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STABILITY OF VARIABLE THICKNESS PLATE MADE OF CONTINUOUS INHOMOGENEOUS MATERIAL

Abstract

In the paper it is assumed that a variable thickness plate is on Fouss-Winkler basis and elasticity modulus is a continuous fucntion of thickness coordinate. The thickness is a fucntion of coordinate of mean plane. The main relations, differential equations are given in approximate statement. The calculation were carried out, typical graphs was constructed for a cylindrical form of stability loss.

Variable thickness plates made of inhomogeneous material are widely used in many fields of engineering industry, civil engineering and etc.

Let's consider a rectangular plate of variable thickness made of continuous-elastic material that is subject to the action of uniformly distributed loads in its mean plane. The axis x and y are arranged in the mean plane, and the axis z is perpendicular to them. Assume that the elasticity modulus is a fucntion of thickness coordinates $E = E_0 f_1(z)$, and Poisson coefficient ν is a constant value (that is not principal), and the thickness of the plate varies in the form $h = h_0 f_2(x, y)$, where

$$E_0 = \frac{E_1 + E_2}{2}; \quad h_0 = \frac{h_1 + h_2}{2}.$$

Here E_1 and E_2 are the most and least values of the elasticity modulus, respectively, h_1 and h_2 are analogous values of thickness.

At the stability moment the plate accepts curved equilibrium state close to original.

Infinitesimal deformations and stresses are connected with relations (the validity of Kirchhoff-Love conjecture is preserved)

$$\delta\sigma_1 = \frac{E_0 f_1(z)}{1 - v^2}\left[(\varepsilon_1 + \nu\varepsilon_2) - z(\chi_1 + \nu\chi_2)\right] ,$$

$$\delta\sigma_2 = \frac{E_0 f_1(z)}{1 - \nu^2}\left[(\nu\varepsilon_1 + \varepsilon_2) - z(\nu\chi_1 + \chi_2)\right] , \qquad (1)$$

$$\delta\tau = \frac{E_0 f_1(z)}{1 + \nu}(\varepsilon_3 - z\chi_3) .$$

It is easy to establish that variations of efforts and moments are of the following form:

$$\delta T_{11} = A_1 \left(\varepsilon_1 + \nu \varepsilon_2 \right) - A_2 \left(\chi_1 + \nu \chi_2 \right),$$

$$\delta T_{22} = A_1 \left(\nu \varepsilon_1 + \varepsilon_2 \right) - A_2 \left(\nu \chi_1 + \chi_2 \right), \tag{2}$$

$$\delta T_{12} = \left(1 - \nu\right) A_1 \varepsilon_3 - \left(1 - \nu\right) A_2 \chi_3$$

here we accept the following denotation

$$A_1 = \frac{E_0 h_0}{2\left(1 - \nu^2\right)} \int\limits_{-f_2(x,y)}^{+f_2(x,y)} f_1\left(z\right) dz \ ; \qquad A_2 = \frac{E_0 h_0^2}{4\left(1 - \nu^2\right)} \int\limits_{-f_2(x,y)}^{+f_2(x,y)} f_1\left(z\right) z dz \ ; \tag{3}$$

Similarly for M_{ij} we get:

$$\delta M_{11} = A_2 \left(\varepsilon_1 + \nu \varepsilon_2 \right) - A_3 \left(\chi_1 + \nu \chi_2 \right),$$

$$\delta M_{22} = A_2 \left(\nu \varepsilon_1 + \varepsilon_2 \right) - A_3 \left(\nu \chi_1 + \chi_2 \right), \tag{4}$$

$$\delta M_{12} = \left(1 - \nu\right) A_2 \varepsilon_3 - \left(1 - \nu\right) A_3 \chi_3 \ ,$$

where

$$A_3 = \frac{2}{3} D_0 \int\limits_{-f_2(x,y)}^{+f_2(x,y)} f_1\left(z\right) z^2 dz; \tag{5}$$

here D_0 is cylindrical rigidity

$$D_0 = \frac{E_0 h_0^3}{12\left(1 - \nu^2\right)} \ .$$

Without elementary details note that excluding $\varepsilon_1, \varepsilon_2, \varepsilon_3$ by $\delta T_{11}, \delta T_{22}, \delta T_{12}$ we can represent the expression for moments in the following form:

$$\delta M_{11} = D \left(\chi_1 + \nu \chi_2 \right) + D_1 \delta T_{11},$$

$$\delta M_{22} = D \left(\nu \chi_1 + \chi_2 \right) + D_1 \delta T_{22}, \tag{6}$$

$$\delta M_{12} = D \left(1 - \nu\right) \chi_3 + D_1 \delta T_{12},$$

where

$$D = \frac{A_2^2}{A_1} - A_3; \quad D_1 = A_2 \cdot A_1^{-1}$$

Introducing the stress functions Φ by the relations

$$\frac{\delta T_{11}}{E_0 h_0}=\frac{\partial^2\Phi}{\partial x^2};\quad \frac{\delta T_{22}}{E_0 h_0}=\frac{\partial^2\Phi}{\partial y^2};\ \frac{\delta T_{12}}{E_0 h_0}=\frac{\partial^2\Phi}{\partial x\partial y};\tag{7}$$

and using the equilibrium equation, deformation compatibility condition and moments equation as a result we get the following differential equation with regard to inhomogeneous resistance and deformation compatibility condition

