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THE REMOVABLE SETS OF THE SOLUTIONS OF DEGENERATE PARABOLIC EQUATIONS

Abstract

In the paper, the necessary and sufficient condition of removability of a compact with regard to a first boundary-value problem for second order degenerate divergent parabolic equations in the space of bounded functions is established.

Introduction. Let E_n and R_{n+1} be Euclidian spaces of the points $x=(x_1,...,x_n)$ and $(x,t)=(x_1,...,x_n,t)$, respectively. $\Omega\subset E_n$ be a bounded domain with boundary $\partial\Omega$.

Q_T is a cylindrical domain $\Omega\times(0,T)$, lying in the Euclidian space R^{n+1}, $S_T=\partial\Omega\times[0,T]$, $Q_0=\{(x,t):x\in\Omega,t=0\}$. Let's consider in Q_T the parabolic equation

$$Lu=\frac{\partial u}{\partial t}-\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(a_{ij}\left(x,t\right)\frac{\partial u}{\partial x_j}\right)=f\left(x,t\right), \tag{1}$$

$$u|_{\Gamma(Q_T)}=0, \tag{2}$$

where $\Gamma\left(Q_T\right)$ is a parabolic boundary consisting of lateral surface S_T and lower base. Concerning to coefficients we assume, that $\|a_{ij}\|\left(x\right)$ is a real symmetrical matrix with the measurable elements in Q_T and for all $(x,t)\in Q_T$ and $\xi\in E_n$ the following condition is fulfilled

$$\gamma\sum_{i=1}^{n}\lambda_i\left(x,t\right)\xi_i^2\leq\sum_{i,j=1}^{n}a_{ij}\left(x,t\right)\xi_i\xi_j\leq\gamma^{-1}\sum_{i=1}^{n}\lambda_i\left(x,t\right)\xi_i^2, \tag{3}$$

where $\gamma\in(0,1]$ is a constant, $\lambda_i\left(x,t\right)=\left(|x|_\alpha+\sqrt{|t|}\right)^{\alpha_i}, |x|_\alpha=\sum\limits_{i=1}^{n}|x_i|^{\bar{\alpha}_i}$, $\bar{\alpha}_i=\dfrac{2}{2+\alpha_i}$, $\alpha=(\alpha_1,...,\alpha_n)$, $\alpha_i\geq0$, $i=1,...,n$, and $0\leq\alpha_i<\dfrac{2}{n-1}$, $i=1,...,n$.

Concerning the right-hand side we assume, that $f\left(x,t\right)\in L_2\left(Q_T\right)$.

Denote by $\alpha^-=\min\{\alpha_1,...,\alpha_n\}$, $\alpha^+=\max\{\alpha_1,...,\alpha_n\}$. Denote by $A\left(Q_T\right)$ the set of smooth functions $u\left(x,t\right)\in C^\infty\left(\bar{Q}_T\right)$ for which it is possible to find the domain $\Omega\left(u\right)$ such that $\bar{\Omega}\left(u\right)\subset\Omega$ and $suppu\in\Omega\left(u\right)\times[0,T]$. Let's denote by $\mathring{W}_{2,\alpha}^{1,0}\left(Q_T\right)$ and $\mathring{W}_{2,\alpha}^{1,1}\left(Q_T\right)$ the completion of $A\left(Q_T\right)$ by norms

$$\|u\|_{\mathring{W}_{2,\alpha}^{1,0}(Q_T)}=\left[vrai\max_{t\in[0,T]}\int_\Omega u^2dx+\sum_{i=1}^{n}\int_{Q_T}\lambda_i\left(x,t\right)\left(\frac{\partial u}{\partial x_i}\right)^2dxdt\right]^{1/2}$$

and

$$\|u\|_{\overset{\circ}{W}{}^{1,1}_{2,\alpha}(Q_T)}=\left(\int\limits_{Q_T}\left(u^2+\sum_{i=1}^{n}\int\limits_{Q_T}\lambda_i\left(x,t\right)\left(\frac{\partial u}{\partial x_i}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2\right)dxdt\right)^{1/2}$$

respectively.

Then denote by $u_t=\dfrac{\partial u}{\partial t}$, $u_i=\dfrac{\partial u}{\partial x_i}$, $i=1,...,n.$

The set of all bounded functions in Q_T we'll denote by $M\left(Q_T\right)$.

The function $u\left(x,t\right)\in\overset{\circ}{W}{}^{1,0}_{2,\alpha}\left(Q_T\right)$ is called a generalized solution of problem (1)-(2), if for any function $\eta\left(x,t\right)\in\overset{\circ}{W}{}^{1,1}_{2,\alpha}\left(Q_T\right)$ and for any $t_1\in[0,T]$ the integral identity is fulfilled

$$\int\limits_{\Omega}u\left(x,t_1\right)\eta\left(x,t_1\right)dx-\int\limits_{Q_{t_1}}u\eta_t dxdt+\int\limits_{Q_{t_1}}\sum_{i,j=1}^{n}a_{ij}\left(x,t\right)u_i\eta_j dxdt=\int\limits_{Q_{t_1}}f\eta dxdt, \qquad (4)$$

where $Q_{t_1}=\Omega\times(0,t_1)$. Everywhere further the notation $C\left(\cdot\right)$ means, that the positive constant C depends only on constants of the brackets. Let's introduce the spaces $\overset{\circ}{W}{}^{1,0}_{p,\alpha}$ and $L_{p,\omega}\left(Q_T\right)$. Let $\omega\left(x,t\right)$ be a measurable function in Q_T, finite and positive for a.e. $(x,t)\in Q_T$. Denote by $L_{p,\omega}\left(Q_T\right)$ a Banach space of the functions $u\left(x,t\right)$, given on Q_T, with the finite norm:

$$\|u\|_{L_{p,\omega}(Q_T)}=\left(\int\limits_{Q_T}\left(\omega\left(x,t\right)\right)^{p/2}\left|u\left(x,t\right)\right|^p dxdt\right)^{1/p},\ 1<p<\infty.$$

