Vagif Ya. GULMAMEDOV

ON REGULAR SOLVABILITY OF ONE INITIAL-BOUNDARY VALUE PROBLEM FOR THIRD ORDER OPERATOR-DIFFERENTIAL EQUATIONS

Abstract

In the paper sufficient conditions are obtained for regular solvability of boundary value problem for one class of third order operator differential equations, whose principle part contains normal operator.

Let H be a separable Hilbert space, operator A be a normal operator with completely continuous inverse operator A^{-1} in H. Suppose that the operator A has polar expansion $A = UC$, where U is unitary operator, C is a positive definite self-adjoint operator, i.e., C where $C = C^* > dE (d > 0)$, where E is a unit-operator in the space H.

We suppose that spectrum of the operator A is contained in the angular sector

$$S_\varepsilon = \{\lambda : |\arg \lambda| \leq \varepsilon\}, \ 0 \leq \varepsilon < \frac{\pi}{6}.$$

The operators A and C have the following representations:

$$A\cdot = \sum_{k=1}^{\infty} \lambda_k (\cdot, e_k) e_k, \ C\cdot = \sum_{k=1}^{\infty} |\lambda_k| (\cdot, e_k) e_k,$$

where λ_k are eigenvalues of the operator A and $a\{e_k\}$ is corresponding orthonormalized system of eigenvectors. By means of operator C we construct scale of Hilbert spaces $H_\gamma (\gamma \geq 0)$ in the following form: $H_\gamma = D(A^\gamma)$. H_γ is a Hilbert space with respect to the scalar product

$$(x, y)_\gamma = (C^\gamma x, C^\gamma y), \ x, y \in D(A^\gamma).$$

For $\gamma = 0$ we assume that $H_0 = H$ and $(x, y)_0 = (x, y), x, y \in H$. Further, denote by $L_2(R; H_\gamma)$ and $L_2(R_+; H_\gamma), (R = (-\infty; \infty), R_+ = (0; \infty))$ spaces of all vector functions $f(t)$ with values from the space H_γ, strongly measurable and for which the Bochner integrals are finite

$$\|f(t)\|_{L_2(R;H)}=\left(\int\limits_{-\infty}^{+\infty}\|f(t)\|_H^2\,dt\right)^{\frac{1}{2}}<\infty,$$

$$\|f(t)\|_{L_2(R_+;H)}=\left(\int\limits_{0}^{+\infty}\|f(t)\|_H^2\,dt\right)^{\frac{1}{2}}<\infty,$$

respectively.

For $\gamma=0$, suppose $L_2(R;H_0)=L_2(R;H)$ and $L_2(R_+;H_0)=L_2(R_+;H)$.

Suppose that vector function $u(t)\in L_2(R;H)$ has generalized derivatives till the third order, moreover, $u^{(3)}(t)\in L_2(R;H)$. Then following the book [1], we define the following Hilbert space

$$W_2^3(R;H)=\left\{u(t)\left|u^{(3)}(t)\in L_2(R;H), C^3u(t)\in L_2(R;H)\right.\right\}$$

with the norm $\|u\|_{W_2^3(R;H)}=\left(\left\|u^{(3)}\right\|^2_{L_2(R;H)}+\left\|C^3u\right\|^2_{L_2(R;H)}\right)^{\frac{1}{2}}$. In the same way we define the space $W_2^3(R_+;H)$:

$$W_2^3(R_+;H)=\left\{u(t)\left|u^{(3)}(t)\in L_2(R_+;H), C^3u(t)\in L_2(R_+;H)\right.\right\}$$

with the norm

$$\|u\|_{W_2^3(R_+;H)}=\left(\left\|u^{(3)}\right\|^2_{L_2(R_+;H)}+\left\|C^3u\right\|^2_{L_2(R_+;H)}\right)^{\frac{1}{2}}.$$

From the theorem of intermediate derivative it follows that the subspace

$$W_2^3(R;H;0;1)=\left\{u\left|u\in W_2^3(R_+;H), u(0)=u'(0)=0\right.\right\}$$

is also a Hilbert space.

