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ON COMPLETENESS OF ELEMENTARY SOLUTIONS OF THE FOURTH ORDER HOMOGENOUS OPERATOR-DIFFERENTIAL EQUATIONS OF THE ELLIPTIC TYPE

Abstract

The conditions, providing, completeness of the decreasing elementary solutions of one class of fourth order operator-differential equations are found. In the work it is proved existence of the regular solution of corresponding homogeneous operator-differential equation, when the boundary conditions contain operators, and it is proved a completeness of some arbitrary chains, constructed by these boundary conditions.

Let $\mathcal{H}$ be a separable Hilbert space, A positively defined self-adjoint operator in $\mathcal{H}$. It is known that, the domain of the operator $A^\gamma (\gamma > 0)$ becomes a Hilbert Space $\mathcal{H}_\gamma$ with respect to the scalar products $(x,y)_\gamma = (A^\gamma x, A^\gamma y), x,y \in D(A^\gamma)$. We'll denote by $L_2(R_+;\mathcal{H}_\gamma)$ a set of all measurable Bohner vector-functions with the values from $\mathcal{H}_\gamma$, for which $\|f\| = \left(\int\limits_0^\infty \|f(t)\|_\gamma^2 \, dt\right)^{1/2} < \infty$. Further, let $L(X,Y)$ define a set of linear restrictions of the operators acting from the Hilbert Space X to another Y; $\sum(.)$ be a spectrum of the operator $(.)$; $\sum_\infty$ be an ideal of the completely continuous operators in $L(\mathcal{H},\mathcal{H})$; $\sum_p = \left\{A : A \in \sum_\infty, \sum\limits_{n=1}^\infty s_n^p(A) < \infty\right.$ where $s_n(A)$ $-s$ are numbers of the operator $A\}$; in future everywhere u', u'', u''' and $u^{(4)}$ are derivatives in the since of distributions theory [1].

Now let's introduce the following sets:

$$W_2^4(R_+;\mathcal{H}) = \{u : u \in L_2(R_+;\mathcal{H}_4), u^{(4)} \in L_2(R_+;\mathcal{H})\},$$

$$\overset{\circ}{W}{}_2^4(R_+;\mathcal{H}) = \{u : u \in W_2^4(R_+;\mathcal{H}), u(0) = u'(0) = u''(0) = u'''(0) = 0\},$$

$$W_2^{4,T,k}(R_+;\mathcal{H}) = \left\{u : u \in W_2^4(R_+;\mathcal{H}), u(0) = Tu''(0), u'(0) = Ku'''(0),\right.$$

$$\left. T \in L(\mathcal{H}_{3/2},\mathcal{H}_{7/2}), K \in L(\mathcal{H}_{1/2},\mathcal{H}_{5/2})\right\}.$$

Each of these sets provided with norm

$$\|u\|_{W_2^4} = \left(\|u\|^2_{L_2(R_+;\mathcal{H}_4)} + \left\|u^{(4)}\right\|^2_{L_2(R_+;\mathcal{H})}\right)^{1/2},$$

becomes a Hilbert space [1,p.29].

Now we'll pass to the statement of the problems, which we are studying. Let $B_1, B_2, B_3 \in L(\mathcal{H};\mathcal{H})$, then a domain of the operator bundle

$$P(\lambda) = \lambda^4 E + \lambda^3 B_3 A + \lambda^2 B_2 A^2 + \lambda B_1 A^3 + A^4 \qquad (1)$$

coincides with the space $\mathcal{H}_4$; here E single operator in $\mathcal{H}$. In the theorem on the completeness of decreasing elementary solutions of the equation

$$P(d/dt)u = u^{(4)} + B_3Au''' + B_2A^2u'' + B_1A^3u' + A^4u = 0 \qquad (2)$$

by fulfilling the boundary conditions:

$$u(0) - Tu''(0) = \varphi, \varphi \in \mathcal{H}_{7/2}, u'(0) - Ku'''(0) = \psi, \psi \in \mathcal{H}_{5/2} \qquad (3)$$

in the corresponding space of solutions of problem (2), (3) in supposition $A^{-1} \in \sum_p$.

