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ON ONE MODELING ALGORITHM OF RANDOM MOVING OF PARTICLES ALONG A RING WITHOUT OVERTAKING

Abstract

The various models of continuous moving of particles along a circle are considered. Two approaches of investigation are suggested:

1. The modeling algorithms are described in detail. The developed algorithms of computer modeling as distinct from the other known algorithms essentially lower the modeling time, admit to allow for all lower models and give general principle of approach to such type problems.

2. Along with modeling method on computer by analytical approach for stochastic systems, the class of probable states which may contain the system, is described. The optimal number of particles minimizing mean wait of particles at set point is found.

1. Introduction. In recent years the study of mathematical models of queue dynamic systems is of particular interest. The models of system with moving objects are one of important directions in queuing theory. The interest to these investigations is caused by broad practical application: transport systems (vertical, motor, ship, airport, space), telecommunication networks (telephone, videophone, bank, computer), service controls (client, technology), complicated technical aggregates (assembly, disassembly, checkup) and a lot of other problems. In this direction at the present time along with analytical methods of investigations one successively uses the methods of computer modeling of various system which allow to describe adequately the studied process and to obtain the numerical results of characteristics that we are interested in for different states of system.

The transport problems are important class of queueing system. The traffic control problem can be referred to these systems. The ways diagrams, waiting period of requirements, capacity of system, service time, downtime, queue length, rate of moving of particles and so on are basic characteristics of models of transport problems. A lot of these concepts are defined in the paper [1].

The simplest mathematical models of moving particles on straight line are considered in [2,8]. The generalization of these models to the case of moving along closed trajectory and the control problems are considered in [4-6] where the basic characteristics of models of moving of the particles along circular route are also studied. In these papers the control strategies by traffic road are studied. It is shown that for some states of system under definite conditions, the deliberate or random delay of service facility decreases efficiency factor – mean waiting period of requirements before service.

Analogy of the known "paradox of waiting period" arising in queue theory is considered in [9]. In particular, it is discussed the Pollachek-Hinchin formula for mean waiting period in the system $M/G/1$ indicating the fact that this time may be arbitrary large at indefinitely small loading of server if dispersion of service time is large in comparison with its mathematical expectation.

2. Mathematical models. The model of moving of s particles without overtaking along a closed contour is considered. For simplicity of statement, a circumference of length 1 as contour is considered. The particles are numbered $1, 2, \ ..., \ s$ counterclockwise, and they commute along the circumference counterclockwise with the given rates V_1 and V_2 $(V_1 < V_2)$. The distance between particles is regulated by the given quantities Q_1 or Q_2 $(Q_1 < Q_2)$. The rate of particles is controlled so that the distance between these particles changes in the interval $[Q_1, Q_2]$.

Following [10] we denote by $\rho_{i,t}$ the distance from particle i to particle $i+1$ along track direction, $i = 1, 2, \ ..., \ s-1$ at the moment t, by $\rho_{s,t}$ the distance between particles 1 and s, by $V_{i,t}$ the rate of particle i in time t. If $\rho_{i,t} = Q_2$ then $V_{i,t} = V_2$ at $\rho_{i,t} = Q_1$, $V_{i,t} = V_2$.

We select a point on circumference and announce it an initial point, i.e., zero point. At the initial moment of moving $t_0 = 0$:

$$\rho_{i,t0} = Q_1, \quad i = 1, 2, \ ..., \ s-1;$$
$$\rho_{s,t0} = 1 - (s-1)\, Q_1.$$

The particles $1, 2, ..., \ s-1$ have the rate V_1, and the particle s has the rateV_2 .

After a lapse of time

$$\Delta t_1 = \frac{Q_2 - Q_1}{V_2 - V_1}$$

the distance $\rho_{s-1,t0+\Delta t1}$ increases from Q_1 to Q_2. At this moment the rate of the particles $s-1$ is switched from V_1 to V_2, since the distance between the particles $s-1$ and s gets the value Q_2.

