$$\varepsilon_r=\frac{3}{2}\alpha_\Delta T(r,t)-\frac{3\alpha}{2r^2}\int\limits_{r_0}^{r} r_\Delta T(r,t)\,dr+C_1(t)-\frac{C_2(t)}{r^2}, \tag{14}$$

$$\varepsilon_\varphi=\frac{3\alpha}{2r^2}\int\limits_{r_0}^{r} r_\Delta T(r,t)\,dr+C_1(t)+\frac{C_2(t)}{r^2}. \tag{15}$$

After finding ε_r and ε_φ, the stress components σ_r and σ_φ are found by formulae (9) and (10):

$$\sigma_r=2G_0\left[-\frac{3\alpha}{2r^2}\int\limits_{r_0}^{r} r_\Delta T(r,t)\,dr+3C_1(t)-\frac{C_2(t)}{r^2}-\right.$$

$$\left.-\int\limits_0^t R(t-\tau)\left(-\frac{3\alpha}{2r^2}\int\limits_{r_0}^{r} r_\Delta T(r,\tau)\,dr+3C_1(\tau)-\frac{C_2(\tau)}{r^2}\right)d\tau\right], \tag{16}$$

$$\sigma_\varphi=2G_0\left[\frac{3\alpha}{2r^2}\int\limits_{r_0}^{r} r_\Delta T(r,t)\,dr+3C_1(t)+\frac{C_2(t)}{r^2}-\frac{3}{2}\alpha_\Delta T(r,t)-\right.$$

$$\left.-\int\limits_0^t R(t-\tau)\left(\frac{3\alpha}{2r^2}\int\limits_{r_0}^{r} r_\Delta T(r,\tau)\,dr+3C_1(\tau)+\frac{C_2(\tau)}{r^2}-\frac{3}{2}\alpha_\Delta T(r,t)\right)d\tau\right]. \tag{17}$$

For definition of the unknown functions $C_1(t)$ and $C_2(t)$ we'll use boundary conditions (7) (for continuous disk) and (8) (for disk with hole of radius a).

First of all let's consider the case of continuous disk.

At that $r_0 = 0$. As $\lim\limits_{r\to 0}\frac{1}{r}\int\limits_0^r r_\Delta T(r,t)\,dr = 0$, from satisfaction of the second boundary condition (7), taking into account (13) it follows, that $C_2(t) = 0$. The first condition (7), using (16) helps us to define the function $C_1(t)$:

$$C_1(t) = \frac{\alpha}{2b^2}\int\limits_0^b r_\Delta T(r,t)\,dr.$$

At that the expressions for stresses σ_r and σ_φ for continuous disk will be:

$$\sigma_r = 2G_0\left[-\frac{3\alpha}{2r^2}\int\limits_0^r r_\Delta T(r,t)\,dr + \frac{3\alpha}{2b^2}\int\limits_0^b r_\Delta T(r,t)\,dr - \right.$$

$$\left. -\int\limits_0^t R(t-\tau)\left(-\frac{3\alpha}{2r^2}\int\limits_0^r r_{\Delta'}T(r,\tau)\,dr + \frac{3\alpha}{2b^2}\int\limits_0^b r_\Delta T(r,\tau)\,dr\right)d\tau\right], \tag{18}$$

$$\sigma_\varphi = 2G_0\left[\frac{3\alpha}{2r^2}\int\limits_0^r r_{\Delta'}T(r,t)\,dr - \frac{3}{2}\alpha_\Delta T(r,t) + \frac{3\alpha}{2b^2}\int\limits_0^b r_\Delta T(r,t)\,dr - \right.$$

$$\left. -\int\limits_0^t R(t-\tau)\left(\frac{3\alpha}{2r^2}\int\limits_0^r r_\Delta T(r,\tau)\,dr - \frac{3}{2}\alpha_{\Delta'}T(r,\tau) + \frac{3\alpha}{2b^2}\int\limits_0^b r_\Delta T(r,\tau)\,dr\right)d\tau\right]. \tag{19}$$