In the given paper we investigate solvability of the following boundary value problem:

$$\frac{d^3u(t)}{dt^3}-A^3u(t)+A_1\frac{d^2u(t)}{dt^2}+A_2\frac{du(t)}{dt}+A_3u(t)=f(t), t\in R_+=(0;+\infty), \quad (1)$$

$$u(0)=u'(0)=0, \quad (2)$$

where the derivatives are understood in the sence of distribution theorem [1], operator A is normal, A_1,A_2,A_3 are linear operators in $H, f(t)\in L_2(R_+;H)$, $u(t)\in W_2^3(R_+;H)$. Later on we suppose that for operator-coefficients A,A_2,A_3 the following conditions are satisfied

1) A is normal operator with completely inverse operator A^{-1} with
the spectrum in the angular sector $S_\varepsilon = \{\lambda : |\arg \lambda| \le \varepsilon\}$, where $0 \le \varepsilon < \dfrac{\pi}{6}$.
2) The operators $B_j = A_j A^{-j} (j = 1, 2, 3)$ are bounded in H.

It follows from condition (1) that operator A has polar expansion $A = UC = CU$, where C is a positive definite shelf-adjoint operator and U is a unitary operator in H.

Definition 1.1. *If vector function $u(t) \in W_2^3(R_+; H)$ satisfies equation (1) almost everywhere, then we call it a regular solution of equation (1).*

Definition 1.2. *If for any $f(t) \in L_2(R_+; H)$ there exists a regular solution of equation (1) which satisfies boundary conditions (2) in the sense*

$$\lim_{t\to+0} \|u(t)\|_{5/2} = 0, \ \lim_{t\to+0} \|u(t)\|_{3/2} = 0,$$

and inequality

$$\|u\|_{W_2^3(R_+;H)} \le const \, \|f\|_{L_2(R_+;H)} ,$$

is fulfilled for it, then problem (1), (2) is called regular solvable.

In the given paper we find algebraic conditions expressed by the coefficients of operator differential equation (1), which allow to establish regular solvability of problem (1), (2), when operator A in the principal part of equation is a normal operator. Note that when $A = C$ is a sell-adjoint positive definite operator, these conditions were known even for differential equations of higher odd order (see [2]). For discontinuous coefficients when $n = 3$ and $A = C$ is positive definite self-adjoint operator. These conditions were found in the paper [3]. For equations of arbitrary odd order, conditions for solvability are in the paper [4]. In the author's paper [5] analogous boundary value problems with normal principle part for equations of even order $n = 2k$ were investigated.

Let us solve the boundary value problem (1), (2). Denote by

$$P_0 u = \frac{d^3 u}{dt^3} - A^3 u, u \in W_2^3(R_+; H), \tag{3}$$

$$P_1 u = A_1 \frac{d^2 u}{dt^2} + A_2 \frac{du}{dt} + A_3 u, u \in W_2^3(R_+; H), \tag{4}$$

$$P u = P_0 u + P_1 u, u \in W_2^3(R_+; H). \tag{5}$$

The following lemma is proved similar to the lemma of the paper [5].

Lemma 1. *Let conditions (1), (2) be satisfied. Then operators P_0, P_1 and consequently P are continuous operators acting from space $W_2^3(R_+; H)$ to $L_2(R_+; H)$.*

The following theorem is valid.

