Abdurrahim F. GULIYEV, Sakina H. HASANOVA

ON SOME QUALITY PROPERTIES OF SOLUTIONS OF THE SECOND ORDER QUASILINEAR EQUATIONS

Abstract

A quasilinear parabolic equation of the second order is considered. A lemma on increase is proved for positive solutions of this equation. Sufficient conditions for the regularity of boundary points are established.

Let R_{n+1} be $(n+1)$ dimensional Euclidean space of points $(t,x)=(t,x_1,...,x_n)$; D be a domain in R_{n+1}, ∂D, and $\Gamma(D)$ be a boundary and parabolic boundary of the domain D, respectively; $C^{t_1t_2}_{x^0,R}$ be a cylinder $t_1<t<t_2$, $\left|x-x^0\right|<R$; $A_{l_m}=\{(\tau,\xi)\,|F_{s,\beta}(t-\tau,x-\xi)\geq l_m^{-s}\}$, $A_{l_m,\nu_m}=A_{l_m}\cap\{t\leq-\nu_m\}$, $\tilde{A}_{l_m,\nu_m}=A_{l_m}\cap\{t>-\nu_m\}$, $m=1,2,..$; $D^c=R_{n+1}\backslash D$; $\gamma_{s,\beta}(E)$ be a parabolic (s,β) - capacity of the set $E\subset R_{n+1}$, generated by the kernel

$$F_{s,\beta}(t,x)=\begin{cases} t^{-s}\exp\left(-\frac{|x|^2}{4\beta t}\right), & for \quad t>0 \\ 0, & for \quad t\leq 0\ . \end{cases}$$

In domain D we consider a quasilinear parabolic equation of the form

$$Lu=\sum_{i,j=1}^{n}a_{ij}(t,x,u,u_x)\,u_{ij}-u_t=0, \tag{1}$$

where $u_x=(u_1,...,u_n)$, $u_i=\frac{\partial u}{\partial x_i}$, $u_{ij}=\frac{\partial^2 u}{\partial x_i\partial x_j}$; $i,j=1,...,n$, $\|a_{ij}(t,x,z,v)\|$ is a real symmetric matrix whose elements are measurable in D for any fixed $z\in E_1$, $v\in E_n$, moreover

$$\sup_{(t,x)\in D}a_{ii}(t,x,z,v)=M_1<\infty, \tag{2}$$

$$\inf_{(t,x)\in D}\min_{|\xi|=1}\sum_{i,j=1}^{n}a_{ij}(t,x,z,v)\,\xi_i\xi_j=\alpha_1>0. \tag{3}$$

We'll assume that the maximum principle is fulfilled for the operator L (see [1]).

The goal of the paper is the obtaining a lemma on increase of positive solutions of second order equations and also obtaining a sufficient condition in terms of Wiener type convergence of a series for M_1,α_1-regularity of boundary point. The obtained condition enables to establish M_1,α_1-regularity of boundary points of wider domains than in [2]. For heat equations analogous results in cylindrical domains were established by F.N.Tikhonov [3], for a domain bounded by two straight lines parallel to the axis x, and curves $x=\varphi_1(t)$ and $x=\varphi_2(t)$ sufficient and necessary conditions very close to each other were obtained by I.G.Petrovskii [4]. In sequel, a regularity criterion of a boundary point for second order parabolic equations in terms of potential were obtained in the papers [5], [6]. In terms of parabolic capacity for divergent structure parabolic equations a regularity criterion of a boundary point was obtained in the papers [7], [8]. These criteria are complete analogies of a Wiener criterion for Laplace equation. For non-divergent structure parabolic equations the necessary and sufficient conditions for the regularity of a boundary point were obtained in [9].

1. Estimation of potential type function. Let the numbers $s > 0$, $\beta > 0$ and $a > 5$ be given. Consider the cylinders

$$C_m = C^{-l_m,0}_{0,2a\sqrt{\frac{\beta s}{e}l_m}}, \ m = 1, 2, \dots .$$

By S_m we denote a lateral surface of the cylinder C_m. Here and further we'll assume that the following conditions are fulfilled

$$\frac{l_{m+1}}{\nu_m} \ln \frac{l_m}{\nu_m} \overset{m\to 0}{\longrightarrow} 0, \ l_{m+1} < \nu_m < l_m, \ m = 1, 2, \dots . \tag{4}$$