Now, let's define the functions $C_1(t)$ and $C_2(t)$ for disk with hole of radius a. At that $r_0 = a$. Using (16) and (8) we obtain the following expressions for $C_1(t)$ and $C_2(t)$:

$$C_1(t) = \frac{\alpha}{2(b^2-a^2)}\int\limits_a^b r_\Delta T(r,t)\,dr;\; C_2(t) = \frac{3\alpha a^2}{2(b^2-a^2)}\int\limits_a^b r_\Delta T(r,t)\,dr.$$

Taking into account these formulae for $C_1(t)$ and $C_2(t)$ in correlations (13)-(17), define the required quantities $u, \varepsilon_r, \varepsilon_\varphi, \sigma_r, \sigma_\varphi$ for disk with central hole of radius $r_0 = a$.

Let's return to the solution of problem (3)-(7) (or (8)) under the conditions $\omega_1(T) \neq 1$, $\omega_2(T) \neq 1$. The functions $\omega_1(T)$ and $\omega_2(T)$ we'll represent in the form $\omega_1 = 1 - \omega_i^0(T)$ $(i = 1, 2)$.

Under zero approximation we take $\omega_1^0(T) = \omega_2^0(T) = 0$. From here, it follows, that $\omega_1(T) = \omega_2(T) = 1$. At that the definition problem $u^{(0)}, \varepsilon_r^{(0)}, \varepsilon_\varphi^{(0)}, \sigma_r^{(0)}, \sigma_\varphi^{(0)}$ is a problem of linear theory of thermoviscoelasticity of first type [2], the solution of which is obtained above. The problem of any k-th approximation consists of the following correlations:

$$\frac{\sigma_r^{(k)}}{2G_0} = 2\varepsilon_r^{(k)} + \varepsilon_\varphi^{(k)} - 3\alpha_\Delta T - \int\limits_0^t R(t-\tau)\left(2\varepsilon_r^{(k)} + \varepsilon_\varphi^{(k)} - 3\alpha_\Delta T\right)d\tau -$$

$$-\left[\omega_1^0(T)\left(2\varepsilon_r^{(k-1)}+\varepsilon_\varphi^{(k-1)}-3\alpha_\Delta T\right)-\right.$$

$$\left.-\int_0^t R(t-\tau)\,\omega_2^0(T)\left(2\varepsilon_r^{(k-1)}+\varepsilon_\varphi^{(k-1)}-3\alpha_\Delta T\right)d\tau\right] \tag{20}$$

$$\frac{\sigma_r^{(k)}}{2G_0}=2\varepsilon_\varphi^{(k)}+\varepsilon_r^{(k)}-3\alpha_\Delta T-\int_0^t R(t-\tau)\left(2\varepsilon_\varphi^{(k)}+\varepsilon_r^{(k)}-3\alpha_\Delta T\right)d\tau-$$

$$-\left[\omega_1^0(T)\left(2\varepsilon_\varphi^{(k-1)}+\varepsilon_r^{(k-1)}-3\alpha_\Delta T\right)-\right.$$

$$\left.-\int_0^t R(t-\tau)\,\omega_2^0(T)\left(2\varepsilon_r^{(k-1)}+\varepsilon_\varphi^{(k-1)}-3\alpha_\Delta T\right)d\tau\right] \tag{21}$$

$$\frac{\partial\sigma_r^{(k)}}{\partial r}+\frac{\sigma_r^{(k)}-\sigma_\varphi^{(k)}}{r}=0, \tag{22}$$

$$\varepsilon_r^{(k)}=\frac{\partial u^{(k)}}{\partial r},\quad \varepsilon_\varphi^{(k)}=\frac{u^{(k)}}{r}, \tag{23}$$

$$\sigma_r^{(k)}/_{r=b}=0,\quad u^{(k)}/_{r=0}=0\quad \text{(continuous disk)}, \tag{24}$$

$$\sigma_r^{(k)}/_{r=a}=0,\quad \sigma_r^{(k)}/_{r=b}=0\quad \text{(disk with hole } a). \tag{25}$$