Theorem 1. *Let condition 1) be satisfied. Then operator P_0 is isomorphically maps the space $W_2^3(R_+;H;0;1)$ into $L_2(R_+;H)$.*

Proof. It follows from lemma 1 that the operator P_0. Acting from space $W_2^3(R_+;H;0;1)$ to $L_2(R_+;H)$ is continuous. On the other hand, $P_0u = 0$ the equation to $u \in W_2^3(R_+;H;0;1)$ is equivalent to boundary problem

$$P_0 = \frac{d^3u}{dt} - A^3u, u \in W_2^3(R_+;H), \tag{6}$$

$$u(0) = u'(0) = 0. \tag{7}$$

It is known that a general solution of homogeneous equation (6) from space $W_2^3(R_+;H)$ has the form:

$$u(t) = e^{\omega_1 tA}\varphi_1 + e^{\omega_{2tA}}\varphi_2$$

where $\varphi_1, \varphi_2 \in H_{5/2}$ and $\omega_1 = \overline{\omega_2} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \left(\omega_1^3 = \omega_2^3 = 1\right)$.

Using boundary conditions (7) we obtain that $\varphi_1 = \varphi_2 = 0$, i.e. $u(t) = 0$.

Thus, operator P_0 vanishes only at zero in space $W_2^3(R_+;H;0;1)$. Let us show that its image coincides with whole space $L_2(R_+;H)$.

To this end we consider the equation

$$P_0u_1 = f, \ f \in L_2(R_+;H), u_1 \in W_2^3(R_+;H;0;1),$$

which is equivalent to the boundary value problem:

$$\frac{d^3u_1}{dt^3} - A^3u_1 = f(t), t \in R_+, \tag{8}$$

$$u_1(0) = u_1'(0) = 0. \tag{9}$$

If we denote by $\tilde{f}(\xi), \xi \in R$ Fourier transformation of vector functions $f(t)$ then $u_1(t) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \tilde{u}_1(\xi) \ \hat{f}(\xi)e^{it\zeta}d\zeta, t \in R$, where $\tilde{u}_1(\xi) = \left((-i\xi)^3 - A^3\right)^{-1}, \xi \in R$, satisfies equation (8) almost everywhere in R.

Let us show that $u_1(t)$ belongs to space $W_2^3(R_+;H)$. According to Plancheral theorem it suffices to show that $\xi^3\hat{u}_1(\xi) \in L_2(-\infty;\infty), A^3\hat{u}_1(\xi) \in L_2(-\infty;\infty)$.

Since

$$\|u_1\|^2_{W_2^3(R;H)} = \left\|u_1^{(3)}\right\|^2_{L_2(R;H)} + \left\|A^3u_1\right\|^2_{L_2(R;H)} =$$

$$= \left\|\xi^3\hat{u}_1(\xi)\right\|^2_{L_2(R;H)} + \left\|A^3\hat{u}_1(\xi)\right\|^2_{L_2(R;H)}.$$

Then we have from (10) that it suffices to estimate $\left\|A^3\hat{u}_1(\xi)\right\|^2_{L_2(R;H)}$.

Since

$$\left\|A^3\hat{u}_1\right\|^2_{L_2(R;H)} \le \sup_{\xi\in R}\left\|A^3\left(-i\xi^3E+A^3\right)^{-1}\right\|^2_{H\to H}\left\|\hat{f}(\xi)\right\|_{L_2(R;H)} \tag{10}$$

Then we obtain from the spectral expansion of operator A :

$$\left\|A^3\left((-i\xi)^3E-A^3\right)^{-1}\right\|^2_{H\to H)} \le \sup_k\left|\lambda_k^3\left(-i\xi^3+\lambda_k^3\right)^{-1}\right| \le$$

$$\le \sup_k\frac{|\lambda_k|^6}{|\lambda_k|^6+\xi^6-2|\lambda_k|^3|\xi|^3\sin 3\varepsilon} \le \tag{11}$$

$$\le \sup_k\frac{\zeta^6}{\left(|\lambda_k|^6+\xi^6\right)(1-\sin 3\varepsilon)} \le \frac{1}{1-\sin 3\varepsilon}.$$