Let (τ, ξ) be an arbitrary point belonging to $E_m = A_{l_m,\nu_m} \backslash D$. Fix this point, the number m and consider the function

$$g(t, x) = F_{s,\beta}(t - \tau, x - \xi) \ .$$

Estimate

$$\sup_{(t,x)\in S_m} g(t, x) \ .$$

By the inequality $|a - b| \geq ||a| - |b||$ we have

$$\sup_{(t,x)\in S_m} g(t, x) \leq (t - \tau)^{-s} \exp\left(-\frac{(|x| - |\xi|)^2}{4\beta(t - \tau)}\right) \leq (t - \tau)^{-s} \times$$

$$\times \exp\left(-\frac{\left(2a\sqrt{\frac{\beta s}{e}l_m} - 2\sqrt{\frac{\beta s}{e}l_m}\right)^2}{4\beta(t - \tau)}\right) = (t - \tau)^{-s} \exp\left(-\frac{sl_m(a - 1)^2}{e(t - \tau)}\right) \ .$$

The function $t^{-s} \exp\left(-\frac{sl_m(s - l_m)^2}{ez}\right)$ is an increasing function with respect to z at $0 < z < l_m$. Since $0 < t - \tau < l_m$ then

$$\sup_{(t,x)\in S_m} g(t, x) \leq l_m^{-s} \exp\left(\frac{-sl_m(a - 1)^2}{el_m}\right) = l_m^{-s} \exp\left(-\frac{s(a - 1)^2}{e}\right). \tag{5}$$

Now, estimate

$$\inf_{(t,x)\in C_{m+1}} g(t, x) \ .$$

By the inequality $|a - b| \leq |a| + |b|$ we have

$$\inf_{(t,x)\in C_{m+1}} g(t, x) \leq (-\tau)^{-s} \exp\left(-\frac{|\xi|^2}{4\beta(-\tau)}\right) \exp\left(\frac{|\xi|^2}{4\beta(-\tau)} - \frac{|x - \xi|^2}{4\beta(t - \tau)}\right) \geq$$

$$\geq l_m^{-s}\exp\left(\frac{|\xi|^2}{4\beta(-\tau)}-\frac{|\xi|^2}{4\beta(t-\tau)}-\frac{|x|^2}{4\beta(t-\tau)}-\frac{2|x||\xi|}{4\beta(t-\tau)}\right)\geq$$

$$\geq l_m^{-s}\exp\left(\frac{|\xi|^2}{4\beta}\left(\frac{1}{-\tau}-\frac{1}{t-\tau}\right)-\frac{|x|^2}{4\beta(t-\tau)}-\frac{2|x||\xi|}{4\beta(t-\tau)}\right)\geq$$

$$\geq l_m^{-s}\exp\left(\frac{|\xi|^2}{4\beta}\frac{t}{(-\tau)(t-\tau)}-\frac{|x|^2}{4\beta(t-\tau)}-\frac{2|x||\xi|}{4\beta(t-\tau)}\right)\geq$$

$$\geq l_m^{-s}\exp\left(\frac{4\beta s(-\tau)\,l_m\frac{l_m}{-\tau}(-l_{m+1})}{4\beta(-\tau)(-l_{m+1}+\nu_m)}-\frac{4a^2\beta s l_{m+1}}{4e\beta(-l_{m+1}+\nu_m)}-\right.$$

$$\left.-\frac{2a\sqrt{\beta s l_{m+1}}\,2\sqrt{\beta s(-\tau)\ln\frac{l_m}{-\tau}}}{2\beta\sqrt{e}\,(-l_{m+1}+\nu_m)}\right)\geq$$

$$\geq l_m^{-s}\exp\left(-\frac{s\frac{l_{m+1}}{\nu_m}\ln\frac{l_m}{\nu_m}}{1-\frac{l_{m+1}}{\nu_m}}-\frac{a^2 s\frac{l_{m+1}}{\nu_m}}{e\left(1-\frac{l_{m+1}}{\nu_m}\right)}-\frac{2as\sqrt{\frac{l_{m+1}}{\nu_m}\ln\frac{l_m}{\nu_m}}}{\sqrt{e}\left(1-\frac{l_{m+1}}{\nu_m}\right)}\right)=l_m^{-s}\exp\left(-\frac{s}{e}J_1\right)\ ,$$

where

$$J_1=e\frac{\frac{l_{m+1}}{\nu_m}\ln\frac{l_m}{\nu_m}}{1-\frac{l_{m+1}}{\nu_m}}+\frac{a^2\frac{l_{m+1}}{\nu_m}}{1-\frac{l_{m+1}}{\nu_m}}+\frac{2a\sqrt{e\frac{l_{m+1}}{\nu_m}\ln\frac{l_m}{\nu_m}}}{1-\frac{l_{m+1}}{\nu_m}}\ .$$

