MECHANICS

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INVESTIGATION OF A PLASTIC STATE OF ACCELERATEDLY ROTATING NON-HOMOGENEOUS DISKS

Abstract

The general solutions of distribution of stresses in non-homogeneous acceleratedly rotating disk are obtained. The non-homogeneity in the form of power low changing of elasticity module in radial direction is considered. The ability of appearing plastic flow is determined basing on Tresk plasticity low.

In the paper [1-3] the distribution of stresses and permutations in homogeneous elastic disks of constant and variable thickness were studied. In the present paper the stress-strain state of a disk made of non-homogeneous material is investigated. Investigating the obtained stresses field the possibility of appearing the elastic flow is determined.

Non-uniformly rotating circular ring disk of variable thickness is considered. We accept that the stress state of the disk has radial symmetry and corresponds to the plane stress state.

The equilibrium equation of non-uniformly rotating disk have the following form [1-4]:

$$\frac{d}{dr}\left(\sigma_r h\right)+\frac{h}{r}\left(\sigma_r-\sigma_\theta\right)+\rho\omega^2 hr=0 \tag{1}$$

$$\frac{d}{dr}\left(h\tau_{r\theta}\right)+\frac{2h}{r}\tau_{r\theta}-\rho\alpha hr=0 \tag{2}$$

Here σ_r, σ_θ are normal stresses, $\tau_{r\theta}$ is tangential velocity, ρ is a density of material, h is a thickness of disk by radius, ω is angular speed, α is angular acceleration of disk. In the considered problem the material is assumed non-homogeneous in such sense that the module of elasticity

$$E=A\cdot r^\gamma;\quad G=B\cdot r^\gamma, \tag{3}$$

where γ is a constant, the coefficients A and B are also constant. For the plane stress-state the equation state of deformation compatibility is led to the following equation:

$$\frac{d}{dr}\left(\frac{\sigma_\theta-\nu\sigma_r}{E}\right)+\left(\sigma_\theta-\sigma_r\right)\frac{1+\nu}{E}=0. \tag{4}$$

Let's determine the stress function $\varphi(r)$ with the relations

$$hr\sigma_r = \varphi, \qquad h\sigma_\theta = \frac{d\varphi}{dr} + \rho h\omega^2 r^2. \tag{5}$$

Substituting expressions (5) for stress components in equation (4) we reduce it to the following equation for the function of the stresses φ:

$$r\frac{d}{dr}\left[\frac{1}{Eh}\left(\frac{d\varphi}{dr} - \frac{\nu\varphi}{r} + h\rho\omega^2 r^2\right)\right] + \frac{1+\nu}{Eh}\left(\frac{d\varphi}{dr} - \frac{\varphi}{r} + h\rho\omega^2 r^2\right) = 0 \tag{6}$$

In particular case when

$$h(r) = h_0 \cdot r^n \tag{7}$$

equation (6) is led to the following form

$$r^2\frac{d^2\varphi}{dr^2} + r(1-n-\gamma)\frac{d\varphi}{dr} - [1-\nu(n+\gamma)]\varphi + \rho\omega^2 r^{3+n} h_0 (3+\nu-\gamma) = 0 \tag{8}$$

The general solution of equation (8) has the following form:

$$\varphi = C_1 r^{k_1} + C_2 r^{k_2} + C_3 r^{3+n}, \tag{9}$$

where

$$C_3 = \frac{-\rho\omega^2(3+\nu-\gamma)}{8+3n-3\gamma+\nu\gamma}.$$

The exponents of degrees k_1 and k_2 are the roots of the square equation

$$x^2 \mp (n+\gamma)x - [1-\nu(n+\gamma)] = 0, \tag{10}$$

and the values C_1 and C_2 are arbitrary constants of integration that are determined from the conditions on a contour. On the basis of equations 5) we can find the radial and circular components of stress in the form

$$\sigma_r = C_1 r^{k_1-1} + C_2 r^{k_2-1} - C_3 r^2$$

$$\sigma_\varphi = C_1 k_1 r^{k_1-1} + C_2 k_2 r^{k_2-1} - 3C_3 r^2 \tag{11}$$

In the frame of linear theory equation (2) is integrated independently. The solution of equation (2) after the substitution (7) subject to the boundary condition $\tau_{r\theta} = 0$ will be

$$\tau_{r\theta} = \begin{cases} \frac{\rho\alpha b^2}{n+4}\left[\left(\frac{r}{b}\right)^2 - \left(\frac{b}{r}\right)^{n+2}\right], & n \neq -4 \\ \\ \rho\alpha r^2 \ln\left(\frac{r}{b}\right), & n = -4 \end{cases} \tag{12}$$

At solution of the problem the two forms of boundary conditions are considered.