Thus, $u_1(t)\in W_2^3(R;H)$. Denote by $u_1(t)(t\in R)$ contraction of vector function, in semiaxis $R_+=(0;\infty)$ in $\upsilon(t)$. Obviously, $\upsilon(t)\in W_2^3(R_+;H)$ and $\upsilon(t)$ satisfies equation (8) almost everywhere in R_+. Then general solution of equation (8) can be written in the form

$$u(t)=\upsilon(t)+e^{\omega_1 tA}\varphi_1+e^{\omega_2 tA}\varphi_2,$$

where vectors $\varphi_1,\varphi_2\in H_{5/2}$ and they are defined from boundary conditions (9):

$$\varphi_1=\frac{1}{\omega_2-\omega_1}\left(A^{-1}\upsilon'(0)-\omega_2\upsilon(0)\right)\in H_{5/2},$$

$$\varphi_2=\frac{1}{\omega_1-\omega_2}\left(A^{-1}\upsilon'(0)-\omega_1\upsilon(0)\right)\in H_{5/2}.$$

Since operator P_0 acting from $W_2^3(R_+;H;0;1)$ to $L_2(R_+;H)$ is bounded, then we obtain that according to Banach theorem on the inverse operator that if the operator P_0^{-1} exists and bounded, then P_0 realizes isomorphism between the spaces $W_2^3(R_+;H;0;1)$ and $L_2(R_+;H)$.

The theorem is proved.

Lemma 2. *Let condition (1) be satisfied. Then for any* $u(t)\in W_2^3(R_+;H;0;1)$ *the following inequality holds*

$$\|P_0u_1\|^2_{L_2(R_+;H)} \ge (1-\sin 3\varepsilon)\|u\|^2_{W_2^3(R_+;H)}. \tag{12}$$

Proof. Let $u(t)\in W_2^3(R_+;H;0;1)$. Then it is obvious that

$$\|P_0u\|^2_{L_2(R_+;H)}=\left\|\frac{d^3u}{dt^3}-A^3u\right\|^2_{L_2(R_+;H)}=\left\|\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}+$$

$$+\left\|\frac{d^3u}{dt^3}-A^3u\right\|^2_{L_2(R_+;H)}-2\operatorname{Re}\left(\frac{d^3u}{dt^3},A^3u\right)_{L_2(R_+;H)}= \tag{13}$$

$$=\|u\|^2_{W_2^3(R_+;H)}-2\operatorname{Re}\left(\frac{d^3u}{dt^3},A^3u\right)_{L_2(R_+;H)}.$$

For $u(t)\in W_2^3(R_+;H;0;1)$ $(u(0)=u'(0)=0)$ after integration by parts we obtain

$$\left(\frac{d^3u}{dt^3},A^3u\right)_{L_2(R_+;H)}=-\int_0^{+\infty}\left(A^{*^3}u,\frac{d^3u}{dt^3}\right)_H dt=-\left(A^{*^3}u,\frac{d^3u}{dt^3}\right)_{L_2(R_+;H)}. \tag{14}$$

Hence

$$2\operatorname{Re}\left(\frac{d^3u}{dt^3},A^3u\right)_{L_2(R_+;H)}=\left(\frac{d^3u}{dt^3},A^3u\right)_{L_2(R_+;H)}+\left(A^3u,\frac{d^3u}{dt^3}\right)_{L_2(R_+;H)}=$$

$$-\left(A^{*^3}u,\frac{d^3u}{dt^3}\right)_{L_2(R_+;H)}+\left(A^3u,\frac{d^3u}{dt^3}\right)_{L_2(R_+;H)}=$$

$$=\left((A^3-A^{*^3})u,\frac{d^3u}{dt^3}\right)_{L_2(R_+;H)}=\left((E-A^{*^3}A^{-3})A^{-3}u,\frac{d^3u}{dt^3}\right)_{L_2(R_+;H)}.$$