By condition (4) there exists such m_0, that for $m\geq m_0$, $J_1<1$.
And thus

$$\inf_{(t,x)\in C_{m+1}} g(t,x)\geq l_m^{-s}e^{-\frac{s}{e}}. \tag{6}$$

Granting, that (τ,ξ) is an arbitrary point from E_m by (5) and (6) we have

$$\sup_{\substack{(\tau,\xi)\in E_m\\(t,x)\in S_m}} F_{s,\beta}(t-\tau,x-\xi)\leq l_m^{-s}e^{-\frac{s(a-1)^2}{e}}, \tag{7}$$

$$\sup_{\substack{(\tau,\xi)\in E_m\\(t,x)\in S_m}} F_{s,\beta}(t-\tau,x-\xi)\leq l_m^{-s}e^{-\frac{s}{e}}\ . \tag{8}$$

Now, let E_m be a B-set and measure μ be defined on it. The function

$$U(t,x)=\int\limits_{E_m} F_{s,\beta}(t-\tau,x-\xi)\,d\mu(\tau,\xi)$$

is said to be a potential type function. Then we obtain from (7) and (8)

$$\sup_{(t,x)\in S_m} U\leq l_m^{-s}e^{-\frac{s(a-1)^2}{e}}\mu(E_m)\ , \tag{9}$$

$$\inf_{(t,x)\in C_{m+1}} U\geq l_m^{-s}e^{-\frac{s}{e}}\mu(E_m)\ . \tag{10}$$

Remark. It follows from the ways stated above by means of which we get inequalities (7) and (8), that these inequalities are true also in the case

$$\frac{l_{m+1}}{\nu_m}\ln\frac{l_m}{\nu_m}<K,\quad l_{m+1}<\nu_m<l_m,\quad m=1,2,\dots\ ,$$

where K is some positive number depending on s,β and a.

2. Lemma on increase. Introduce the following definition. Let $s>0$ and $\beta>0$ be given. Let $E-B$ be a set in R_{n+1}. Consider on E all possible measures μ such that

$$\int\limits_E F_{s,\beta}\left(t-\tau,x-\xi\right)d\mu\left(\tau,\xi\right)\leq 1\quad for\quad (t,x)\notin\bar{E}\ . \tag{*}$$

Assume

$$\gamma_{s,\beta}\left(E\right)=\sup\mu\left(E\right),$$

where the upper bound is taken on all possible measures satisfying the condition (*).

Call the number $\gamma_{s,\beta}$ a parabolic (s,β)-capacity of the set E.

Now let's pass to the formulation and proof of the lemma on increase of positive solutions of equation (1).

Lemma. *Let the numbers $s>0$, $\beta>0$, $a>5$ be given and the number m be fixed. Let C_m have the above indicated sense. Let $D\subset R_{n+1}$ be a domain with eigen boundary Γ and $C_{m+1}\cap D\neq\varnothing$. Let Γ_m be the part of the eigen boundary D which is located strictly interior to C_m. Let the operator L be strictly defined in D and for this operator it is fulfilled the condition*

$$\beta\leq\alpha_1\ \ and\ \ s\geq\frac{M_1}{2\beta}\ , \tag{11}$$

where α_1, M_1 are the constants of inequalities (2),(3). Let $u\left(t,x\right)$ be a subparabolic function for this operator continuous in $\bar{D}$, positive in D and vanishing in Γ_m. Then if condition (4) is fulfilled, then

$$\sup_{D\cap C_m}u>\left(1+\eta l_m^{-s}\gamma_{s,\beta}\left(E_m\right)\right)\sup_{D\cap C_{m+1}}u, \tag{12}$$

where $\eta>0$ is a constant dependent on s,β and a.