After a lapse of the following time Δt_1 the distance $\rho_{s-1,t0+\Delta t1}$ achieves the value Q_2 and the rate of the particle $s-2$ is switched to V_2. This process continues until there comes the moment t^* when ρ_{s,t^*} decreases to the rate value Q_1 (fig.1).

After this, the system reaches a state in which k_1 number particles have the rate V_1, and k_2 number particles have the rate V_2. Then one particle (in the given case s) will switche the rate from V_2 to V_1 and $k_1 + 1$ particles will have the rate V_1, and $k_2 - 1$ particles will have the rate V_2 and the system in this condition spends the time interval

$$t_1 = \frac{1 - (k_1 + 1)\, Q_1 - (k_2 - 1)\, Q_2}{V_2 - V_1}$$

later on one particle is switched from the rate V_1 to the rate V_2 in the system within the time interval

$$t_2 = \frac{k_1 Q_1 + k_2 Q_2 - 1}{V_2 - V_1}.$$

k_1 particles have the rate V_1, and k_2 particles have the rate V_2. This process is repeated at each period

$$t = t_1 + t_2$$

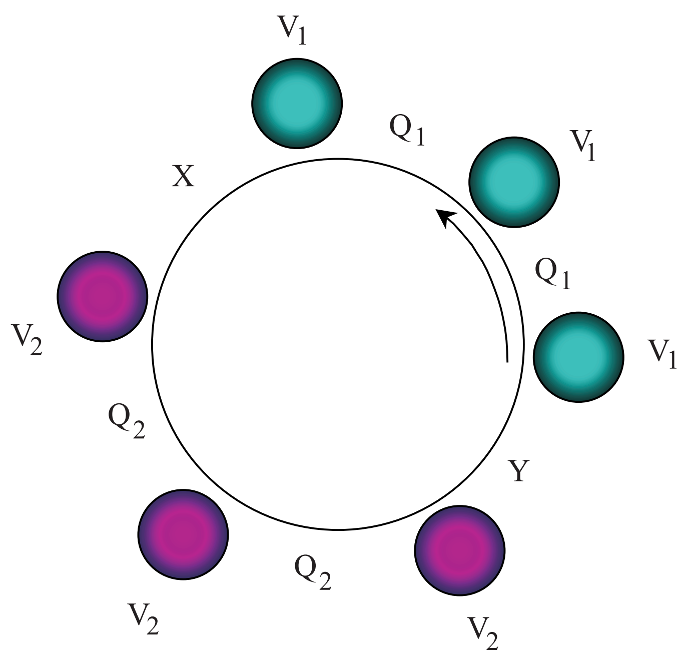

Fig. 1. The particles arrangment along moving route in $AES(k_1, k_2)$.

Thus, the particle does not change the rate if $Q_1 < \rho_{i,t} < Q_2$, but changes it only at the end of the segment $[Q_1; Q_2]$. Such models of moving particles imitate the moving of transport units in different systems. In these models the particles increase the rate if the distance to the next particle growths, and it is taken off when decreasing this distance. In practice it is observed in transport systems.

As efficiency factors of system the mean waiting period of particle is chosen at arbitrary point of trajectory which is determined in the following manner. The point is randomly (uniformly) chosen on the circumference and the efficiency coefficient is determined as mean waiting period of particle at the chosen point. In some classifications this coefficient is called virtual waiting period. The mean waiting period is determined as total waiting period of particles at the chosen point, divided into amount of such particles.

The different steady-state condition arising in these systems are described in the paper [10]. The optimal number of moving particles minimizing mean waiting period at the chosen point is found for such systems.

In the present paper it is assumed an approach for behaviour modeling of above-mentioned system by computers which will be base one for further models. Here the adequacy of the constructed model and possibility of visual trace of the states of the system will be defined. When modeling the basic characteristics of the system will be computed, as well as the moving of particles along the circle will be graphically (on a monitor of computer) demonstrated.