The solution of this problem we'll represent in the form:

$$u^{(k)}=\bar{u}^{(k)},\ \varepsilon_r^{(k)}=\bar{\varepsilon}_r^{(k)}=\frac{\partial\bar{u}^{(k)}}{\partial r},\ \varepsilon_\varphi^{(k)}=\bar{\varepsilon}_\varphi^{(k)}=\frac{\bar{u}^{(k)}}{r}, \tag{26}$$

$$\sigma_r^{(k)}=\bar{\sigma}_r^{(k)}-2G_0\left[\omega_1^0(T)\left(2\varepsilon_r^{(k-1)}+\varepsilon_\varphi^{(k-1)}-3\alpha\Delta T\right)-\right. \tag{27}$$

$$\left.-\int_0^t R(t-\tau)\,\omega_2^0(T)\left(2\varepsilon_r^{(k-1)}+\varepsilon_\varphi^{(k-1)}-3\alpha_\Delta T\right)d\tau\right],$$

$$\sigma_\varphi^{(k)}=\bar{\sigma}_\varphi^{(k)}-2G_0\left[\omega_1^0(T)\left(2\varepsilon_\varphi^{(k-1)}+\varepsilon_r^{(k-1)}-3\alpha\Delta T\right)-\right. \tag{28}$$

$$\left.-\int_0^t R(t-\tau)\,\omega_2^0(T)\left(2\varepsilon_\varphi^{(k-1)}+\varepsilon_r^{(k-1)}-3\alpha_\Delta T\right)d\tau\right].$$

Taking into account (26)-(28) in correlations (21)-(25) we'll obtain the following problem for definition of $\bar{u}^{(k)},\bar{\varepsilon}_r^{(k)},\bar{\varepsilon}_\varphi^{(k)},\bar{\sigma}_r^{(k)},\bar{\sigma}_\varphi^{(k)}$:

$$\bar{\sigma}_r^{(k)}=2G_0\left[2\bar{\varepsilon}_r^{(k)}+\bar{\varepsilon}_\varphi^{(k)}-3\alpha\Delta T-\int_0^t R(t-\tau)\left(2\bar{\varepsilon}_r^{(k)}+\bar{\varepsilon}_\varphi^{(k)}-3\alpha\Delta T\right)d\tau\right], \tag{29}$$

$$\bar{\sigma}_{\varphi}^{(k)} = 2G_0 \left[2\bar{\varepsilon}_{\varphi}^{(k)} + \bar{\varepsilon}_r^{(k)} - 3\alpha\Delta T - \int_0^t R(t-\tau) \left(2\bar{\varepsilon}_{\varphi}^{(k)} + \bar{\varepsilon}_r^{(k)} - 3\alpha\Delta T \right) d\tau \right], \quad (30)$$

$$\frac{\partial \bar{\sigma}_r^{(k)}}{\partial r} + \frac{\bar{\sigma}_r^{(k)} - \bar{\sigma}_{\varphi}^{(k)}}{r} - 2G_0 F^{(k-1)}(r,t) = 0, \quad (31)$$

$$\bar{\varepsilon}_r^{(k)} = \frac{\partial \bar{u}^{(k)}}{\partial r}, \quad \bar{\varepsilon}_{\varphi}^{(k)} = \frac{\bar{u}^{(k)}}{r}, \quad (32)$$

$$\bar{\sigma}_r^{(k)}|_{r=b} = 2G_0 Q^{(k-1)}(r,t)|_{r=b}, \quad \bar{u}^{(k)}|_{r=0} = 0, \quad (33)$$

$$\bar{\sigma}_r^{(k)}|_{r=a} = 2G_0 Q^{(k-1)}(r,t)|_{r=a}, \quad \bar{\sigma}_r^{(k)} = 2G_0 Q^{(k-1)}(r,t)|_{r=b}. \quad (34)$$

Here the following denotation is taken:

$$F^{(k-1)}(r,t) = \frac{\partial Q^{(k-1)}(r,t)}{\partial r} \frac{1}{2} \left[\omega_1^0(T) \left(\varepsilon_{\varphi}^{(k-1)} - \varepsilon_r^{(k-1)} \right) - , \right.$$

$$\left. - \int_0^t R(t-\tau)\, \omega_2^0(T) \left(\varepsilon_{\varphi}^{(k-1)} - \varepsilon_r^{(k-1)} \right) d\tau \right],$$

$$Q^{(k-1)}(r,t) = \omega_1^0(T) \left(2\varepsilon_r^{(k-1)} + \varepsilon_{\varphi}^{(k-1)} - 3\alpha\Delta T \right) -$$

$$- \int_0^t R(t-\tau)\, \omega_2^0(T) \left(2\varepsilon_r^{(k-1)} + \varepsilon_{\varphi}^{(k-1)} - 3\alpha\Delta T \right) d\tau.$$