To this end, at first we shall consider the operator-differential equation:

$$P(d/dt) = u^{(4)} + B_3Au''' + B_2A^2u'' + B_1A^3u' + A^4u = f,\ t \in R_+ \qquad (4)$$

by fulfilling the boundary conditions

$$u(0) = Tu''(0),\ u'(0) = Ku'''(0) \qquad (5)$$

where almost everywhere $f(t) \in \mathcal{H}, u(t) \in \mathcal{H}$.

The questions on the completeness of the elementary solutions in the case when the operators are in the boundary conditions are investigated for example, in the work [5] for second order equations.

Definition 1. *Problem (4), (5) is called regular solvable, if for each vector-function* $f(t) \in L_2(R_+; H)$ *there exists a unique vector-function* $u(t) \in \in W_2^{4,T,K}(R_+; H)$*, which satisfies equation (4) almost everywhere in* R_+*, boundary conditions (5) are fulfilled in the sense of convergence of the space* $H_{7/2}, H_{5/2}$ *and it holds the inequality*

$$\|u\|_{W_2^4} \le const \, \|f\|_{L_2}. \qquad (6)$$

Let's find conditions, providing regular solvability of problem (4), (5).

First of all, we shall consider the equation

$$P_0(d/dt)u = u^{(4)} + A^4u = f \qquad (7)$$

where $f(t) \in L_2(R_+; \mathcal{H})$. Let's denote by $\mathcal{P}_0$ the operator, acting from space $W_2^{4,T,K}(R_+; \mathcal{H})$ in $L_2(R_+; \mathcal{H})$ by the following way:

$$\mathcal{P}_0 u = P_0(d/dt)u, u \in W_2^{4,T,K}(R_+; \mathcal{H}).$$

It's true.

Theorem 1. *Let* $C = A^{7/2}TA^{-3/2}, S = A^{5/2}KA^{-1/2}$*, these operators are commutative, i.e.* $CS = SC$ *and point* $-1 \notin \sum(CS - S + C)$*. Then operator* $\mathcal{P}_0$ *realizes an isomorphism from the space* $W_2^{4,T,K}(R_+; H)$ *on* $L_2(R_+; H)$*.*

Proof. The condition $-1 \notin \sum(CS - S + C)$ implies that homogeneous $P_0(d/dt) = 0$ has just a zero solution from the space $W_2^{4,T,K}(R_+; \mathcal{H})$, but at any $f(t) \in L_2(R_+; \mathcal{H})$

equation (7) has solution from the space $W_2^{4,T,K}(R_+;\mathcal{H})$, representable in the form

$$u(t)=\frac{1}{4\sqrt{2}}\int_0^\infty\left((1+i)e^{-\frac{1+i}{\sqrt{2}}|t-s|A}+(1-i)e^{-\frac{1-i}{\sqrt{2}}|t-s|A}\right)A^{-3}f(s)ds-$$
$$-\frac{i}{4\sqrt{2}}e^{-\frac{1+i}{\sqrt{2}}tA}A^{-7/2}(CS-S+C+E)^{-1}\times$$
$$\times[((C+iE)(S-iE)+(E+iC)(S+iE))\times$$
$$\times A^{1/2}\int_0^\infty e^{-\frac{1+i}{\sqrt{2}}tA}f(s)ds+2(E+iC)(S-iE)A^{1/2}\int_0^\infty e^{-\frac{1-i}{\sqrt{2}}sA}f(s)ds]+ \tag{8}$$
$$+\frac{i}{4\sqrt{2}}e^{-\frac{1-i}{\sqrt{2}}tA}A^{-7/2}(CS-S+C+E)^{-1}\times$$
$$\times\left[2(E-iC)(S+iE)A^{1/2}\int_0^\infty e^{-\frac{1+i}{\sqrt{2}}sA}f(s)ds+\right.$$
$$\left.+((E-iC)(S-iE)+(E+iC)(E-iS))\,A^{1/2}\int_0^\infty e^{-\frac{1-i}{\sqrt{2}}sA}f(s)ds\right].$$