During modeling $t \in [0,T]$ where T is given time $(7200-14400)$ and $t \in [T_0, T_0+\Delta T]$ where ΔT is a little time interval $(300-600)$ when the behaviour of system is observed in graphical condition.

The following initial parameters are given in the model:

- modeling time - T;
- path trajectory – length of circumference - $l0$;
- rates of particles - $V_1,\ V_2$;
- threshold values of distances between particles - Q_1, Q_2;
- amount of particles in system - s
- intensity of Poisson flow of requirements - λ.

The values of these components of the system usually do not change during one modeling cycle.

It is introduced two-dimensional vector of the values $ost\,(n,3)$ for representing the states of observation points, where n is amount of observation points, $ost\,(j,1)$ are coordinates of j -th observation point, $ost\,(j,2)$ is time of last requirement service in the observation point, $ost\,(j,3)$ is current time of count of observation point.

Further, two-dimensional vector of particles states $A\,(s,4)$ is introduced, $A\,(i,1)$ is rate of particle i at current time t, $A\,(i,2)$ are current coordinates of particle i , $A\,(i,3)$ are old coordinates of particle i before movement, $A\,(i,4)$ is the number of observation point to which particle approachs towards.

The vectors $kost\,(n,2)$ and $kr\,(s,4)$ serve for graphical data storage for the models of II type.$kost\,(n,2)$ are graphic coordinates and states of observation points; $kr\,(s,4)$ are old and new graphical coordinates and states of particles.

The system is observed at the moments

$$t_0, t_1, t_2, ..., t_n, ...$$

where $t_0 = 0$, when there occur changes in the system.

We call changes such moments $t_n = t_{n-1} + \Delta t_n$ when the rates of particles are switched from V_1 to V_2 or vice-versa. Therefore, such particles $i_1, i_2, ..., i_k$ are chosen which move opposite the previous particle. The intervals Δt_{i_k} :

$$\Delta t_{i_k} = \left| \frac{A(i_k+1,2) - A(i_k,2) - q}{V_2 - V_1} \right|$$

are defined where

$$q = \begin{cases} Q_1, & if \quad A(i_k+1,1) < A(i_k,1) \\ Q_2, & if \quad A(i_k+1,1) > A(i_k,1) \end{cases}$$

when the distances between particles reach threshold values, i.e., there comes a time to switch the rate of this particle i_k.

We choose time slice Δt_n as minimal from computed intervals

$$\Delta t_n = \min \left\{ \frac{Q_1}{V_1}; \Delta t_{i_1}; \Delta t_{i_2}; ...; \Delta t_{i_k} \right\}$$

on the expiry of that there occurs change in the system in the form of movement forward of all particles

$$A(i,2) = A(i,3) + A(i,1)\,\Delta t_n$$

and switching the rate of particle (or particles) whose coordinates achieve threshold values Q_1 or Q_2. There occur changes in the system between these moments, these changes are computed by the analytical way.

To the observation point a flow of requirements comes at the moments

$$t'_0, t'_1, t'_2, ..., t'_k, ...$$

For the Poisson flow the intervals between entries

$$\xi_i = t'_i - t_{i-1}, i = 1, 2, ..., k, ...$$

where $t'_0 = 0$, have exponential distribution. It is known that if $F(\xi)$ is a function of exponential distribution, then the function $F^{-1}[F(\xi)]$ is uniform distribution [3]. Therefore, for realization of the values ξ_i we use the following representation

$$\xi_i = -\frac{1}{\lambda} \ln(1 - r_i), i = 1, 2, ..., k, ...$$

where $r_i \in (0;1)$ is a random variable having uniform distribution in the interval $(0,1)$, λ is intensity of Poisson flow.

Then $t'_i = t'_{i-1} + \xi_i, i = 1, 2, ..., k, ...$

At the moments of passing of particles through observation point the amount of requirements accumulated in this point is determined. At that it is sufficient to remember only approach time of last requirement which was not served; that essentially facilitates the modeling process. At this stage the total waiting period of requirements is computed from approach moment to service moment and the mean waiting period of requirements, i.e. efficiency of system - W is determined.