Case I:

$$u = 0 \quad \text{at} \quad r - a$$
$$\sigma_r = 0 \quad \text{at} \quad r = b \tag{13}$$
$$\tau_{r\theta} = 0 \quad \text{at} \quad r = b$$

Case II:

$$\sigma_r = 0 \quad \text{at} \quad r = a,\ r = b$$
$$\tau_{r\theta} = 0 \quad \text{at} \quad r = b \tag{14}$$

Now we consider only the first case of boundary conditions.

For the constants C_1 and C_2 we'll obtain the following expressions

$$C_1 = \frac{\rho\omega^2}{\left[8 - 3(\gamma + n) - \nu(\gamma + n)\right](1 - \nu k_2)(k_1 - (\gamma + n))\, b^{k_1}} \times$$
$$\times \frac{\rho\omega^2}{-(1 - \nu k_1)(k_2 - \gamma - n)\, b^{k_2} \left\{ a^{3 - 2k_1 + k_2} b^{k_1} (k_1 - \gamma - n)^2 (1 - \nu k_2) \right.} \times$$
$$\times \frac{\rho\omega^2}{\left[3(1 - \nu^2) - (\gamma + n)(1 + \nu^2)\right] - a^{3 - k_1} b^2 (k_1 - \gamma - n)(k_2 - \gamma - n)(1 - \gamma k_1)} \times$$
$$\times \frac{\rho\omega^2}{\left[3(1 - \nu^2) - (n + \gamma)(1 + \nu^2)\right] + a^{k_2 - k_1} b^{1 - \nu k_2}} \times$$
$$\times \frac{\rho\omega^2}{\left[3 + \nu - (\gamma + n)(1 - \nu k_1)(3 - n - \gamma)\, b^2\right] - (3 - 3\nu^2 - n - \gamma)} \times$$
$$\times \frac{\rho\omega^2}{\left. (1 + \nu^2)(k_1 - n - \gamma)\, a^{3 - k_1} b^{k_1 - 1} \right\}} \tag{15}$$

$$C_2 = \frac{\rho\omega^2 (k_2 - \gamma)(ab)^2}{8 - 3(n + \gamma) - \nu(n + \gamma)} \times$$

$$\times \frac{(3 + \nu - n - \gamma)(1 - \gamma k_1)(3 - n - \gamma)\, b^2 - \left(3(1 - \nu^2) - n - \gamma\right)(1 + \nu^2)(k_1 - n - \gamma)\, a^{3 - k_1} b^{k_1 - 1}}{(3 - n - \gamma)(1 - \nu k_2)(k_1 - n - \gamma)\, a^{2 + k_2 - k_1} b^{k_1 + 1} - (1 - k\nu_1)(k_2 - n - \gamma)\, b^{k_2 + 1} a^2} \tag{16}$$

For future investigation it is necessary to investigate the signs and value's of the roots k_1 and k_2 of equation (10).

Analysis shows that we have the cases:

At

$$n + \gamma > \frac{1}{\nu}, \quad k_1 \cdot k_2 > 0 \tag{17}$$

$$k_1=-\frac{n+\gamma}{2}+\sqrt{\left(\frac{n+\gamma}{2}\right)^2+\left[(1-\nu)(n+\gamma)\right]}<0$$

$$k_2=-\frac{n+\gamma}{2}-\sqrt{\left(\frac{n+\gamma}{2}\right)^2+\left[(1-\nu)(n+\gamma)\right]}<0$$

At

$$(n+\gamma)<\frac{1}{\nu},\quad k_1\cdot k_2<0 \tag{18}$$

$$k_1>0,\qquad k_2<0$$

$$n+\gamma=\frac{1}{\nu},\qquad k_1\cdot k_2=0$$

At

$$k_1=0,\quad k_2=-\frac{1}{\nu} \tag{19}$$

The expressions for the main stresses are given in the form

$$(\sigma_1,\sigma_2)=\frac{1}{2}(\sigma_r+\sigma_\theta)\pm\sqrt{\frac{1}{4}(\sigma_r-\sigma_\theta)^2+\tau_{r\theta}^2}$$

$$\sigma_3=\sigma_z=0 \tag{20}$$

We use the obtained stress components for determining the values ω and α that involve in disk material of plastic flow at some value of radius.

We accept the Tresk yield condition. The elastic deformations begin at some radius when

$$\max\left\{|\sigma_1-\sigma_2|,\ |\sigma_2-\sigma_3|,\ |\sigma_3-\sigma_1|\right\}=2k=Y \tag{21}$$

where Y is a yield point on expansion.