Problem (29)-(34) differs from the problem at zero approximation only by availability of additional “volume” and ”surface” forces. Let’s take into account (32) in (29) and (30) and we’ll use the obtained expressions for $\bar{\sigma}_r^{(k)}, \bar{\sigma}_{\varphi}^{(k)}$ in equation (31). At that for the definition of the quantity $\bar{u}^{(k)}$ we’ll have the equation:

$$\frac{\partial^2 \bar{u}^{(k)}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}^{(k)}}{\partial r} - \frac{\bar{u}^{(k)}}{r^2} = \frac{3}{2}\alpha \frac{\partial (\Delta T)}{\partial r} + \frac{1}{2}\frac{\partial f^{(k-1)}(r,t)}{\partial r}, \quad (35)$$

where

$$f^{(k-1)}(r,t) = \int_{r_0}^{r} \left[F^{(k-1)}(r,t) + \int_0^t \Gamma(t-\tau)\, F^{(k-1)}(r,\tau)\, d\tau \right] dr,$$

the function $\Gamma(t)$ is a resolvent of $R(t)$. The solution of equation (35) is the following:

$$\bar{u}^{(k)} = \frac{3}{2}\alpha \frac{1}{r} \int_{r_0}^{r} r\Delta T(r,t)\, dr + \frac{1}{2r} \int_{r_0}^{r} r f^{(k-1)}(r,t)\, dr + C_1^{(k)} + \frac{C_2^{(k)}(t)}{r}. \quad (36)$$

From here we define the components $\bar{\varepsilon}_r^{(k)}, \bar{\varepsilon}_\varphi^{(k)}$:

$$\bar{\varepsilon}_r^{(k)} = \frac{3}{2}\alpha\Delta T(r,t) - \frac{3\alpha}{2r^2}\int_{r_0}^{r} r\Delta T(r,t)\,dr + \frac{1}{2}f^{(k-1)}(r,t) -$$

$$-\frac{1}{2r^2}\int_{r_0}^{r} rf^{(k-1)}(r,t)\,dr + C_1^{(k)} + \frac{C_2^{(k)}(t)}{r^2}, \tag{37}$$

$$\bar{\varepsilon}_\varphi^{(k)} = \frac{3\alpha}{2r^2}\int_{r_0}^{r} r\Delta T(r,t)\,dr + \frac{1}{2r^2}\int_{r_0}^{r} rf^{(k-1)}(r,t)\,dr + C_1^{(k)}(t) + \frac{C_2^{(k)}(t)}{r^2}. \tag{38}$$

Using (37) and (38) in correlations (29) and (30) we find the expressions for $\bar{\sigma}_r^{(k)}$ and $\bar{\sigma}_\varphi^{(k)}$:

$$\bar{\sigma}_r^{(k)} = G_0\left[-\frac{3\alpha}{r^2}\int_{r_0}^{r} r\Delta T(r,t)\,dr + 2f^{(k-1)}(r,t) - \frac{1}{r^2}\int_{r_0}^{r} rf^{(k-1)}(r,t)\,dr+\right.$$

$$+6C_1^{(k)}(t) - \frac{2C_2^{(k)}(t)}{r^2} - \int_0^t R(t-\tau)\left(-\frac{3\alpha}{r^2}\int_{r_0}^{r} r\Delta T(r,\tau)\,dr + 2f^{(k-1)}(r,\tau) -\right.$$

$$\left.\left.-\frac{1}{r^2}\int_{r_0}^{r} rf^{(k-1)}(r,\tau)\,d\tau + 6C_1^{(k)}(\tau) - \frac{2C_2^{(k)}(\tau)}{r^2}\right)d\tau\right], \tag{39}$$