In II type model single fragments of modeling time are chosen and all changes are graphically imitated on the monitor. At that one can observe, for example, the modeling time, switching the rate of particles (in the form of color change, service of requirements and so on). The programs allowing to depict moving of particles on display and to observe the changes in the system are developed. The graph expressing the efficiency of system – change of mean waiting period of requirements at the current modeling time is considered in the model.

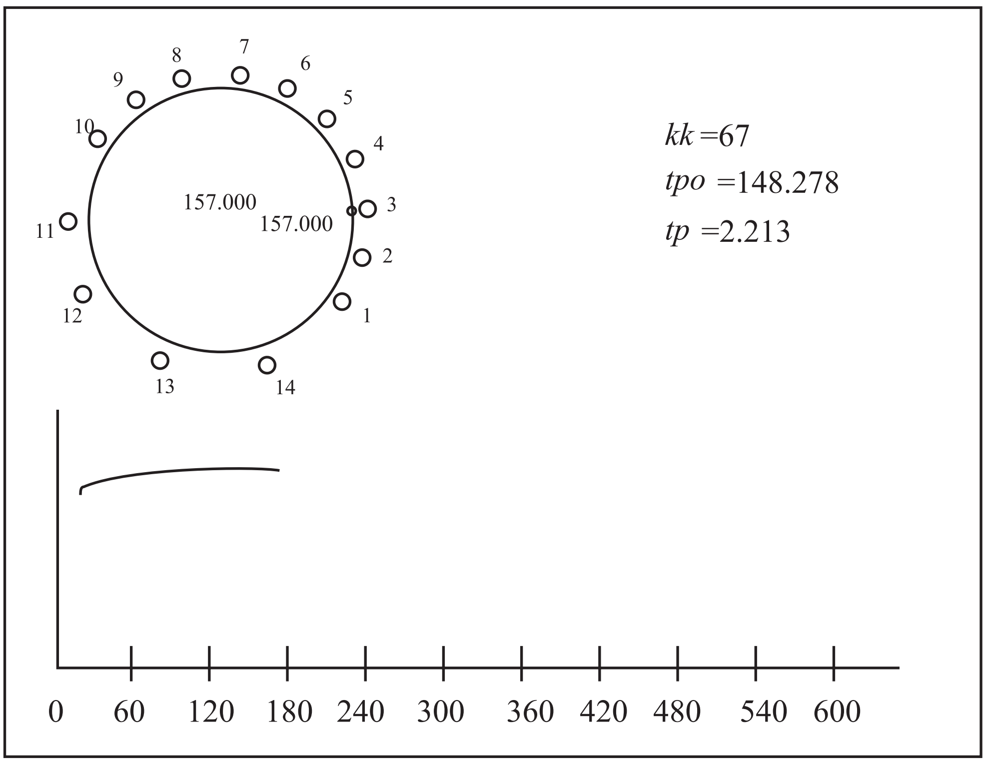

Fig.2. Modeling fragment of system in computer.

With the help of formulae for computing of the system efficiency stated in the paper [10] we calculate them for various states with above mentioned initial data.

The amount of particles $s = 18$, then

$$W_2 = \sum_{i=1}^{s} \frac{Q_1}{l_0} \frac{Q_1}{2V_1} = \sum_{i=1}^{18} \frac{(l_0/18)^2}{l_0} \frac{1}{2V_1} = \sum_{i=1}^{18} \frac{l_0}{2M_1 \cdot 18^2} = \frac{2\pi r}{2V_1 \cdot 18} = 2,617.$$

When $s = 10$. Then

$$W_2 = \sum_{i=1}^{s} \frac{Q_2}{l_0} \frac{Q_2}{2V_2} = \sum_{i=1}^{10} \frac{(l_0/10)^2}{l_0} \frac{1}{2V_2} = \frac{\pi r}{10 \cdot V_1} = 2,355.$$

As we see the results of modeling give the same results, i.e. the efficiency of the system is uniquely determined at various states. In other words, the complete adequacy of superposed computer models to the mathematical one is achieved. For simple models the results of modeling coincide with the theoretically results obtained in the paper [10].