Thus,

$$\left|2\operatorname{Re}\left(\frac{d^3u}{dt^3},A^3u\right)_{L_2(R_+;H)}\right|\le$$

$$\le\left\|\left(E-A^{*^3}A^{-3}\right)A^3u\right\|_{L_2(R_+;H)}\left\|\frac{d^3u}{dt^3}\right\|_{L_2(R_+;H)}\le \tag{15}$$

$$\le\left\|E-A^{*^3}A^{-3}\right\|_{H\to H}\cdot\left\|A^3u\right\|_{L_2(R_+;H)}\cdot\left\|\frac{d^3u}{dt^3}\right\|_{L_2(R_+;H)}.$$

Using spectral expansion of the operator A we obtain:

$$\left\|E-A^{*^3}A^{-3}\right\|\le\sup_k\left|1-\left(\frac{\overline{\lambda}_k}{\lambda_k}\right)^3\right|=\sup_k\left|1-\left(\frac{e^{-3i\varphi_k}}{e^{3i\varphi_k}}\right)^3\right|=$$

$$=\sup_k\left|e^{-3i\varphi_k}-e^{3i\varphi_k}\right|=2\sup_k\sin 3\varphi_k\le 2\sin 3\varepsilon.$$

Taking into account the last inequality in (15) and using Cauchy inequality we obtain:

$$\left| 2\operatorname{Re}\left(\frac{d^3u}{dt^3}, A^3u \right)_{L_2(R_+;H)} \right| \le \sin 3\varepsilon \left\| u \right\|^2_{W_2^3(R_+;H)} \tag{16}$$

Then equality (13) subject to inequality (16) implies that

$$\left\| P_0 u \right\|^2_{L_2(R_+;H)} = \left\| u \right\|^2_{W_2^3(R_+;H)} - \left| 2\operatorname{Re}\left(\frac{d^3u}{dt^3}, A^3u \right)_{L_2(R_+;H)} \right| \ge$$

$$\ge \left\| u \right\|^2_{W_2^3(R_+;H)} - \sin 3\varepsilon \left\| u \right\|^2_{W_2^3(R_+;H)} = (1 - \sin 3\varepsilon) \left\| u \right\|^2_{W_2^3(R_+;H)} .$$

The lemma is proved.

Lemma 3. *For any $u(t) \in W_2^3(R_+; H; 0; 1)$ the following inequalities hold*

$$\left\| A^2 \frac{du}{dt} \right\|_{L_2(R_+;H)} \le \frac{2^{\frac{1}{3}}}{3^{\frac{1}{2}}} (1 - \sin 3\varepsilon)^{-\frac{1}{2}} \left\| P_0 u \right\|_{L_2(R_+;H)} , \tag{17}$$

$$\left\| A \frac{d^2u}{dt^2} \right\|_{L_2(R_+;H)} \le \frac{2^{\frac{1}{3}}}{3^{\frac{1}{2}}} (1 - \sin 3\varepsilon)^{-\frac{1}{2}} \left\| P_0 u \right\|_{L_2(R_+;H)} , \tag{18}$$

$$\left\| A^3 u \right\|_{L_2(R_+;H)} \le (1 - \sin 3\varepsilon)^{-\frac{1}{2}} \left\| P_0 u \right\|_{L_2(R_+;H)} . \tag{19}$$

Proof. Since $A = UC$, where U and C are permutable and $u(t) \in W_2^3(R_+; H; 0; 1)$, $u(0) = u'(0) = 0$ then after integration by parts we obtain:

$$\left\| A^2 \frac{du}{dt} \right\|^2_{L_2(R_+;H)} = \left\| C^2 \frac{du}{dt} \right\|^2_{L_2(R_+;H)} = \int_0^{+\infty} \left(C^2 \frac{du}{dt}, C^2 \frac{du}{dt} \right)_H dt =$$

$$= -\left(C^3 u, C \frac{d^2u}{dt^2} \right)_{L_2(R_+;H)} \le \left\| C^3 u \right\|_{L_2(R_+;H)} \cdot \left\| C \frac{d^2u}{dt^2} \right\|_{L_2(R_+;H)} =$$

$$= \left\| A^3 u \right\|_{L_2(R_+;H)} \left\| A \frac{d^2u}{dt^2} \right\|_{L_2(R_+;H)} .$$