$\overset{\circ}{W}{}^{1,0}_{p,\alpha}\left(Q_T\right)$ is a Banach space of the functions $u\left(x,t\right)$, given on Q_T, with the finite norm:

$$\|u\|_{W^{1,0}_{p,\alpha}(Q_T)}=\left[\int\limits_{Q_T}\left|u\right|^p dxdt+\sum_{i=1}^{n}\int\limits_{Q_T}\left(\lambda_i\left(x,t\right)\right)^{p/2}\left|u_i\right|^p dxdt\right]^{1/p},$$

$1<p<\infty$. Now, similar to $\overset{\circ}{W}{}^{1,0}_{2,\alpha}\left(Q_T\right)$ it is introduced the subspace $\overset{\circ}{W}{}^{1,0}_{p,\alpha}\left(Q_T\right)$ for $p\in(1,\infty)$. The space adjoint to $\overset{\circ}{W}{}^{1,0}_{p,\alpha}\left(Q_T\right)$ we'll denote by $\overset{*}{W}{}^{1,0}_{p,\alpha}\left(Q_T\right)$. Let $E\subset Q_T$ be some compact. Let's denote by $A_E\left(Q_T\right)$ the totality of all functions $u\left(x,t\right)\in C^{\infty}\left(\bar{Q}_T\right)$ and for each of them there exists some neighbourhood of the compact E, in which $u\left(x,t\right)=0$. The function $u\left(x,t\right)\in W^{1,0}_{2,\alpha}\left(Q_T\setminus E\right)$ is called a generalized solution of the equation $Lu=f\left(x\right)$ in $Q_T\setminus E$, vanishing on $\Gamma\left(Q_T\right)$, if integral indentity (4) is fulfilled for any function $\eta\left(x\right)\in A_E\left(Q_T\right)$.

The compact E is called removable with regard to first boundary-value problem for the operator L in the space $M\left(Q_T\right)$, if any generalized solution of the equation $Lu=0$ in $Q_T\setminus E$, vanishing on $\Gamma\left(Q_T\right)$ and belonging to the space $M\left(Q_T\right)$, is identically equal to zero.

The purpose of the present paper is finding necessary and sufficient condition of removability of the compact E in above indicated sense.

The questions of removability for uniformly elliptic equations of divergent structure have been considered in the papers [1], [2], [3] for uniformly degenerate elliptic equations in the papers [4], [5], for non uniformly degenerate elliptic equations in the paper [6]. Let's remind also the paper [7]. Removability of the compacts for solution of parabolic equations is well studied in the class of bounded and Hőlder functions [8], [9], [10], [11]. At that the condition of removability is described subiect to the type of equation in terms of equality to zero of corresponding Hausdorff capacities and measures.

Let's consider, that the coefficients of the operator L are continued in $R_{n+1}\setminus Q_T$ with preservation of conditions (3). For this it is sufficient, for example, to suppose $a_{ij}(x,t)=\delta_{ij}\lambda_i(x,t)$ for $(x,t)\in R_{n+1}\setminus Q_T$, $i,j=1,...,n$, where δ_{ij} is a Kroneker's symbol. For $x^0\in E_n,\ R>0$ and $k>0$ we'll denote by $\mathcal{E}_{R,\,k}\left(x^0\right)$ the ellipsoid $\left\{x:\sum\limits_{i=1}^{n}\dfrac{\left(x_i-x_i^0\right)^2}{R^{\alpha_i}}<(kR)^2\right\}\cdot\mathcal{E}$ is such an ellipsoid, that $\bar{Q}_T\subset\mathcal{E}$, and $B(\mathcal{E})$ is a set of functions, satisfying the uniform $\overline{\mathcal{E}}$ Lipshit's condition in 0 and vanishing close to $\partial\mathcal{E}$.

We'll say, that the function $u(x,t)\in\overset{\circ}{W}{}_{2,\alpha}^{1,0}(\mathcal{E})$ is non negative on the set $H\subset\mathcal{E}$ in the sense of functions $\overset{\circ}{W}{}_{2,\alpha}^{1,0}(\mathcal{E})$ such that $\{u_m(x,t)\}, m=1,2,...$, for $u_{(m)}(x,t)\in B(\mathcal{E})$, $u_{(m)}(x,t)\geq 0$ for $(x,t)\in H$ and $\lim\limits_{m\to\infty}\|u_m(x,t)-u(x,t)\|_{W_{2,\alpha}^{1,0}(\mathcal{E})}=0$.

We'll say that the function $u(x,t)\in\overset{\circ}{W}{}_{2,\alpha}^{1,0}(Q_T)$ is non negative on $\Gamma(Q_T)$ in the sense of $\overset{\circ}{W}{}_{2,\alpha}^{1,0}(Q_T)$ if there exists sequence of the functions $\{u_{(m)}(x,t)\}$, $m=1,2,...$, such, that $u_{(m)}(x,t)\in C^{(1)}\left(\bar{Q}_T\right)$, $u_{(m)}(x,t)\geq 0$ for $(x,t)\in\Gamma(Q_T)$ and $\lim\limits_{m\to\infty}\|u_m(x,t)-u(x,t)\|_{W_{2,\alpha}^{1,0}(Q_T)}=0$. Similarly it is possible to define the inequalities $u(x,t)\geq const,\ u(x,t)\geq v(x,t),\ u(x,t)\leq 0$ and also $u(x,t)=1$ on the set H in the sense of $\overset{\circ}{W}{}_{2,\alpha}^{1,0}(\mathcal{E})$.