The developed algorithms of computer modeling unlike the other known algorithms essentially decrease modeling time, allow to take into account all nuances of models and give the general principles of approach to such type problems. Note that I and II type models are generalized in the same program. In this program it is sufficient to turn off or turn on subprograms of imitation of moving of particles, this imitation is created with help of one button at modeling time.

The different modification of the given model are considered for practical application:

- In practice the switching of rates doesn't occur instantaneously but with some acceleration. From these reasonings acceleration in switching of rates is added to the model and this model is studied in different states.

- Unlike ideal modeling in practice the service process occupies definite time and therefore, the accounting of service time approaches the model to more real condition.

- In the model we assume the dimension of service facility as infinite. But in practice usually it is of finite dimension, therefore the program for modeling of process with the finite particles volume is developed at which some requirements can remain in the observation point for service, that is reflected in the results of efficiency of system.

- The models with finite volume of bunker were considered in observation point after filling of which incoming requirements are lost.

3. Analytical results.

Let us consider the stochastic model of moving of particles along the ring with one-moment external action.

Let at some moment $t^* \in (0;T)$ the rate of some particle moving with the rate V_2 be forcedly (contrary to rules of modeling) switched to V_1 and released. The system further is regulated by above mentioned rights of determinate model.

In the state (AES1) the system is not subjected to exterior effect since in this state all particles are commuted with the rate V_1 and therefore the exterior effect does not change the efficiency of the system

$$W_1^* = W_1.$$

In the state (AES2) with the probability

$$p_2 = \frac{s_2 - 1}{s_2}$$

one of particles for which

$$\rho_{j,t^*} = Q_2,\ j = 1, 2, ..., s_2 - 1$$

is satisfied is chosen for delay

After delay this particle is instantaneously switched to the rate V_2 (see Fig.1).

With the probability

$$p_2^* = \frac{1}{s_2}$$

the exterior effect can delay the particle s_2 for which

$$Q_1 < \rho_{s_2,t^*} = 1 - (s_2 - 1)\, Q_2 \le Q_2$$

is satisfied.

In this case during

$$\Delta t^* = \frac{Q_2 - \rho_{s_2,t^*}}{V_2 - V_1} = \frac{sQ_2 - 1}{V_2 - V_1}$$

the particle s_2 moves with the rate V_1 and later will switch to the rate V_2 since

$$\rho_{s_2,t^*+\Delta t^*} = Q_2.$$

During this time the distance to the previous particle decreases

$$\rho_{s_2-1,t^*+\Delta t^*} = s_2 Q_2 - 1 > Q_1$$

which does not attain the limit value Q_1 for switching the rate to V_1.

Thus, the system with the probability $1/s_2$ can be in state when only one particle of s_2 in the time interval $[t^*;\ t^* + \Delta t^*]$ will move with the rate V_1 and further the system passes to the state (AES2).

Theorem 1. *In the condition (AES2) the efficiency of the system with delay as $T \to \infty$ is equal to efficiency of determinate model*

$$\lim_{T\to\infty} W_2^* = W_2.$$

Proof. For computation of W_2^* we divide modeling interval into 3 subinterval

$$[0;T] = [0;t^*) + [t^*;t^* + \Delta t^*) + [t^* + \Delta t^*;T]\,.$$

In the first and last subintervals as it was shown above the efficiencies of the systems coincide. It remains to compare coefficients in the subinterval $[t^*;t^* + \Delta t^*)$. In this subinterval with the probability $1/s_2$ we obtain the difference for ΔW_2^*

$$W_2^* = W_2 + \frac{1}{s_2}\Delta W_2^*.$$

If we throw out the same intervals from the both systems, where the efficiencies of systems coincide, then we obtain the following estimation for ΔW_2^*

$$\Delta W_2^* = -\frac{Q_2^2}{2V_2} - \frac{Y^2}{2V_2} + \\ + \int\limits_0^{\Delta t^*} \frac{\left(Q_2 - t\left(V_2 - V_1\right)\right)^2}{2V_2} dt + \int\limits_0^{\Delta t^*} \frac{\left(Y + t\left(V_2 - V_1\right)\right)^2}{2V_1} dt.$$