Then for any $\varepsilon > 0$, the following inequality holds

$$\left\| A^2 \frac{du}{dt} \right\|^2_{L_2(R_+;H)} \le \frac{\varepsilon}{2} \left\| A^3 u \right\|^2_{L_2(R_+;H)} + \frac{1}{2\varepsilon} \left\| A \frac{d^2u}{dt^2} \right\|^2_{L_2(R_+;H)} . \tag{20}$$

On the other hand, analogously proceeding we obtain that for $u(t) \in W_2^3(R_+; H; 0; 1)$ the following inequality holds:

$$\left\| A \frac{d^2u}{dt^2} \right\|^2_{L_2(R_+;H)} \le \frac{\sigma}{2} \left\| A^2 \frac{du}{dt} \right\|^2_{L_2(R_+;H)} + \frac{1}{2\sigma} \left\| \frac{d^3u}{dt^3} \right\|^2_{L_2(R_+;H)} . \tag{21}$$

Taking into account inequality (21) in (20) we have:

$$\left\|A^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)} \leq \frac{\varepsilon}{2}\left\|A^3u\right\|^2_{L_2(R_+;H)} + \frac{1}{4\varepsilon\sigma}\left\|\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)} + \frac{\sigma}{4\varepsilon}\left\|A^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)}.$$

Assuming in the last inequality: $2\varepsilon^2 = \sigma^{-1}$ we obtain that for $\varepsilon > 0$ the inequality

$$\left\|A^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)} \leq \frac{\varepsilon}{2}\left(\left\|A^3u\right\|^2_{L_2(R_+;H)} + \left\|\frac{d^3u}{dt^3}\right\|^2_{L_2(R_+;H)}\right) + \frac{1}{8\varepsilon^3}\left\|A^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)}$$

holds.

Hence we have:

$$\left(1-\frac{1}{8\varepsilon^3}\right)\left\|A^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)} \leq \frac{\varepsilon}{2}\left\|u\right\|^2_{W_2^3(R_+;H)},$$

If we suppose $\varepsilon > \frac{1}{2}$ we obtain:

$$\left\|A^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)} \leq \frac{4\varepsilon^4}{8\varepsilon^3-1}\left\|u\right\|^2_{W_2^3(R_+;H)}.$$

If we minimize the right-hand side of the last inequality with respect to $\varepsilon > \frac{1}{2}$ we obtain that for $\varepsilon = \frac{1}{\sqrt[3]{2}}\left(\varepsilon > \frac{1}{2}\right)$, the following inequality holds

$$\left\|A^2\frac{du}{dt}\right\|^2_{L_2(R_+;H)} \leq \frac{2^{\frac{2}{3}}}{3}\left\|u\right\|^2_{W_2^3(R_+;H)}, \text{ i.e. } \left\|A^2\frac{du}{dt}\right\|_{L_2(R_+;H)} \leq \frac{2^{\frac{2}{3}}}{3^{\frac{1}{2}}}\left\|u\right\|_{W_2^3(R_+;H)}.$$

Applying lemma 2 (inequality (12)) to the last inequality, we obtain:

$$\left\|A^2\frac{du}{dt}\right\|_{L_2(R_+;H)} = \frac{2^{\frac{1}{3}}}{3^{\frac{1}{2}}}(1-\sin 3\varepsilon)^{-\frac{1}{2}}\left\|P_0u\right\|_{L_2(R_+;H)}$$

Thus, inequality (17) is proved. Inequality (18) is proved in the same way. Inequality (19) follows from inequality (12). The lemma is proved.

Let us prove a theorem on regular solvability of problem (1), (2).