It is easy to check, that first number satisfies equation (7) and belongs to the space $W_2^4(R_+;\mathcal{H})$ (see [2,3,]). Further, from the inequality [6, p.208]

$$\left\|A^{1/2}\int_0^\infty[\exp(-tA)]\,f(t)dt\right\|_{\mathcal{H}}\le\frac{1}{\sqrt{2}}\|f\|_{L_2}, \tag{9}$$

$$\left\|A^{1/2}[\exp(-tA)]\,\psi\right\|_{L_2}\le\frac{1}{\sqrt{2}}\|\psi\|,\psi\in\mathcal{H}, \tag{10}$$

implies the inequality:

$$\left\|A^{1/2}\int_0^\infty\left[\exp(-\frac{1\pm i}{\sqrt{2}}tA)\right]f(t)dt\right\|_{\mathcal{H}}\le\frac{1}{\sqrt[4]{2}}\|f\|_{L_2}, \tag{11}$$

$$\left\|A^4\left[\exp(-\frac{1\pm i}{\sqrt{2}}tA)\right]\psi\right\|_{L_2}\le\frac{1}{\sqrt[4]{2}}\|\psi\|_{7/2},\psi\in\mathcal{H}_{7/2}. \tag{12}$$

Consequently, the second and the third number in equality (8) also belong to the space $W_2^4(R_+;\mathcal{H})$.

Fulfilment of boundary conditions (5) can be checked directly. Boundedness of the operator $\mathcal{P}_0$ follows from the inequality

$$\|\mathcal{P}_0u\|_{L_2}^2=\left\|u^{(4)}+A^4u\right\|_{L_2}^2\le2\|u\|_{W_2^4}^2 \tag{13}$$

Thus, an operator $\mathcal{P}_0$ is bounded and one-to-one acts from the space $W_2^4(R_+;\mathcal{H})$ on $L_2(R_+;\mathcal{H})$ and by the Banach's theorem on the inverse operator realizes isomorphism by these spaces.

The theorem is proved.

This theorem implies that $\|\mathcal{P}_0 u\|_{L_2}$is a norm of the space $W_2^4(R_+;\mathcal{H})$, equivalent to the original norm $\|u\|_{W_2^4}$.

Now we'll research problem (4), (5).

Theorem 2. *Let conditions of theorem 1 be fulfilled and*

$$\sum_{j=1}^{3} \|B_j\| \, N_{T,K,j} < 1,$$

where

$$N_{T,K,j} = \sup_{0\neq u \in W_2^{4,T,K}(R+;\mathcal{H})} \left(\left\| A^{4-j} u^{(j)} \right\|_{L_2} / \|\mathcal{P}_0 u\|_{L_2} \right). \tag{14}$$

Then problem (4), (5) is regular solvable.

Proof. Let's write problem (4), (5) in the form of operator equation $(\mathcal{P}_0 + \mathcal{P}_1)\, u = f$, where $f(t) \in L_2(R+;\mathcal{H}), u(t) \in W_2^{4,T,K}(R+;\mathcal{H})$. $\mathcal{P}_1 u = \sum\limits_{j=1}^{3} B_j A^{4-j} u^{(j)}$ for $u \in W_2^{4,T,K}(R+;\mathcal{H})$. Since, the operator $\mathcal{P}_0$ has a bounded inverse $\mathcal{P}_0^{-1}$ by theorem 1, acting from $L_2(R+;\mathcal{H})$ on $W_2^{4,T,K}(R+;\mathcal{H})$, then after substitution $u = \mathcal{P}_0^{-1} \upsilon$ we shall obtain the following equation in $L_2(R+;\mathcal{H})$:

$$(E + \mathcal{P}_1 \mathcal{P}_0^{-1})\upsilon = f.$$

On the other hand

$$\left\| \mathcal{P}_1 \mathcal{P}_0^{-1} \upsilon \right\|_{L_2} = \|\mathcal{P}_1 u\|_{L_2} \le \sum_{j=1}^{3} \|B_j\| \left\| A^{4-j} u^{(j)} \right\|_{L_2} \le$$

$$\le \sum_{j=1}^{3} \|B_j\| \, N_{T,k,j} \, \|\mathcal{P}_0 u\| = \sum_{j=1}^{3} \|B_j\| \, N_{T,K,j} \, \|\upsilon\|_{L_2}.$$