Let $h(x,t)\in\overset{\circ}{W}{}_{2,\alpha}^{1,0}(Q_T)$, $f^0(x,t)\in L_2(Q_T)$, $f^i(x,t)\in L_{2,\lambda_i^{-1}}(Q_T)$, $i=1,...,n$, be given functions. Let's consider the first boundary-value problem

$$Lu=f^0(x,t)+\sum_{i=1}^{n}\frac{\partial f^i(x,t)}{\partial x_i},\ (x,t)\in Q_T, \tag{5}$$

$$(u(x,t)-h(x,t))\in\overset{\circ}{W}{}_{2,\alpha}^{1,0}(Q_T). \tag{6}$$

The function $u(x,t)\in\overset{\circ}{W}{}_{2,\alpha}^{1,0}(Q_T)$ is called a generalized solution of problem (5)-(6) if for any function $\eta(x)\in\overset{\circ}{W}{}_{2,\alpha}^{1,0}(Q_T)$ the integral identity (4) is fulfilled with corresponding right-side, i.e. in (4) the right-hand side will be in the form:

$$\int\limits_{Q_{T_1}}\left[f^0(x,t)\,\eta dxdt-\sum_{i=1}^{n}f^i(x,t)\,\eta_i\right]dxdt.$$

Lemma 1. *If with respect to coefficients of the operator L conditions (3) are fulfilled, then the first boundary-value problem has a unique generalized solution $u(x,t) \in \overset{\circ}{W}{}^{1,0}_{2,\alpha}(Q_T)$ for $h(x) \in W^{1,0}_{2,\alpha}(Q_T)$, $f^0(x) \in L_2(Q_T)$, $f^i(x) \in L_{2,\lambda_i^{-1}}(Q_T)$, $i=1,...,n$.*

Proof. We'll show the scheme of the proof. First of all we prove the estimation for $u(x,t) \in \overset{\circ}{W}{}^{1,0}_{2,\alpha}(Q_T)$

$$\int\limits_{Q_{t_1}} u^2 dxdt \le C_1(\alpha,\Gamma) \int\limits_{Q_{t_1}} \sum_{i=1}^{n} \lambda_i(x,t)\, u_i^2 dxdt, \tag{7}$$

as $t_1 \in [0,T]$. This estimation is obtained from representation of $u(x,t)$ in the form of the integral:

$$u(x_1,x',t') = \int\limits_{-R}^{x_1} u_1(\tau,x',t')\,d\tau, \text{ where } x' = (x_2,...,x_n),\ t' \in (0,t_1).$$

Then we'll apply Hőlder's inequality and estimate the integrals with weight, using condition on the weight α_i

$$\int\limits_{-R}^{x_1} \frac{d\tau}{\lambda_1(\tau,x',t')} \le \int\limits_{-R}^{x_1} \frac{d\tau}{\left[\sqrt{|t'|} + |\tau|^{\bar\alpha_1} + \sum\limits_{i=2}^{n} |x_i|^{\bar\alpha_i}\right]^{\alpha_i}} \le \int\limits_{-R}^{R} \frac{d\tau}{|\tau|^{2\alpha_1/(1+\alpha_1)}},$$

and as $\dfrac{2\alpha_1}{2+\alpha_1} < 1$, then $\int\limits_{-R}^{R} \dfrac{d\tau}{\lambda_1(\tau,x',t')} \le C_2(\alpha,R)$.

To prove the existence and uniqueness of the solution we first of all pass to the smooth domains and smooth coefficients. We use the weight estimations of type (7) and then passing to the limit in the norm $\overset{\circ}{W}{}^{1,0}_{2,\alpha}(Q_T)$ we obtain the required.

For example, let's show the uniqueness of the solution. Let's assume, that there exist two solutions $u^{(1)}(x,t)$ and $u^{(2)}(x,t)$ of our problem. Let $u(x,t) = u^{(1)}(x,t) - u^{(2)}(x,t)$. Then for any function $\eta(x,t) \in \overset{\circ}{W}{}^{1,0}_{2,\alpha}(Q_T)$ which vanishes to zero at $t=T$ the following identity is fulfilled:

$$-\int\limits_{Q_T} u\eta_t dxdt + \int\limits_{Q_T} \sum_{j=1}^{n} a_{ij}(x,t)\, u_i \eta_j dxdt = 0. \tag{8}$$

Let's fix an arbitrary $\delta \in (0, T_0+T)$ and suppose, that $\eta(x,t)$ vanishes at 0 and $t \le -T_0$ and $t \ge T-\delta$. Denote for $h \in (0,\delta]$ $\eta_{\bar h}(x,t) = \dfrac{1}{h}\int\limits_{t-h}^{t} \eta(x,t)\,d\tau$ and substitute in (8) $\eta = \eta_{\bar h}$. We'll obtain

$$-\int\limits_{Q_T} u\,(\eta_{\bar h})_t\, dxdt + \int\limits_{Q_T} \sum_{i,j=1}^{n} a_{ij}(x,t)\, u_i\,(\eta_{\bar h})_j\, dxdt = 0. \tag{9}$$

Taking into account $(\eta_{\bar h})_t = (\eta_t)_{\bar h}$, $(\eta_{\bar h})_j = (\eta_j)_{\bar h}$ and that

$$-\int\limits_{Q_T} u\,(\eta_t)_{\bar h}\,dxdt = -\int\limits_{Q_{T-\delta}} u_h \eta_t dxdt = \int\limits_{Q_{T-\delta}} (u_h)_t\,\eta dxdt,$$

$$\int\limits_{Q_T}\sum_{i,j=1}^n a_{ij}(x,t)\,u_i\,(\eta_j)_{\bar h}\,dxdt = \int\limits_{Q_{T-\delta}}\sum_{i,j=1}^n (a_{ij}(x,t)\,u_i)_h\,\eta_j dxdt,$$

from (9) we have

$$\int\limits_{Q_T} (u_h)_t\,\eta dxdt + \int\limits_{Q_{T-\delta}}\sum_{i,j=1}^n (a_{ij}(x,t)\,u_i)_h\,\eta_j dxdt = 0, \tag{10}$$

here $u_h(x,t) = \frac{1}{h}\int\limits_t^{t+h} u(x,\tau)$. From the paper [12] it follows, that (10) is true for any function $\eta(x,t) \in \overset{\circ}{W}{}^{1,0}_{2,\alpha}(Q_{T-\delta})$. Thus, substituting to (10) $\eta(x,t) = u_h(x,t)$ and tending h to zero, we'll obtain

$$\frac{1}{2}\int\limits_{\Omega} u^2(x,T-\delta)\,dx + \int\limits_{Q_{T-\delta}}\sum_{i,j=1}^n a_{ij}(x,t)\,u_i u_j dxdt = 0.$$

From condition (3) we'll obtain

$$\int\limits_{\Omega} u^2(x,T-\delta)\,dx = 0.$$

As δ is arbitrary from $(0,T_0+T)$, we'll obtain $\int\limits_{Q_T} u^2(x,t)\,dxdt = 0$ i.e. $u(x,t) = 0$ almost everywhere. The uniqueness is proved.