Opening the integrals we obtain

$$\Delta W_2^* = -\frac{Q_2^2}{2V_2} - \frac{Y^2}{2V_2} + \\ + \frac{1}{2V_2}\frac{-1}{3\left(V_2 - V_1\right)}\left(Q_2 - t\left(V_2 - V_1\right)\right)^3\Big|_0^{\Delta t^*} + \frac{1}{2V_1}\frac{1}{3\left(V_2 - V_1\right)}\left(Y + t\left(V_2 - V_1\right)\right)^3\Big|_0^{\Delta t^*}.$$

Substituting the value Δt^* to his own place and simplifying the expression we obtain

$$\Delta W_2^* = -\frac{Q_2^2 + Y^2}{2V_2} + \frac{-\left(Y^3 - Q_2^3\right)}{6V_2\left(V_2 - V_1\right)} + \frac{Q_2^3 + Y^3}{6V_1\left(V_2 - V_1\right)} = \\ = \frac{Q_2^3 - Y^3}{6V_1\left(V_2 - V_1\right)}\left(\frac{1}{V_2} + \frac{1}{V_1}\right) - \frac{Q_2^2 + Y^2}{2V_2},$$

where

$$Y = 1 - \left(s_2 - 1\right) Q_2 \le Q_2.$$

For computation of efficiency of the system with delay we pass to the limit as $T \to \infty$

$$\lim_{T\to\infty} W_2^* = \lim_{T\to\infty}\left(\frac{T - \Delta t^*}{T} W_2 + \frac{1}{s}\frac{\Delta t^*}{T}\Delta W_2^*\right) = W_2.$$

Thus, for the state (AES2) in the limit relation the efficiencies of the considered systems coincide.

Let us consider the condition $AES\left(k1, k2\right)$. For studying the exterior effect in this condition we consider the efficiency of system more detailed.

As it was shown above, in the condition $AES\left(k1, k2\right)$ the amount of particles commuting with the rates V_1 and V_2 can change. The system can hit to the condition $C\left(k_1, k_2\right)$ or $C\left(k_1 + 1, k_2 - 1\right)$. Allowing for the equations of the states $AES\left(k1, k2\right)$ the system in the condition $C\left(k_1, k_2\right)$ will spend the time interval

$$t_1 = \frac{k_1 Q_1 + k_2 Q_2 - 1}{V_2 - V_1}.$$

Further, the rate V_2 of one particle is switched to V_1 and the system comes to the condition $C\left(k_1 + 1, k_2 - 1\right)$ and will be in this condition the time interval

$$t2 = \frac{1 - \left(k_1 + 1\right) Q_1 - \left(k_2 - 1\right) Q_2}{V_2 - V_1}.$$

Then it again will pass to the condition $C(k_1, k_2)$. This process will be repeated at each time interval

$$t = t_1 + t_2 = \frac{Q_2 - Q_1}{V_2 - V_1}.$$

Therefore, it is enough to study the behaviour of the system at the time interval $[0, t]$.

For definiteness we assume that in delay moment there are k_2 particles with the rate V_2 in the system. At some moment $t_k^* \in [0, T]$ rate V_2 of one particle is forcedly switched to V_1 and released. If this particle is one of j particles for which

$$\rho_{j,t_k^*} = Q_2$$

is satisfied, then the rate of this particle is instantly switched to V_2 (see Fig.1), i.e., moving condition does not change. In other words, with the probability

$$p_k = \frac{k_2 - 1}{k_2} = 1 - \frac{1}{k_2}$$

the efficiency of system will coincide with efficiency of determinate model.