Theorem 2. *Let conditions 1), 2) be satisfied, moreover, inequality*

$$\frac{2^{\frac{1}{3}}}{3^{\frac{1}{2}}}\left(\|B_1\| + \|B_2\|\right) + \|B_3\| < (1-\sin 3\varepsilon)^{\frac{1}{2}}$$

holds. Then problem (1), (2) is regular solvable.

Proof. We rewrite problem (1), (2) in the form $Pu = f$, $u \in W_2^3(R_+;H;0;1)$, $f \in L_2(R_+;H)$ i.e. $(P_0 + P_1)u = f$. If we change $P_0u = v$, we obtain equation $\left(E + P_1P_0^{-1}\right)v = f$ in space $L_2(R_+;H)$. Since

$$\left\|P_1P_0^{-1}v\right\|_{L_2(R_+;H)} = \left\|P_1u\right\|_{L_2(R_+;H)} \leq \left\|A_1\frac{d^2u}{dt^2}\right\|_{L_2(R_+;H)} +$$

$$+\left\|A_2\frac{du}{dt}\right\|_{L_2(R_+;H)}+\|A_3u\|_{L_2(R_+;H)}\leq\left\|A_1A^{-1}\right\|\cdot\left\|A\frac{d^2u}{dt^2}\right\|_{L_2(R_+;H)}+$$

$$+\left\|A_2A^{-2}\right\|\cdot\left\|A^2\frac{du}{dt}\right\|_{L_2(R_+;H)}+\left\|A_3A^{-3}\right\|\cdot\left\|A^3u\right\|_{L_2(R_+;H)},$$

then applying lemma 3 in the last inequality we obtain:

$$\left\|P_1P_0^{-1}\upsilon\right\|_{L_2(R_+;H)}=\left[\frac{2^{\frac{1}{3}}}{3^{\frac{1}{2}}}\left(\|B_1\|+\|B_2\|\right)+\|B_3\|\right]\times$$

$$\times(1-\sin 3\varepsilon)^{-\frac{1}{2}}\|\upsilon\|_{L_2(R_+;H)}<\|\upsilon\|_{L_2(R_+;H)},$$

i.e. norm of operator $P_1P_0^{-1}$ is less than unity. Therefore operator $E+P_1P_0^{-1}$ is invertible in the space $L_2(R_+;H)$ and $u(t)=P_0^{-1}\left(E+P_1P_0^{-1}\right)^{-1}f(t)$. Thence, $\|u\|_{W_2^3(R_+;H)}\leq const\cdot\|f\|_{L_2(R_+;H)}$.

The theorem is proved.

References

[1]. Lions J.L., Majenec E. N*onhomogenous boundary value problems and their applications.* Moscow, "Mir", 1971, 371 p. (Russian)

[2]. Aliyev A.R. *On solvability of boundary value problem for third order operator differential equations with discontinuous coefficients.* Proc. Of IMM of AS Azerb., 1997, v.VII(XV) , pp.18-25. (Russian)

[3]. Aliev A.R. *On solvability of one class of boundary problems for operator differential equations of odd order with variable coefficients.* Doklady NAN Azerb., 2001, v. LVII, No4-6, pp.3-8. (Russian)

[4]. Mirzoyev S.S. *On conditions of correct solvability of boundary problems for operator differential equations.* Soviet Math. Dokl., 1983, v.273, No2, pp.292-295. (Russian)

[5]. Gulmamedov V.Ya.// *On correct solvability of boundary value problems for the even order operator-differential equations.* Transactions of NAS of Azerb. Baku, 1999, v.XIX, No1-2, pp.70-75.

Vagif Ya. Gulmamedov

Azerbaijan State Economic University.

6, Istiglaliyyat str., AZ1001, Baku, Azerbaijan.

E-mail: babek_1958@rambler.ru

Tel.: (99412) 927 769 (off.)

Received September 10, 2002; Revised September 29, 2003.

Translated by Azizova R.A.