The lemma is proved.

Lemma 2. *Let with respect to coefficients of the operator L conditions (3) be fulfilled and $\mathcal{E}_{R,1}(0)\times(0,T)\subset Q_T$. Then for any generalized solution $u(x,t)\in \overset{\circ}{W}{}^{1,0}_{2,\alpha}(Q_T)$ of equation (1) in Q_T the inequality of Harnack's type is true*

$$\sup_{\mathcal{E}_{R,1}(0)\times(0,T)} u \le C_1(\gamma,\alpha,\eta)\inf_{\mathcal{E}_{R,1}(0)\times(0,T)} u, \tag{11}$$

and if at that $(y,t)\in\partial\mathcal{E}_{R,2}(0)\times(0,T)$ and $\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\subset Q_T$, then inequality of type (11) is true in $\mathcal{E}_{R,1}(y)\times(0,T)$.

The proof of this lemma carried out closer to ideas of the paper [13] taking into account the features of the weight $\lambda_i(x,t)$. For example, for a function from $A_E(Q_T)$, where $E=\mathcal{E}_{R,1}(0)\times(t_1,t_2)$ and $\sigma=\frac{2k-1}{k}$ at R_0 such, that $R\le R_0$, the inequality is true

$$\left(\oint\limits_E u^{2\sigma}dxdt\right)^{1/\sigma} \le C_1\left(\max_{t_1\le t\le t_2}\oint\limits_{\mathcal{E}_{R,1}(0)} u^2dx+\right.$$

$$+R^2 \oint\limits_E \sum_{i=1}^{n} \lambda_i (x,t)\, u_i^2 dxdt \Bigg)$$

here $\oint\limits_E u^{2\sigma} dxdt = \frac{1}{mesE} \int\limits_E udxdt$. Then to the integral identity we substitute $\eta(x,t) = u^{\frac{\beta+1}{2}}$ as $\beta \neq 0, -1$ and $\eta(x,t) = -\ln u$, as $\beta = -1$.

Further estimations are done similar to [13].

Lemma 3. *Let with respect to coefficients of the operator L conditions (3) be fulfilled. Then there exists $p_0(\alpha, n)$ such, that if $p > p_0$, $h(x) \in W_{p,\alpha}^{1,0}(Q_T)$, $f^0(x,t) \in L_p(Q_T)$, $f^i(x,t) \in L_{p,\lambda_i^{-1}}(Q_T)$, then any generalized solution $u(x,t) \in \overset{\circ}{W}{}_{2,\alpha}^{1,0}(Q_T)$ of equation (1) is continuous by Hőlder in each strictly inner subdomain Q_T.*

This lemma is a corollary of lemma 2. And the proof is standard.

Lemma 4. *Let with respect to coefficients L conditions (3) be fulfilled, and $u(x,t) \in \overset{\circ}{W}{}_{2,\alpha}^{1,0}(Q_T)$ be generalized solution of first boundary-value problem (5)-(6) at $f^i(x,t) \equiv 0$, $i = 0, ..., n$. Then, if $h(x)$ is bounded on $\Gamma(Q_T)$ in the sense of $W_{2,\alpha}^{1,0}(Q_T)$, then for the solution $u(x,t)$ the following maximum principle is true*

$$\inf_{\Gamma(Q_T)} h \leq \inf_{Q_T} u \leq \sup_{Q_T} u \leq \sup_{\Gamma(Q_T)} h,$$

where $\inf\limits_{\Gamma(Q_T)} h \left(\sup\limits_{Q_T} h \right)$ is the least upper (lower) bound of such numbers a, for which $h(x) \geq a$ $\quad (h(x) \leq a)$ on $\Gamma(Q_T)$ in the sense of $W_{2,\alpha}^{1,0}(Q_T)$.

The lemma is proved similarly to the maximum principle for parabolic equations.

Let's suppose, that the coefficients of the operator L are smooth in $\bar{Q}_T$ and the function $V(x,t)$ is finite by norm of the space $W_{2,\alpha}^{1,0}(R_{n+1} \setminus \Pi)$ such, that

$$LV = 0 \text{ in } R_{n+1} \setminus \Pi, \;\; V|_{t=0} = 0, \;\; V|_{t=0} = 0, \;\; V|_{\Gamma(R_{n+1} \setminus \Pi)} = 1,$$

where Π is a step domain with a smooth boundary (i.e. with smooth boundaries of the bases of components of it's cylinders). The existence of such a function follows from lemma 1 and [12]. The number

$$cap_L(\Pi) = \int\limits_{\gamma} \left(\frac{\partial V}{\partial \nu} - V \cos(n,t) \right) ds,$$

where γ is an arbitrary closed piece-wise smooth surface, containing inside of itself Π, is called L capacity Π. In this definition the integral doesn't depend on γ and as γ we'll take $\partial\Pi$. If instead of R_{n+1} we'll take the ellipsoid $\mathcal{E}_T = \mathcal{E}_{R,K}(0) \times (0,T)$ and $\Pi \subset \mathcal{E}_T$ is some compact, then $cap_L^{(\mathcal{E}_T)}(\Pi)$ is similarly defined, which is called L capacity of the compact Π with regard to $\mathcal{E}_T$.