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+2\frac{\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2}\right)+2\frac{\partial D}{\partial y}\left(\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial x^2\partial y}\right)+$$
$$+\frac{\partial^2 D}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2 D}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)+2\left(1-\nu\right)\frac{\partial^2 D}{\partial x\partial y}\ \frac{\partial^2 w}{\partial x\partial y}+$$
$$+E_0 h_0\left(\frac{\partial^2 D_1}{\partial x^2}\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2 D_1}{\partial y^2}\frac{\partial^2\Phi}{\partial y^2}-2\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2\Phi}{\partial x\partial y}\right)+$$
$$+T_{11}\frac{\partial^2 w}{\partial x^2}+2T_{12}\frac{\partial^2 w}{\partial x\partial y}+T_{22}\frac{\partial^2 w}{\partial y^2}+k\left(1+\varphi\left(x,y\right)\right)w=0\tag{8}$$

Deformation compatibility condition:

$$\frac{E_0 h_0}{1-\nu^2}\left[\left(\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}\right)\frac{1}{A_1}+2\left(\frac{\partial^3\Phi}{\partial x^3}+\frac{\partial^3\Phi}{\partial x\partial y^2}\right)\frac{\partial}{\partial x}\left(\frac{1}{A_1}\right)+\right.$$
$$\left.+2\left(\frac{\partial^3\Phi}{\partial y^3}+\frac{\partial^3\Phi}{\partial x^2\partial y}\right)\frac{\partial}{\partial y}\left(\frac{1}{A_1}\right)+\left(\frac{\partial^2\Phi}{\partial x^2}-\nu\frac{\partial^2\Phi}{\partial y^2}\right)\frac{\partial^2}{\partial x^2}\left(\frac{1}{A_1}\right)+2\left(1-\nu\right)\frac{\partial^2\Phi}{\partial x\partial y}\left(\frac{1}{A_1}\right)\right]+$$
$$+\frac{\partial^2}{\partial y^2}\left(\frac{A_2}{A_1}\right)\frac{\partial^2 w}{\partial x^2}-2\frac{\partial^2}{\partial x\partial y}\left(\frac{A_2}{A_1}\right)\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2}{\partial x^2}\left(\frac{A_2}{A_1}\right)\frac{\partial^2 w}{\partial x^2}=0\tag{9}$$

The solution of equations (8) and (9) is very complicated and we can hardly find effective results that can be applied in engineering practice. Therefore we usually use approximate statement of solutions of concrete problems.

In approximate statement $\delta T_{ij}=0\quad (i,j=1,2)$ the stability equation is very simplified and takes the following form:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+2\frac{\partial D}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2}\right)+2\frac{\partial D}{\partial y}\left(\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial x^2\partial y}\right)+$$
$$+\frac{\partial^2 D}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2 D}{\partial y^2}\ \left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)+2\left(1-\nu\right)\frac{\partial^2 D}{\partial x\partial y}\ \frac{\partial^2 w}{\partial x\partial y}$$
$$+T_{11}\frac{\partial^2 w}{\partial x^2}+2T_{12}\frac{\partial^2 w}{\partial x\partial y}+T_{22}\frac{\partial^2 w}{\partial y^2}+k\left(1+\varphi\left(x,y\right)\right)w=0\tag{10}$$

By solving concrete problems corresponding boundary conditions should be given.

To illustrate the abovementioned we consider the following problem: Cylindrical form of stability loss. Assume $h = h_0\left(1+\varepsilon_1\eta\right);\ E = E_0\left(1+\varepsilon_2\rho^2\right)$, here we accept $\eta = xa^{-1},\ \rho = zh^{-1}$.

In approximate statement for cylindrical form of stability loss ($a >> b$) equation (10) will have the form (consider Winkler resistance):

$$D\frac{d^4w}{dx^4} + 2\frac{dD}{dx}\frac{d^3w}{dx^3} + \frac{d^2D}{dx^2}\frac{d^2w}{dx^2} + kw + T_{11}\frac{d^2w}{dx^2} = 0 \tag{11}$$

where $D = \dfrac{A_2^2}{A_1} - A_3$.