Therefore, by fulfiling the inequality $\sum\limits_{j=1}^{3} \|B_j\| \, N_{T,K,j} < 1$ the operator $E + \mathcal{P}_1 \mathcal{P}_0^{-1}$ is reversible and we can find $u(t)$.

The theorem is proved.

Let's denote by $N_{0,j} = \sup\limits_{0\neq u \in \mathring{W}_2^4(R_+;\mathcal{H})} \left(\left\| A^{4-j} u^{(j)} \right\|_{L_2} / \|P_0 u\|_{L_2} \right), j = 1,2,3.$

Remark 1. It is obvious, that $N_{T,K,j} \ge N_{0,j}$ and

$$N_{0,j} = \left(\left(\frac{4}{4-j} \right)^{4-j} \left(\frac{4}{j} \right)^{j} \right)^{-1/4}, j = 1,2,3$$

[7]. In suppositions $A^{-1} \in \sum_p$ the operator bundle $P(\lambda)$ has a discrete spectrum, and let $\lambda_n \, (n = 1,2,3...)$ be characteristic numbers of bundle $P\,(\lambda)$ from the left-plane Π_-, and $x_{0,n}, x_{1,n}, .., x_{m,n}$ be eigen and joined vectors, responding to the characteristic number λ_n:

$$P(\lambda_n) x_{0,n} = 0,$$

$$\sum_{j=0}^{P} \frac{1}{j!} P^{(j)}(\lambda_n) x_{p-j,n} = 0, p = 1, ..., m.$$

Then the vector-functions

$$u_{p,n}(t) = e^{\lambda_n t}\left(\frac{t^p}{p!}x_{0,n} + \frac{t^{p-1}}{(p-1)!}x_{1,n} + ... + x_{p,n}\right), p = 0, 1, ..., m,$$

belong to the space $W_2^4(R+;\mathcal{H})$ and satisfy equation (2). They will be called elementary solutions of equation (2) [5]. It is obvious, that elementary solutions satisfy the following boundary conditions:

$$u_{0,n}(0) - Tu''_{0,n}(0) = x_{0,n} - \lambda_n^2 Tx_{0,n} \equiv \varphi_{0,n},$$

$$u_{1,n}(0) - Tu''_{1,n}(0) = x_{1,n} - \lambda_n^2 Tx_{1,n} - 2\lambda_n Tx_{0,n} \equiv \varphi_{1,n},$$

$$u_{p,n}(0) - Tu''_{p,n}(0) = x_{p,n} -$$

$$-\lambda_n^2 Tx_{p,n} - 2\lambda_n Tx_{p-1,n} - Tx_{p-2,n} \equiv \varphi_{p,n}, p = 2, ..., m,$$

$$u'_{0,n}(0) - Ku'''_{0,n}(0) = \lambda_n x_{0,n} - \lambda_n^3 Kx_{0,n} \equiv \psi_{0,n},$$

$$u'_{1,n}(0) - Ku'''_{1,n}(0) = \lambda_n x_{1,n} - \lambda_n^3 Kx_{1,n} + x_{0,n} - 3\lambda_n^2 Kx_{0,n} \equiv \psi_{1,n}, \tag{15}$$

$$u'_{2,n}(0) - Ku'''_{2,n}(0) = \lambda_n x_{2,n} - \lambda_n^3 Kx_{2,n} +$$

$$+x_{1,n} - 3\lambda_n^2 Kx_{1,n} - 3\lambda_n Kx_{0,n} \equiv \psi_{2,n},$$

$$u'_{p,n}(0) - Ku'''_{p,n}(0) = \lambda_n x_{p,n} - \lambda_n^3 Kx_{p,n} + x_{p-1,n} -$$