Proof. Fix m and give an arbitrary $\varepsilon>0$ and let the measure μ defined on E_m be such that

$$U\left(t,x\right)=\int\limits_{E_m}F_{s,\beta}\left(t-\tau,x-\xi\right)d\mu\left(\tau,\xi\right)\leq 1$$

exterior to $\bar{E}_m$ and

$$\mu\left(E_m\right)>\gamma_{s,\beta}\left(E_m\right)-\varepsilon\ .$$

Denote $\sup\limits_{D\cap C_m}=M_m$ and introduce the subsidiary function

$$\upsilon\left(t,x\right)=M_m\left[1-U\left(t,x\right)+l_m^{-s}e^{-\frac{s(a-1)^2}{e}}\mu\left(E_m\right)\right]\ .$$

By inequality (11) the function U is subparabolic and therefore υ is subparabolic. Everywhere on the eigen boundary of the domain D we have:

$$u\left(t',x'\right) \leq \lim_{(t,x)\to(t',x')} \upsilon\left(t,x\right) \ ,$$

In fact, the eigen boundary of the domain D consists of $\bar{\Gamma}_m$ and points arranged on S_m and in the lower basis of C_m.

Since $U \leq 1$ exterior to $\bar{E}_m$, then

$$\lim_{(t,x)\to(t',x')\in\bar{\Gamma}_m} \upsilon\left(t,x\right) \geq 0,$$

when $u|_{\bar{\Gamma}_m} = 0$ ($u|_{\Gamma_m}$ and so, by continuity $u|_{\bar{\Gamma}_m} = 0$). Further, on the lower basis of C_m at the points of the boundary arranged at the positive distance from Γ_m and so at the positive distance from E_m, the function U equals zero and thus, $\upsilon > M_m$ when $u \leq M_m$. Finally, on S_m by inequality (9)

$$U \leq l_m^{-s} e^{-\frac{s(a-1)^2}{e}} \mu\left(E_m\right)$$

and so, $\upsilon \geq M_m$ but $u \leq M_m$.

Consequently by the maximum principle $u \leq \upsilon$ in D

$$\sup_{D\cap C_{m+1}} u \leq \sup_{D\cap C_{m+1}} \upsilon \leq M_m \left[1 - \inf_{D\cap C_{m+1}} U + l_m^{-s} e^{-\frac{s(a-1)^2}{e}} \mu\left(E_m\right)\right]$$

and by inequality (10)

$$\sup_{D\cap C_{m+1}} u \leq M_m \left[1 - l_m^{-s}\left(e^{-\frac{s}{e}} - e^{-\frac{s(a-1)^2}{e}}\right)\left(\gamma_{s,\beta}\left(E_m\right) - \varepsilon\right)\right] .$$

Since this is true for any ε, then we finally get

$$\sup_{D\cap C_m} u \leq \left[1 - \eta l_m^{-s} \gamma_{s,\beta}\left(E_m\right)\right] \sup_{D\cap C_{m+1}} u,$$

where $\eta = e^{-\frac{s}{e}} - e^{-\frac{s(a-1)^2}{e}}$ whence the required inequality (12) follows.

The lemma is proved.

Remark. If in special case in the lemma on increase we take $\nu_m = \frac{l_m}{2}$, $\frac{l_{m+1}}{l_m} = q < 1$ then we get the result indicated in [2].

3. A theorem on the regularity of a boundary point. Now let's pass to the definition of M_1, α_1-regularity of the point.

Let $D \subset R_{n+1}$ be a domain and ∂D its boundary. Let M_1 and α_1 be two positive numbers. The point $\left(t^0, x^0\right) \in \partial D$ is said to be M_1, α_1-regular if the following conditions are fulfilled.

For any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ there will be found such a $\delta > 0$ that whatever is the domain $D' \subset D$, wholly lying on a half-space $t < t_0$, whatever is the uniform parabolic operator L', defined in D' for which $M_1' \leq M_1$, $\alpha_1' \geq \alpha_1$ whatever is the subparabolic for this operator function $u'\left(t,x\right)$ not exceeding a unit in D' and not exceeding zero at the intersection of eigen boundary D' and ε_1-vicinity of the point (t^0, x^0) (if it is not empty), it is fulfilled the following inequality

$$u'(t,x) \leq \varepsilon_2.$$

Theorem 1. *In order that the point* $(t^0, x^0) \in \partial D$ *be* M_1, α_1*-regular, it is sufficient that*

$$\sum_{m=1}^{\infty} l_m^{-\frac{M_1}{2\alpha_1}} \gamma_m = \infty, \tag{13}$$

where $\gamma_m = \gamma_{\frac{M_1}{2\alpha_1},\alpha_1}(A_{l_m,\nu_m} \backslash D)$.