Now, define D. For this we first are to define A_1, A_2, A_3

$$A_1 = \frac{E_0h_0}{2\left(1-\nu^2\right)}\int\limits_{-(1+\varepsilon_1\eta)}^{+(1+\varepsilon_1\eta)}\left(1+\varepsilon_2\rho^2\right)d\rho = \frac{E_0h_0}{2\left(1-\nu^2\right)}\left[2\left(1+\varepsilon_1\eta\right) + \frac{2\varepsilon_2}{3}\left(1+\varepsilon_1\eta\right)^3\right];$$

$$A_2 = \frac{E_0h_0^2}{4\left(1-\nu^2\right)}\int\limits_{-(1+\varepsilon_1\eta)}^{+(1+\varepsilon_1\eta)}\left(1+\varepsilon_2\rho^2\right)\rho d\rho = \frac{E_0h_0^2}{4\left(1-\nu^2\right)}\{\frac{1}{2}[\left(1+\varepsilon_1\eta\right)^2 - \left(1+\varepsilon_1\eta\right)^2] +$$

$$+\frac{\varepsilon_2}{4}\left[\left(1+\varepsilon_1\eta\right)^4 - \left(1+\varepsilon_1\eta\right)^4\right]\Big\} = 0;$$

$$A_3 = \frac{2}{3}D_0\int\limits_{-(1+\varepsilon_1\eta)}^{+(1+\varepsilon_1\eta)}\left(1+\varepsilon_2\rho^2\right)\rho^2 d\rho = \frac{2}{3}D_0[\frac{2}{3}[\left(1+\varepsilon_1\eta\right)^3 + \frac{2\varepsilon_2}{5}\left(1+\varepsilon_1\eta\right)^5] =$$

$$= \frac{4}{9}D_0[\left(1+\varepsilon_1\eta\right)^3 + \frac{3\varepsilon_2}{5}\left(1+\varepsilon_1\eta\right)^5];$$

Or

$$D = \frac{-4}{9}D_0\left[\left(1+\varepsilon_1\eta\right)^3 + \frac{3\varepsilon_2}{5}\left(1+\varepsilon_1\eta\right)^5\right] \tag{12}$$

Using Bubnov-Galerkin method, considering (12) in equation (11) we get (for hinge fastening $w = f_0\sin\dfrac{\pi}{a}x$)

$$\sigma_{kp} - \tilde{K} = \sigma_{kp}^0 K\left(\varepsilon_1, \varepsilon_2, X\right) \tag{13}$$

here $\sigma_{kp}^0 = \dfrac{\pi^2 D_0}{a^2h_0^2}$ is corresponding value of critical load for homogeneous plate of constant thickness

$$K = 1 + \varepsilon_1 + \frac{2\pi^2-21}{6\pi^2}\varepsilon_1^2 +$$

$$+\frac{3\varepsilon_2}{5}\left(1 + 2\varepsilon_1 + \frac{2\pi^2-8}{\pi^2}\varepsilon_1^2 + \frac{\pi^2-8}{\pi^2}\varepsilon_1^3 + \frac{12\pi^4-160\pi^2+195}{60\pi^4}\varepsilon_1^4\right) \tag{14}$$

By means of (14) we can study the stability of inhomogeneous plate of variable thickness at cylindrical form of stability loss.

For $\varepsilon_1 \neq 0, \quad \varepsilon_2 = 0$ we get the case of homogeneous plate of variable thickness with regard to linear resistance of basis

$$\bar{\sigma}_{kp} = \sigma_{kp}^0 \left(1 + \varepsilon_1 + \frac{2\pi^2 - 21}{6\pi^2}\varepsilon_1^2\right) .$$

For $\varepsilon_1 = 0, \quad \varepsilon_2 \neq 0$ we get the case of inhomogeneous plate of constant thickness with regard to external resistance

$$\bar{\sigma}_{kp} = \sigma_{kp}^0 \left(1 + \frac{3\varepsilon_2}{5}\right) .$$

Numerical calculation was carried out for concrete values of ε_1 and ε_2 where the results are given in the form of tables and typical graphs. The results of calculations show that discount of inhomogeneity for a variable thickness plate essentially influences on the size of critical parameters

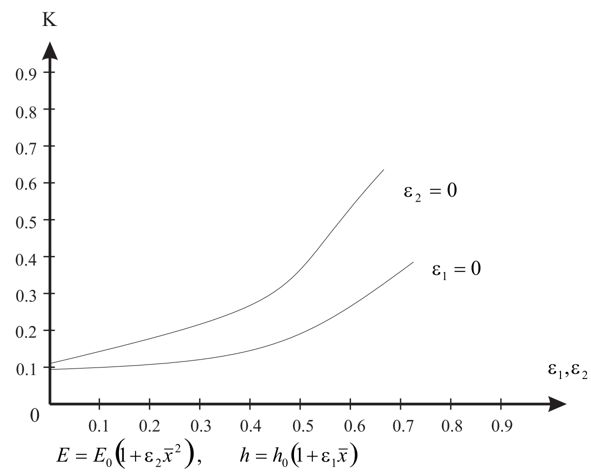

Fig.1. *Bilateral hinge supported rectangular plate inhomogeneous by thickness.*

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