Lemma 5. *There exist constants C_1 and C_2, dependent on γ, α, Ω such that*

$$C_1(\gamma,\alpha,\Omega)\, cap_{L_1}(\Pi) < cap_{L_2}(\Pi) < C_2(\gamma,\alpha,\Omega)\, cap_{L_1}(\Pi).$$

Proof. Let w and v be solutions of the equations $L_1 w = 0$ and $L_2 v = 0$. Then ∂/ν means a conormal derivative for corresponding operator, ν_t is direction consine of the normal with axist t, equaled to 1, if integration is taken by lower cover, and equaled to -1, if integration is taken by upper cover of Π. Let $\Pi \subset \Pi_T$, $\Pi_T = \{(t,x) : 0 < t < T\}$. To the identity $\int\limits_{\Pi_T \setminus \Pi} (wL_2 v + vL_1 w)\, dxdt = 0$ we'll apply Green's formula and using properties w and v, we'll obtain

$$J(v,w) = \int\limits_{\partial\Pi} \left(\frac{\partial v}{\partial \nu_2} + \frac{\partial w}{\partial \nu_1} - vw\nu_t \right) ds,$$

where

$$J(v,w) = \sum_{i,j=1}^{n} \int\limits_{\Pi_T \setminus \Pi} \left(a_{ij} v_{x_i} w_{x_j} + b_{ij} v_{x_j} w_{x_i} \right) dxdt + \int\limits_{R^n} v(x,T)\, w(x,T)\, dx,$$

here a_{ij} and b_{ij} are coefficients of the operators L_1 and L_2. Estimating $J(v,w)$ from below, we'll obtain

$$J(v,w) \geq \frac{1}{2} \left(cap_{L_1}(\Pi) + cap_{L_2}(\Pi) \right).$$

Using condition (3) and Hőlder's inequality, estimating $J(v,w)$ from above, we'll obtain the estimation

$$cap_{L_1}(\Pi) + cap_{L_2}(\Pi) \leq 4\gamma^2 \sqrt{cap_{L_1}(\Pi) \cdot cap_{L_2}(\Pi)}$$

this proves the lemma.

The lemma is proved.

Let μ be a measure, given on $\mathcal{E}_T$. We'll say, that the function $u(x,t) \in L_1(\mathcal{E}_T)$ is a weak solution of the equation $Lu = -\mu$, equaled to zero on $\Gamma(\mathcal{E}_T)$, if any function $\varphi(x,t) \in \overset{\circ}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T) \cap C\left(\overline{\mathcal{E}}_T\right)$, $L\varphi(x,t) \in C(\bar{\eta}_T)$ the integral identity is fulfilled

$$\int\limits_{\mathcal{E}_T} u(x,t)\, L_{\varphi}(x,t)\, dxdt = \int\limits_{\mathcal{E}_T} \varphi d\mu.$$

By lemma 1 at $h = 0$ the exists continuous linear operator H from $\overset{*}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ into $\overset{\circ}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ such that for each $T \in \overset{*}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ the function $u = H(T)$ is a unique in $\overset{\circ}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ generalized solution of the equation $Lu = T$.

The operator H is called Green operator. By lemma 3 this operator at $p > p_0$ reduces $\overset{*}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ into $C\left(\overline{\mathcal{E}}_T\right)$. The function $u(x,t)$ is a weak solution of the equation $Lu = -\mu$, equal to zero on $\Gamma(\mathcal{E}_T)$, if and only if, for any function $\psi(x,t) \in C\left(\overline{\mathcal{E}}_T\right)$ the identity is fulfilled

$$\int\limits_{\mathcal{E}_T} u(x,t)\, \psi(x,t)\, dxdt = \int\limits_{\mathcal{E}_T} H(\psi)\, d\mu. \tag{12}$$

Similarly in [2] it is proved, that for every measure μ on $\mathcal{E}_T$ there exists a unique weak solution of the equation $Lu=-\mu$ equal to zero on $\Gamma(\mathcal{E}_T)$.

We'll say, that the measure $\mu\in \overset{*}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ if there exists the vector $\bar f(x,t)=\left(f^0(x,t),f^1(x,t),...,f^n(x,t)\right),f^0(x,t)\in L_2(\mathcal{E}_T),\ f^i(x,t)\in L_{2,\lambda_i^{-1}}(\mathcal{E}_T)\quad i=1,...,n$, for any function $\varphi(x,t)\in \mathring{W}^{1,0}_{2,\alpha}(\mathcal{E}_T)\cap C\left(\overline{\mathcal{E}}_T\right)$ the integral identity is true

$$\mu(\varphi(x,t))=\int\limits_{\mathcal{E}_T}\varphi d\mu=\int\limits_{\mathcal{E}_T}\left(f^0(x,t)\varphi(x,t)-\sum_{i=1}^n f^i(x,t)\varphi_i\right)dxdt,$$

Hence we get

$$\left|\int\limits_{\mathcal{E}_T}\varphi d\mu\right|\le C\left(\bar f\right)\|\varphi\|_{W^{1,0}_{2,\alpha}(\mathcal{E}_T)}.$$

Lemma 6. *The weak solution $u(x,t)$ of the equation $Lu=-\mu$, equaled to zero on $\Gamma(\mathcal{E}_T)$, belongs to $\mathring{W}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ if and only if $\mu\in\mathring{W}^{1,0}_{2,\alpha}(\mathcal{E}_T)$.*

Proof. First of all we'll show, that if the function $u(x,t)\in\mathring{W}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ satisfies the integral identity

$$\int\limits_{\mathcal{E}_T}u\varphi_t dxdt+\int\limits_{\mathcal{E}_T}\sum_{i,j=1}^n a_{ij}(x,t)u_i\varphi_j dxdt=-\int\limits_{\mathcal{E}_T}\varphi d\mu \tag{13}$$

for any function $\varphi(x,t)\in\mathring{W}^{1,0}_{2,\alpha}(\mathcal{E}_T)\cap C\left(\overline{\mathcal{E}}_T\right)$, then it is a weak solution of the equation $Lu=-\mu$, equaled to zero on $\Gamma(\mathcal{E}_T)$. Really, assuming $\varphi=H(\psi),\ \psi(x,t)\in C(\mathcal{E}_T)$ we obtain