$$-3\lambda_n^2 Kx_{p-1,n} - 3\lambda_n Kx_{p-2,n} - Kx_{p-3,n} \equiv \psi_{p,n},$$

$$p = 3, ..., m.$$

By fulfilling the condition of theorem 2, it is easy to see, that problem (2), (3) has a unique solution from the space $W_2^4(R+;\mathcal{H})$ at any $\varphi \in \mathcal{H}_{7/2}, \psi \in \mathcal{H}_{5/2}$. A set of all such solutions we'll denote by $W_4(P)$.

From the theorem on the intermediate derivatives and about traces implies, that a set $W_4(P)$ is closed subspace of the space $W_2^4(R+;\mathcal{H})$. There it is stated, a problem: when elementary solutions of problem (2) are complete in the space $W_4(P)$? It holds.

Theorem 3. *Let $C = A^{7/2}TA^{-3/2}, S = A^{5/2}KA^{-1/2}$, these operators are commutative, i.e. $CS = SC, -1\sum(CS - S + C), \sum\limits_{j=1}^{3}\|B_j\| N_{T,K,j} < 1$ and one of the conditions is fulfilled:*

a) $A^{-1} \in \sum_\rho(0 < \rho \leq 2)$ or b) $B_j \in \sum_\infty, j = 1, 2, 3, A^{-1} \in \sum_\rho(0 < \rho < \infty)$. Then a system of elementary solutions of problem (2), (3) is complete in the space $W_4(P)$.

Proof. First of all, we shall prove, that the system $\{\varphi_{p,n}, \psi_{p,n}\}$, defined from equality (15) is complete in the space $\mathcal{H}_{7/2} \oplus \mathcal{H}_{5/2}$. If it is not so, then there exists a non-zero vector $(\tilde{\varphi},\tilde{\psi}) \in \mathcal{H}_{7/2} \oplus \mathcal{H}_{5/2}$ such, that $(\tilde{\varphi},\varphi_{p,n})_{7/2} + (\tilde{\varphi},\psi_{p,n})_{5/2} = 0$. Then from the expansion of the main part of resolvent at the neighborhoods of characteristic numbers it follows, that $(A^{7/2}(E-\bar{\lambda}^2 T)P^{-1}(\bar{\lambda}))^* A^{7/2}\tilde{\varphi} + (A^{5/2}(\bar{\lambda}E - \bar{\lambda}^3 K)P^{-1}(\bar{\lambda}))^* A^{5/2}\tilde{\psi}$ will be holomorphic vector-function in the left half-plane Π_-. If $u(t)$ is a solution of problem (2), (3), then it can be represented in the form

$$u(t) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \hat{u}(\lambda)\exp(\lambda t)d\lambda, \tag{16}$$

where

$$\hat{u}(\lambda) = P^{-1}(\lambda)\{\left(\lambda^3 E + \lambda^2 B_3 A + \lambda B_2 A^2 + B_1 A^3\right)u(0) +$$

$$+(\lambda^2 E + \lambda B_3 A + B_2 A^2)u'(0) + (\lambda E + B_3 A)u''(0) + u'''(0)\}.$$

Taking into account Remark 1 from theorem 5 of the work [4] we'll obtain that by fulfilling the condition of the theorem the following statement holds:

1) $P^{-1}(\lambda)$ is represented in the form of ratio of two ρ order entire and minimal type functions at order ρ;

2) there exists a number $\mathcal{E}$ >0 such, that the resolvent $P^{-1}(\lambda)$ is holomorphic at the angles $S_{\pm\varepsilon} = \{\lambda : \lambda = r\exp(\pm i\theta), \pi/2 < \theta < \pi/2 + \mathcal{E}, r > 0\}$ and at the same angles admits the estimations $\left\|A^{7/2}P^{-1}(\lambda)\right\| \le c\,|\lambda|^{-1/2}$, $\left\|A^{5/2}P^{-1}(\lambda)\right\| \le c\,|\lambda|^{-3/2}$;