$$\bar{\sigma}_\varphi^{(k)} = G_0\left[\frac{3\alpha}{r^2}r\Delta T(r,t)\,dr - 3\alpha T(r,t) + \frac{1}{r^2}\int_{r_0}^{r} rf^{(k-1)}(r,t)\,dr+\right.$$

$$+f^{(k-1)}(r,t) + 6C_1^{(k)}(t) + \frac{2C_2^{(k)}(t)}{r^2}-$$

$$-\int_0^t R(t-\tau)\left(\frac{3\alpha}{r^2}\int_{r_0}^{r} r\Delta T(r,\tau)\,dr - 3\alpha\Delta T(r,\tau) +\right.$$

$$\left.\left.+\frac{1}{r^2}\int_{r_0}^{r} rf^{(k-1)}(r,\tau)\,dr + f^{(k-1)}(r,\tau) + 6C_1^{(k)}(\tau) + \frac{2C_2^{(k)}(\tau)}{r^2}\right)d\tau\right]. \tag{40}$$

Define the unknown functions $C_1^{(k)}(t)$ and $C_2^{(k)}(t)$. Using the boundary conditions (33), for continuous disk we find:

$$C_2^{(k)}(t) = 0;\ C_1^{(k)}(t) = \frac{\alpha}{2b^2}\int_{r_0}^{b} r\Delta T(r,t)\,dr - \frac{1}{3}f^{(k-1)}(b,t) +$$

$$+\frac{1}{6b^2}\int_0^b rf^{(k-1)}(r,t)\,dr+Q^{(k-1)}(b,t)+\int_0^t\Gamma(t-\tau)\,Q^{(k-1)}(b,\tau)\,dr. \qquad (41)$$

Find the expressions $C_1^{(k)}(t)$ and $C_2^{(k)}(t)$ for disk with central hole of radius a, applying boundary conditions (34) and correlations (39);

$$C_2^{(k)}(t)=\frac{a^2b^2}{b^2-a^2}\left[f^{(k-1)}(b,t)-f^{(k-1)}(a,t)-\frac{3\alpha}{2b^2}\int_a^b r\Delta T(r,t)\,dr-\right.$$

$$-\frac{1}{2b^2}\int_a^b rf^{(k-1)}(r,t)\,dr+Q^{(k-1)}(a,t)-Q^{(k-1)}(b,t)+$$

$$+\int_0^t\Gamma(t-\tau)\left(Q^{(k-1)}(a,\tau)-Q^{(k-1)}(b,\tau)\right)d\tau, \qquad (42)$$

$$C_1^{(k)}(t)=$$

$$=\frac{1}{3}\left[\frac{C_2^{(k)}(t)}{a^2}-f^{(k-1)}(a,t)+Q^{(k-1)}(a,t)+\int_0^t\Gamma(t-\tau)\,Q^{(k-1)}(a,\tau)\,d\tau\right]. \qquad (43)$$

After definition of unknown functions $C_1^{(k)}(t)$ and $C_2^{(k)}(t)$, formulae (36)-(40) the quantities $\bar{u},\bar{\varepsilon}_r^{(k)},\bar{\varepsilon}_\varphi^{(k)},\bar{\sigma}_r^{(k)},\bar{\sigma}_\varphi^{(k)}$ and also the required solution of the problem of the k -th approximation $u^{(k)}=\bar{u}^{(k)},\bar{\varepsilon}_r^{(k)},\bar{\varepsilon}_\varphi^{(k)}=\bar{\varepsilon}_\varphi^{(k)}$ will be known. We find the quantities $\sigma_r^{(k)}$ and $\sigma_\varphi^{(k)}$ by formulae (27) and (28).

The obtained solution is an exact solution of the k -th approximation. By already known ε_r and ε_φ we find $\varepsilon_z:\varepsilon_z=3\alpha\Delta T-(\varepsilon_r+\varepsilon_\varphi)$.

Finally, we'll remark, that under the conditions $|\omega_1(T)|\le M_1,|\omega_2(T)|\le M_2$, where $M_1,M_2=const$, the uniform convergence of the given here approximations is proved.

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Received December 17, 2003; Revised April 13, 2004.
Translated by Mamedova Sh.N.