$$\int\limits_{\mathcal{E}_T}H(\psi)d\mu=\int\limits_{\mathcal{E}_T}\varphi d\mu=-\int\limits_{\mathcal{E}_T}\sum_{i,j=1}^n a_{ij}(x,t)u_i\varphi_j dxdt-\int\limits_{\mathcal{E}_T}u\varphi_t dxdt=$$

$$=\int\limits_{\mathcal{E}_T}u\sum_{i,j=1}^n\left(a_{ij}(x,t)\varphi_j\right)_i dxdt-\int\limits_{\mathcal{E}_T}u\varphi_t dxdt=$$

$$=\int\limits_{\mathcal{E}_T}uL\varphi dxdt=\int\limits_{\mathcal{E}_T}u(x,t)\psi dxdt,$$

and now using identity (12) we'll obtain the required. The proof of $\mu\in\mathring{W}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ is similar to the paper [6], vice versa, if $u(x,t)$ is a weak solution of the equation $Lu=-\mu$ equaled to zero on $\Gamma(\mathcal{E}_T)$ and $\mu\in\overset{*}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$, then there exist $f^0,\ f^i$ such that

$$\int\limits_{\mathcal{E}_T}\left(f^0\varphi-\sum_{i=1}^n f^i\varphi_i\right)dxdt=\int\limits_{\mathcal{E}_T}\varphi d\mu=\int\limits_{\mathcal{E}_T}uL\varphi dxdt=$$

$$=\int\limits_{\mathcal{E}_T}u\left[\sum_{i,j=1}^n\left(a_{ij}(x,t)\varphi_j\right)_i-\varphi_t\right]dxdt=$$

$$= \int\limits_{\mathcal{E}_T} \left[-\sum_{i,j=1}^{n} a_{ij}(x,t)\, u_i \varphi_j + u\varphi_t \right] dxdt,$$

for any function $\varphi(x,t) \in \overset{\circ}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T) \cap C\left(\overline{\mathcal{E}}_T\right)$, $L\varphi(x,t) \in C\left(\overline{\mathcal{E}}_T\right)$. Then by lemma 1 we obtain, that $u(x,t) \in \overset{\circ}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$.

The lemma is proved.

Let $\delta(x,t)$ be a Dirac's measure, concentrated at the point $(0,0)$, (y,t) be on arbitrary fixed point $\mathcal{E}_T$.

The weak solution $g(x,y,t)$ of the equation $Lg = -\delta(x-y,t)$, equaled to zero on $\Gamma(\mathcal{E}_T)$, is called Green function of the operator L in $\mathcal{E}_T$.

In the case $\mathcal{E}_T = R_{n+1}$ the corresponding function is called fundamental solution of the operator L and is denoted by $G(x,t)$. According to above stated, if $\psi(x,t)$ is an arbitrary function belonging $C\left(\overline{\mathcal{E}}_T\right)$, then the generalized solution $\varphi(x,t) \in \overset{\circ}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ of equation $h\varphi = -\Psi$, can be represented in the form $\varphi(y) = \int\limits_{\mathcal{E}_T} g(x,y,t)\,\Psi(x,t)\,dxdt$. It is possible to show, that $g(x,y,t)$ is non-negative in $\mathcal{E}_T \times \mathcal{E}_T$, such that $g(x,y,t) = g(y,x,t)$.

Lemma 7. *For any charge of the boundary variation μ on $\mathcal{E}_T$ the following integral*

$$u(x,t) = \int\limits_{\mathcal{E}_T} g(x,y,t)\, d\mu(y)$$

exists, is finite a.e. in $\mathcal{E}_T$ and is a weak solution of the equation $Lu = -\mu$, equaled to zero on $\Gamma(\mathcal{E}_T)$.

Proof. Let μ be measure in $\mathcal{E}_T$ and $\psi(x,t) \in C\left(\overline{\mathcal{E}}_T\right)$, $\psi(x,t) \geq 0$ in ε_T. Denote by $\varphi(x,t) \in \overset{\circ}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$ a generalized solution of the equation $L\varphi = -\psi(x,t)$. Then $\varphi(x,t) \in C\left(\overline{\mathcal{E}}_T\right)$ according to Lemma 3 and $\varphi(x,t) \geq 0$ by Lemma 4 and $\varphi(y) = \int\limits_{\mathcal{E}_T} g(x,y,t)\,\psi(x,t)\,dxdy$. By Fubini theorem hence we conclude, that $\int\limits_{\mathcal{E}_T} g(x,y,t)\, d\mu(y)$ exists almost for all $(x,t) \in \mathcal{E}_T$ and

$$\int\limits_{\mathcal{E}_T} H(\psi)\, d\mu(y) = \int\limits_{\mathcal{E}} \varphi(y)\, d\mu(y) = \int\limits_{\mathcal{E}_T \times \mathcal{E}_T}\int g(x,y,t)\,\psi(x,t)\,dxdtd\mu(y) =$$

$$= \int\limits_{\mathcal{E}_T} \psi(x,t)\, u(x,t)\, dxdt. \tag{14}$$

Equality (14) is fulfilled for any non-negative and continuous in $\overline{\mathcal{E}}_T$ function $\psi(x,t)$. Now taking into account (12) we obtain the required statement.

The lemma is proved.

Let's consider the L -capacity potential $u(x,t)$ of the compact Π with respect to $\mathcal{E}_T$. By Schwartz theorem [14] there exists the measure μ on Π such that

$$-\int\limits_{\mathcal{E}_T} u\eta_t dxdt + \int\limits_{\mathcal{E}_T} \sum_{i,j=1}^{n} a_{ij}(x,t)\, u_i \eta_j dxdt = \int\limits_{\mathcal{E}_T} \eta d\mu. \tag{15}$$

As $u(x,t)=1$ on Π in the sense $\overset{\circ}{W}{}^{1,0}_{2,\alpha}(\mathcal{E}_T)$, then the support of the measure μ is located on $\Gamma(\Pi)$. The measure μ is called L -capacity distribution of the compact Π.