3) at the left half-plane there exists a system of rays $\{\Omega\}$, including rays $\Gamma_{\pm\mathcal{E}} = \{\lambda : \lambda = r\exp(\pm i(\pi/2 + \mathcal{E})), r > 0\}$, such that the angle between neighbour rays is less than π/ρ and on these rays of the functions $\left\|A^{7/2}P^{-1}(\lambda)\right\|$ and $\left\|A^{5/2}P^{-1}(\lambda)\right\|$ grow no faster than $|\lambda|^{-1/2}$ and $|\lambda|^{-3/2}$ correspondingly.

Taking into account all of this in equality (16), a contour of integration we can substitute by $\Gamma_{\pm\mathcal{E}}$. Then at $t > 0$

$$(u(t) - Tu''(t),\tilde{\varphi})_{7/2} + (u'(t) - Ku'''(t),\tilde{\psi})_{5/2} =$$

$$= \frac{1}{2\pi i}\int_{\Gamma_{\pm\mathcal{E}}} \left(\left(\lambda^3 E + \lambda^2 B_3 A + \lambda B_2 A^2 + B_1 A^3\right)u(0) +\right.$$

$$+(\lambda^2 E + \lambda B_3 A + B_2 A^2)u'(0) +$$

$$+(\lambda E + B_3 A)u(0) + u'''(0), (A^{7/2}(E - \lambda^2 T)P^{-1}(\lambda))^* A^{7/2}\tilde{\varphi} +$$

$$+(A^{5/2}(\lambda E - \lambda^3 K)P^{-1}(\lambda))^* A^{5/2}\tilde{\psi})\exp(\lambda t)d\lambda.$$

From the Frangmen-Lindelof's theorem we obtain, that integrand function in front of $\exp\lambda t$ is a polynomial, and therefore the integral equals zero at $t > 0$, consequently, $(u(t) - Tu''(t),\tilde{\varphi})_{7/2} + \left(u'(t) - Ku'''(t),\tilde{\psi}\right)_{5/2} = 0, t > 0$.

Passing to the limit at $t \to 0$, by the theorem about traces we shall obtain $(\varphi,\tilde{\psi})_{7/2} + (\psi,\tilde{\psi})_{5/2} = 0, \forall(\varphi,\psi) \in \mathcal{H}_{7/2} \oplus \mathcal{H}_{5/2}$. Therefore, $\tilde{\varphi} = \tilde{\psi} = 0$. Further, from the uniqueness of the solution of problem (2), (3) and from the theorem about traces it holds the inequality

$$c_2(\|\varphi\|_{7/2}^2 + \|\psi\|_{5/2}^2)^{1/2} \le \|u\|_{W_2^4} \le c_1\left(\|\varphi\|_{7/2}^2 + \|\psi\|_{5/2}^2\right)^{1/2}. \tag{17}$$

Since, the system $\{(\varphi_{p,n}, \psi_{p,n})\}$ is complete in $\mathcal{H}_{7/2} \oplus \mathcal{H}_{5/2}$, then for the given $\mathcal{E} > 0$ there exists a number N and numbers $c_p^N(\mathcal{E})$ such that

$$\left(\left\| \varphi - \sum_{n=1}^{N} \sum_{p} c_p^N(\mathcal{E}) \varphi_{p,n} \right\|_{7/2}^2 + \left\| \psi - \sum_{n=1}^{N} \sum_{p} c_p^N(\mathcal{E}) \psi_{p,n} \right\|_{5/2}^2 \right)^{1/2} < \mathcal{E}. \qquad (18)$$

Taking into account equalities (3) and (5) in (17), from inequality (18) we'll obtain

$$\left\| u(t) - \sum_{n=1}^{N} \sum_{p} c_p^N(\mathcal{E}) u_{p,n}(t) \right\|_{W_2^4} < c_1 \mathcal{E}.$$

The theorem is proved.

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