Proof. Without loss of generality, we can take the point $(0,0)$ instead of the point (t^0, x^0). We'll consider the cylinder

$$C_m = C^{-l_m,0}_{0,2a\sqrt{\frac{\beta s}{e} l_m}}, \quad m = 1, 2, ..., \quad a > 5 \ .$$

Let $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$. Let there be a subdomain $D' \subset D$ disposed in a half-space $t > 0$ and the operator L', constants M_1' and α_1' of inequalities (2), (3) for which

$$M_1' \leq M_1, \quad \alpha_1' \geq \alpha_1$$

are satisfied.

Let $u'(t,x)$ be a subparabolic function and for it

$$u'(t,x) \leq 1 \ \ in \ \ D'$$

and

$$u'(t,x)\left.\right|_{\Gamma' \cap O_{\varepsilon_1}(0,0)} \leq 0,$$

where Γ' is the eigen boundary of D', and $O_{\varepsilon_1}(0,0)$ is ε_1-vicinity of the point $(0,0)$ in R_{n+1}. We are to show that there exists $\delta > 0$ dependent on $\varepsilon_1, \varepsilon_2, M_1$ and α_1, on the domain D, but independent of neither D', L' nor u', such that for each point $(t,x) \in D' \cap O_\delta(0,0)$ it is true the inequality

$$u'(t,x) < \varepsilon_2.$$

For each $m = 1, 2, ...$ we denote

$$M_m = \sup_{D' \cap C_m} u'.$$

Now, for each m we consider the cylinders C_m, C_{m+1}. Consider a set of such points $(t,x) \in D' \cap C_m$ in which $u'(t,x) > 0$. In this set we choose the component D_m' containing that intersection point D' with eigen boundary Γ_{m+1} of the cylinder C_{m+1} where the function u' attains the values M_{m+1}. We have

$$\gamma_{\frac{M_1}{2\alpha_1},\alpha_1}\left(A_{l_m,\nu_m} \backslash D_m'\right) \geq \gamma_{\frac{M_1}{2\alpha_1},\alpha_1}(A_{l_m,\nu_m} \backslash D_m) = \gamma_m.$$

Therefore, applying to the cylinders C_m and C_{m+1} to the function D_m' and function u' the lemma on increase at $s = \dfrac{M_1}{2\alpha_1}$ and $\beta = \alpha_1$, we get

$$M_m \geq \left(1 + \eta l_m^{-\frac{M_1}{2\alpha_1}} \gamma_m\right) M_{m+1}$$

and, consequently,

$$M_m - M_{m+1} \geq \eta l_m^{-\frac{M_1}{2\alpha_1}} \gamma_m M_{m+1},$$

whence, summing the both sides by the index m, we get

$$\sup_D u' - u'(0,0) = \sum_{m=1}^{\infty} (M_m - M_{m+1}) > \eta \sum_{m=1}^{\infty} l_m^{-\frac{M_1}{2\alpha_1}} \gamma_m M_{m+1}. \qquad (14)$$

Since the left hand side of inequality (14) is finite, then by the convergence of the series $\sum_{m=1}^{\infty} l_m^{-\frac{M_1}{2\alpha_1}} \gamma_m$ there exists such a subsequence $\{M_{m_k}\}$ of the sequence $\{M_m\}$, that $M_{m_k} \to 0$, as $k \to \infty$. This means that the affirmation of the theorem is true.

The theorem is proved.

Now, let $l_m = l^{-m \ln m}$, $\nu_m = e^{-m \ln m} m^{-\alpha}$, $0 < \alpha < 1$. It is easy to show that the sequences $\{l_m\}_{m=1}^{\infty}$ and $\{\nu_m\}_{m=1}^{\infty}$ chosen by such a way satisfy the condition (4). In this case we have

$$\sum_{m=1}^{\infty} l_m^{-\frac{M_1}{2\alpha_1}} \gamma_{\frac{M_1}{2\alpha_1},\alpha_1} \left(D^c \cap \tilde{A}_{l_m,\nu_m} \right) \leq \sum_{m=1}^{\infty} l_m^{-\frac{M_1}{2\alpha_1}} \gamma_{\frac{M_1}{2\alpha_1},\alpha_1} \left(C_{0,\rho_m}^{-\nu_m,0} \right) \leq$$