By Lemma 7 the L capacity potential $u(x,t)$ is a weak solution of the equation $Lu=-\mu$, equaled to zero on $\Gamma(\mathcal{E}_T)$ and can be represented in the form

$$u(x,t)=\int\limits_{\mathcal{E}_T} g(x,y,t)\,d\mu(z). \tag{16}$$

On the other hand, there exists the sequence of the function $\{\eta^{(m)}(x,t)\}, m=1,2,...,$ such that $\eta^{(m)}(x,t)\in B(\mathcal{E}_T)$, $\eta^{(m)}(x,t)=1$ for $(x,t)\in\Pi$ and $\lim\limits_{m\to\infty}\|\eta^{(m)}-u\|_{W^{1,0}_{2,\alpha}}=0$. Supposing (15) $\eta^{(m)}(x,t)$, instead of $\eta(x,t)$ we obtain, that its right-hand side is equal to $\mu(\Pi)$ at any natural m, when the left-hand side tends to $cap_L^{(\mathcal{E}_T)}(\Pi)$ as $m\to\infty$. So

$$cap_L^{(\mathcal{E}_T)}(\Pi)=\mu(\Pi). \tag{17}$$

Lemma 8. *Let conditions (3) be fulfilled,* $(y,t)\in\Gamma(\mathcal{E}_{R,2}(0)\times(0,T))$, $\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\subset Q_T$, $(x,t)\in\Gamma(\mathcal{E}_{R,1}(y)\times(0,1))$. *Then for Green's function* $g(x,y,t)$ *the estimations are true*

$$C_1(\gamma,\alpha,n)\left[cap_L^{(\mathcal{E}_T)}\left(\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\right)\right]^{-1}\le g(x,y,t)\le$$

$$\le C_2(\gamma,\alpha,n)\left[cap_L^{(\mathcal{E}_T)}\left(\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\right)\right]^{-1}. \tag{18}$$

If $\mathcal{E}_{R,1}(0)\times(0,T)\subset Q_T$, $(x,t)\in\Gamma(\mathcal{E}_{R,1}(0)\times(0,T))$, then

$$C_1(\gamma,\alpha,n)\left[cap_L^{(\mathcal{E}_T)}\left(\overline{\mathcal{E}}_{R,1}(0)\times(0,T)\right)\right]^{-1}\le g(x,0,t)\le$$

$$\le C_2(\gamma,\alpha,n)\left[cap_L^{(\mathcal{E}_T)}\left(\overline{\mathcal{E}}_{R,1}(0)\times(0,T)\right)\right]^{-1}. \tag{19}$$

Proof. First of all, assume that the coefficients of the operator L are sufficiently smooth in $\overline{\mathcal{E}}_T$.

The general case is obtained from this by the passage to the limit. Then at $x\neq y$ the function $g(x,y,t)$ is continuous on x and y, and

$$\lim_{x\to y} g(x,y,t)=\infty. \tag{20}$$

Let's take the positive number a, which will be chosen later, $K_a=\{x: g(x,y,t)\ge a\}$, where (y,t) is an arbitrary fixed point on $\Gamma(\mathcal{E}_{R,2}(0)\times(0,T))$. From (20) it follows, that (y,t) is an inner point of the compact K. Then L -capacity potential K represented in the form (16) is continuous at the point (y,t), and it means it equals unit at this point. So $1=\int\limits_{\mathcal{E}_T} g(x,y,t)\,d\mu_a(z)$, where μ_a,is the L -capacity distribution of the compact K_a.

If will take into account, that the support of the measure μ is located on $\Gamma(K_a)$, and use (17), then we'll obtain

$$\mu_a(K_a) = cap_L^{(\mathcal{E}_T)}(K_a) = \frac{1}{a}. \qquad (21)$$

Now choosing $a = \inf\limits_{(x,t)\in\Gamma\left(\mathcal{E}_{R,1}(0)\times(0,T)\right)} g(x,y,t)$, by maximum principle $\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\subset K_a$. Then from (21) we obtain

$$cap_L^{(\mathcal{E}_T)}\left(\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\right) \le cap_L^{(\mathcal{E}_T)}(K_a) = \frac{1}{a}. \qquad (22)$$

If put $b = \sup\limits_{(x,t)\in\Gamma\left(\mathcal{E}_{R,1}(0)\times(0,T)\right)} g(x,y,z)$, then $\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\supset K_b$ i.e.

$$cap_L^{(\mathcal{E}_T)}\left(\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\right) \ge cap_L^{(\mathcal{E}_T)}(K_b) = \frac{1}{b}. \qquad (23)$$

From (22) and (23) we obtain, that

$$\inf_{(x,t)\in\Gamma\left(\mathcal{E}_{R,1}(0)\times(0,T)\right)} g(x,y,t) \le \left[cap_L^{(\mathcal{E}_T)}\left(\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\right)\right]^{-1} \le$$

$$\le \sup_{(x,t)\in\Gamma\left(\mathcal{E}_{R,1}(0)\times(0,T)\right)} g(x,z,t). \qquad (24)$$

Taking into account estimation (11) of Lemma 2 and (24) we obtain the requared estimation (18). Estimation (19) is proved similarly.

The lemma is proved.

Corollary. *Under the lemma's conditions let* $y\in\Gamma\left(\mathcal{E}_{R,2}(0)\times(0,T)\right), \overline{\mathcal{E}}_{R,1}(y)\times(0,T)\subset Q_T$, $(x,y)\in\Gamma\left(\mathcal{E}_{R,1}(y)\times(0,T)\right)$ *or* $y=0$, $\overline{\mathcal{E}}_{R,1}(0)\times(0,T)\subset Q_T, (x,y)\in\Gamma\left(\mathcal{E}_{R,1}(0)\times(0,T)\right)$. *Then for the fundamental solution of* $G(x,y,t)$ *the estimation*

$$C_1\left[cap_L\left(\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\right)\right]^{-1} \le G(x,y,t) \le$$

$$\le C_1\left[cap_L\left(\overline{\mathcal{E}}_{R,1}(y)\times(0,T)\right)\right]^{-1}$$

is true.