Let's determine the sign of the expression $\sigma_r+\sigma_\theta$ in case when $n+\gamma=\frac{1}{\nu}$, in this case it is easy to establish that $\sigma_r+\sigma_\theta<0$ at $0.5<\frac{a}{b}<0.9$, and for the cases $\frac{a}{b}<0.5$, $(\sigma_r+\sigma_\theta)$ changes the sign from the positive at $r=a$ to negative at $r=b$. For clearness we will be restricted in future to the cases $0.5\le\frac{a}{b}\le 0.9$.

If describe conditions (21) we'll have

$$f_i=|\sigma_1-\sigma_2|=2\sqrt{\frac{1}{4}(\sigma_r-\sigma_\theta)^2+\tau_{r\theta}^2}=$$

$$=2\sqrt{\frac{1}{4}\left(\frac{C_1}{r}-\frac{3C_2}{r^3}\right)+\frac{\rho^2\alpha^2}{16}\left(r^2-\frac{b^4}{r^2}\right)^2}=2k \tag{21'}$$

$$f_2=|\sigma_2|=\left|\frac{1}{2}(\sigma_r+\sigma_\theta)-\left[\frac{1}{4}(\sigma_r-\sigma_\theta)^2+\tau_{r\theta}^2\right]^{\frac{1}{2}}\right|=$$

$$=\left|\frac{1}{2}\cdot\frac{C_1}{r}+\frac{C_2}{r^3}-5\rho\omega^2r^2\right|-$$

$$-\left\{\frac{1}{4}\left(\frac{C_1}{r}-\frac{3C_2}{r^3}\right)^2+\frac{(\rho\alpha)^2}{16}\left(\frac{b^4}{r^2}-r^2\right)^2\right\}^{\frac{1}{2}}=2k \qquad (21'')$$

Substituting the values of constants $A\,(7)$ and $B\,(8)$ in expressions $(21')$ and $(21'')$ and we'll obtain

$$f_1=\sqrt{\left[\rho\omega^2\frac{a^3\left[\frac{a^2}{b^2}+2+3\left(\frac{b}{a}\right)^5\right]}{r\left(\frac{b^2}{a^2}-1\right)}+\frac{3\rho\omega^2a^3b^2\left(3\left(\frac{b}{a}\right)^3-1\right)}{2r^3\left(\frac{b^2}{a^2}-1\right)}\right]^2+\frac{(\rho\alpha r)^2}{4}\left(\frac{b^4}{r^4}-1\right)^2}$$

$$f_2=\left|-\frac{\rho\omega^2}{\gamma}\frac{a^3}{r}\frac{\left(\frac{b^2}{a^2}+2-3\left(\frac{b}{a}\right)^5\right)}{\left(\frac{b^2}{a^2}-1\right)}+\frac{\rho\omega^2b^2a^3\left(3\left(\frac{b}{a}\right)^3-1\right)}{4r^3\left(\frac{b^2}{a^2}-1\right)}-5r^2\rho\omega^2-\right.$$

$$\left.-\frac{1}{2}\sqrt{\left[\rho\omega^2\frac{a^3}{r}\frac{\left(\frac{b^2}{a^2}+2+3\left(\frac{b}{a}\right)^5\right)}{\left(\frac{b}{a}\right)^2-1}+\frac{3\rho\omega^2a^3b^2\left(3\left(\frac{b}{a}\right)^3-1\right)}{2r^3\left(\frac{b^2}{a^2}-1\right)}\right]^2+\frac{\left(\rho\alpha r^2\right)^2}{4}\left(\frac{b^4}{r^4}-1\right)^2}\right|.$$

$$f_2\,(r)=2k$$

will be a yield point if the condition

$$\frac{\rho\omega^2a^3}{8r}\frac{\left(\left(\frac{b}{a}\right)^2+2+3\left(\frac{b}{a}\right)^5\right)}{\left(\frac{b}{a}\right)^2-1}-\frac{\rho\omega^2b^2a^3\left(3\left(\frac{b}{a}\right)^3-1\right)}{4r^3\left(\left(\frac{b}{a}\right)^2-1\right)}+5\rho\omega^2r^2>$$

$$>\frac{1}{2}\sqrt{\left[\frac{\rho\omega^2a^3}{r}\frac{\left(\frac{b^2}{a^2}+2+3\left(\frac{b}{a}\right)^5\right)}{\left(\frac{b}{a}\right)^2-1}+\frac{3\rho\omega^2a^3b^2\left(3\left(\frac{b}{a}\right)^3-1\right)}{2r^3a^2\left(\frac{b^2}{a^2}-1\right)}\right]^2+\frac{\left(\rho\alpha r^2\right)^2}{4}\left(\left(\frac{b}{r}\right)^4-1\right)}$$

is fulfilled.

Otherwise,

$$f_1\,(r)=2k.$$

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