$$\leq C_1 \sum_{m=1}^{\infty} l_m^{-\frac{M_1}{2\alpha_1}} \left(\nu_m \ln \frac{l_m}{\nu_m} \right)^{\frac{M_1}{2\alpha_1}} = C_2 \sum_{m=1}^{\infty} \left(\frac{\nu_m}{l_m} \ln \frac{l_m}{\nu_m} \right)^{\frac{M_1}{2\alpha_1}} \leq C_3 \sum_{m=1}^{\infty} \frac{\ln^{\frac{M_1}{2\alpha_1}} m}{\alpha^{\frac{M_1}{2\alpha_1}}},$$

where $\rho_m = 2\sqrt{\frac{\beta s}{l} \nu_m \ln \frac{l_m}{\nu_m}}$. For $\alpha \frac{M_1}{2\alpha_1} > 1$ the last series converges. Then the sufficient condition of M_1, α_1-regularity of boundary point will have the form

$$\sum_{m=1}^{\infty} \left(e^{m \ln m} \right)^{\frac{M_1}{2\alpha_1}} \gamma_{\frac{M_1}{2\alpha_1},\alpha_1} \left(D^c \cap A_{e^{-m \ln m}} \right) = +\infty$$

and integral representation of this condition will be of the following form

$$\int \left(e^{z \ln z} \right)^{\frac{M_1}{2\alpha_1}} \gamma_{\frac{M_1}{2\alpha_1},\alpha_1} \left(D^c \cap A_{e^{-z \ln z}} \right) dz = +\infty. \qquad (15)$$

Make the substitution of the variable $t = z \ln z$. Hence we have

$$dz = \frac{dt}{\ln z + 1}.$$

It is clear that $\ln z + 1 \tilde{} \ln z$, at $z \geq 2$. Since $\ln z < \ln z + \ln \ln z = \ln t < 2 \ln z$, then $\ln z \tilde{} \ln t$.

Then we get from (15)

$$\int \frac{e^{ts}}{\ln t} \gamma_{\frac{M_1}{2\alpha_1},\alpha_1} \left(D^c \cap A_{e^{-t}} \right) dt = +\infty.$$

So, we proved the following theorem.

Theorem 2. *In order that the point $\left(t^0, x^0\right) \in \partial D$ be M_1, α_1-regular it is sufficient that*

$$\sum_{m=2}^{\infty} \frac{e^{ms}}{\ln m} \gamma_m \left(D^c \cap A_{e^{-m}} \right) = +\infty$$

where

$$\gamma_m = \gamma_{\frac{M_1}{2\alpha_1},\alpha_1}\left(D^c \cap A_{e^{-m}}\right) \ .$$

References

[1]. Krylov N.B. *On the maximum principle for parabolic and elliptic equations.* Izv. AN USSR, ser. Mat., 1978, v. 42, p. 1050-1062.

[2]. Landis E.M. *Second order equality of elliptic and parabolic types.* M., Nauka, 1971, 288 p.

[3]. Tikhonov A.N. *Uniqueness theorems for heat equations.* Mat. Sb., v42, No 2 (1935), pp.199-216.

[4]. Petrovskiy I.G. *Zur ersten Randwertaufgabe der Wärmelteinungsgleichung.* Compos,Math.,v.1. (1935), pp.383-419.

[5]. Landis E.M. *Necessary and sufficient conditions of regularity of a boundary point for Dirichlet problem for heat equation.* Soviet Doklady, 1969, v.185, No 3, pp.517-520.

[6]. Mamedov I.T. *On regularity of boundary points for linear and quasilinear equations of parabolic type.* Sov. Dokl., 1975, v.223, No 3, pp.539-561.

[7]. Evans L.C., Gariepi R.F. *Wiener criterion for the heat equation.* Arch. Rational Mech. And Anal., 1982, v.78, No 4, pp.293-314.

[8]. Garofalo N., Lanconelli E. *Wiener's criterion for parabolic equations with variable coefficients and its consequences.* Trans. Amer. Math. Soc., 1988, v. 308, No 2, pp. 811-836.

[9]. Guliyev A.F. *Capacity conditions of regularity of boundary points for parabolic equations of the second order.* Izv. AN Azerb. SSR., ser. Phyz.-tekhn. i mat. nauk., 1988, No 3, pp. 23-29.

Abdurrahim F. Guliyev, Sakina H. Hasanova
Institute of Mathematics and Mechanics of NAS of Azerbaijan.
9, F.Agayev str., AZ1141, Baku, Azerbaijan.
Tel.: (99412) 439 47 20 (off.)

Received April 21, 2004; Revised October 06, 2004.
Translated by the authors.