Theorem 1. *Let with respect to coefficients of the operator* L *conditions (3) be fulfilled. Then for removability of the compact* $E\subset Q_T$ *with respect to first boundary-value problem for the operator* L *in the space* $M(Q_T)$ *it is necessary and sufficient, that*

$$cap_L(E) = 0. \qquad (25)$$

Proof. Sufficiency. If condition (25) is fulfilled, then by above stated

$$cap_L^{(\mathcal{E}_T)}(E) = 0. \qquad (26)$$

First of all not losing generality, we'll consider the case, when the coefficients L are infinitely differentiable in $\overline{\mathcal{E}}_T$. Fix the arbitrary $\varepsilon>0$ and $M^0=\left(x^0,t^0\right)\in$

$Q_T \backslash E$. By virtue of (26) there exists the neighbourhood Π of the compact E such, that

$$cap_L^{(\mathcal{E}_T)}\left(\bar{\Pi}\right) < \varepsilon. \tag{27}$$

It is possible to take ε so small, that

$$dist\left(M^0, \bar{\Pi}\right) \geq \frac{1}{2} dist\left(M^0, E\right). \tag{28}$$

Denote by $\upsilon_\Pi(x,t)$ and μ_Π the L capacity potential of the compact $\bar{\Pi}$ with respect to $\mathcal{E}_T$ and $\bar{L}$ capacity distribution $\bar{\Pi}$, respectively. According to the above proved

$$\upsilon_\Pi(x,t) = \int\limits_{\mathcal{E}_T} g(x,y,t)\, d\mu(y),$$

and the function $\upsilon_\Pi(x,t)$ is a generalized solution of the equation $L\upsilon = 0$ in $\mathcal{E}_T \backslash \bar{\Pi}$, vanishing on 0 in $\Gamma(\mathcal{E}_T)$ and 1 in $\Gamma(\mathcal{E}_T)$ in sense $W_{2,\alpha}^{1,0}(\mathcal{E}_T)$. Consider $u(x,t) \in M(Q_T)$ is solution of the equation $Lu = 0$ in $Q_T \backslash E$ vanishing on $\Gamma(Q_T)$ and denote $M = \sup\limits_{Q_T}|u|$. The function $\upsilon_\Pi(x,t)$ we'll be non-negative on $\Gamma(Q_T)$ in the sense of $W_{2,\alpha}^{1,0}(Q_T)$. Hence we obtain that the function $u(x,t) - M\upsilon_\Pi(x,t)$ being a generalized solution of the equation $Lu = 0$ in $Q_T \backslash \Pi$, is non-positive on $\Gamma(Q_T \backslash \Pi)$. Then by maximum principle, by lemma 4, $u(x,t) - M\upsilon_\Pi(x,t) \leq 0$ in Q_T and, in particular

$$u\left(M^0\right) \leq M\upsilon_\Pi\left(M^0\right) \leq \sup_{(y,t)\in\Gamma(\Pi)} g\left(x^0,t^0,y\right)\mu_\Pi\left(\bar{\Pi}\right) =$$

$$= M \sup_{(y,t)\in\Gamma(\Pi)} g\left(x^0,t^0,y\right) cap_L^{(\mathcal{E}_T)}\left(\bar{\Pi}\right). \tag{29}$$

By virtue of the function $g(x,y,t)$ at $x \neq y$ and inequality (28) we have

$$\sup_{(y,t)\in\Gamma(\Pi)} g\left(x^0,t^0,y\right) \leq C_3\left(\gamma,\alpha,n,M^0,E\right).$$

So, from (27) and (29) we obtain $u\left(M^0\right) \leq M \cdot C_3 \cdot \varepsilon$. As ε is arbitrary, hence we obtain $u\left(M^0\right) \leq 0$. By the similar arguments with the function $u(x,t) + M\upsilon_\Pi(x,t)$, we obtain that $u\left(M^0\right) \geq 0$. From the last two inequalities, by virtue of arbitrariness of the point M^0, we obtain, that $u(x,t) \equiv 0$ in $Q_T \backslash E$. And thus the sufficiency of condition (25) is proved.

Necessity. Assume, that $cap_L(E) > 0$. Denote by $\mathcal{E}'_T$ such a set, that $\overline{\mathcal{E}}'_T \subset \mathcal{E}_T$, $E \subset \overline{\mathcal{E}}'_T$. Assume $Q_T = \mathcal{E}'_T$. Also by the above stated $cap_L^{(\mathcal{E}'_T)}(E) > 0$. Let $u_E(x,t)$ and ν_E be L -capacity potential of the compact E with respect to $\overline{\mathcal{E}}'_T$ and L -capacity distribution E, respectively.

Similarly to [15], we'll give the equivalent defination of L -capacity of the compact E with respect to $\mathcal{E}'_T$. Let $g(x,t)$ be Green's function of the operator L in $\mathcal{E}'_T$. Let's call the measure μ on E L feasible, if $supp\mu \subset E$ and

$$\cup_\mu^E(x,t) = \int\limits_{\mathcal{E}'_T} g(x-y,t-\tau)\, d\mu(y-\tau) < 1 \text{ for } x \in supp\mu. \tag{30}$$

The quantity $sup\mu(E) = cap_L^{(\mathcal{E}'_T)}(E)$, where the least upper bound is taken by all L -feasible measures is called L capacity of the compact E with respect to $\mathcal{E}'_T$. The L capacity $cap_L(E)$ is defined similarly. At that [15] it is shown, that there exists a unique measure on which the least upper bound $\mu(E)$ by set of all L feasible measures μ is reached. This measure is L capacity distribution of the compact E. By the above proved the function $u_E(x,t)$ is a generalized solution of the equation $Lu_E = 0$ in $\mathcal{E}'_T \backslash E$, equaled to zero on $\Gamma(\mathcal{E}'_T)$. Besides from the maximum principle of Lemma 4, and from (30) it follows, that $u_E(x,t) \in M(\mathcal{E}'_T)$. On the other hand $u_E(x,t) \not\equiv 0$, since $\nu_\Pi(E) > 0$.

The theorem is proved.

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