With the probability

$$p_k^* = \frac{1}{k_2}$$

the delay can find particle j^* for which

$$\rho_{j,t_k^*} = Y < Q_2.$$

Then during

$$\Delta t_k^* = k_2 \frac{Q_2 - Q_1}{V_2 - V_1}$$

the condition of system changes until the system is not regulated and is not laid in condition $AES(k1, k2)$ of determinate model.

The following theorem 2 is valid.

Theorem 2. *In the condition $AES(k1, k2)$ the efficiency of system with delay as $T \to \infty$ is equal to efficiency of determinate model*

$$\lim_{T\to\infty} W_k^* = W_k.$$

Proof. As it was shown above, up to external action and on the expiry Δt_k^* after noise the efficiency of the considered system will coincide with efficiency of determinate model.

Reasoning analogously as for system (AES2) we throw off the same intervals from the both systems. Note that during Δt_k^* in the system with the delay the amount of particles with the rate V_2 will be per unit less than in determinate model, i.e. during the studied time slice the system hits sometimes to the condition $C(k_1+1,\ k_2-1)$, sometimes to $C(k_1+2,\ k_2-2)$. The particle which is subjected to delay will move with the rate V_1 being at the distance $Y < Q_2$ from it instead of achieve the next one.

Then for efficiency difference we have

$$\Delta W_k^* = \frac{Y^2}{2V_1} - \int_0^{\tau} \frac{(Q_2 - t(V_2-V_1))^2}{2V_2} dt,$$

where

$$\tau = \frac{Q_2 - Y}{V_2 - V_1}.$$

Opening ithis ntegral and simplifying the expression we obtain

$$\Delta W_k^* = \frac{Y^2}{2V_1} - \frac{Y^3 - Q_2^3}{6V_2(V_2-V_1)}.$$

Now we can write the efficiency formula for system with delay

$$W_k^* = \frac{T - k_2\tau}{T} Wk + \frac{1}{k_2}\frac{k_2\tau}{T}\Delta W_k^*.$$

Passing to the limit as $T \to \infty$ we obtain

$$\lim_{T\to\infty} W_k^* = W_k.$$

And now we consider the best moving condition for the determinate system. As it was shown above, the best moving condition is saturated state when the efficiency of the system – the mean waiting time of service is lesser than the remaining steady movement condition.

Let us assume that at the moment t^* the rate of some particle j is forcedly switched from V_2 to V_1 and released. Since the distance $\rho_{j,t} = Q_1 + \varepsilon_j < Q_2$ then the rate of this particle will not at once switched to V_2. On the expiry $\dfrac{\varepsilon_j}{V_2 - V_1}$ the distance of the previous particle $j-1$ to the particle j decreasing will be

$$\rho_{j-1,\Delta t_1^*} = Q_1,$$

where

$$\Delta t_1^* = t^* + \frac{\varepsilon_j}{V_2 - V_1}.$$

By the law of particles behaviour in the system the rate of the particle $j-1$ is also switched to V_1 and the system will be in this state some more time $\dfrac{\varepsilon_{j-1}}{V_2 - V_1}$ until the previous particle $j-2$ is also switched to V_1. The switching of rate to V_1

will occur at each $\dfrac{\varepsilon_k}{V_2 - V_1}$ interval $k = j,\ j-1,...$ period. This process will be continued while the distance

$$\rho_{j,t} = Q_2.$$

But this occurs not before than penultimate particle with the rate V_2 is not switched to V_1. On the expiry of time

$$\Delta t_s^* = \frac{1 - \sum\limits_{i=1}^{s_s} \varepsilon_i}{V_2 - V_1}$$

the system will hit to the state $AES\,(k_1, k_2)$ and will operate by the law of determinate model.

Then the efficiency formula for saturated system will be in the following form

$$W_s^* = \frac{t^* W_s + \Delta t_s^* W_s^* + (T - (t^* + \Delta t_s^*))\, W_k}{T}$$

where t^* is the time interval before noise effect; Δt_s^* is passage time from saturated state to commute state; $T - (t^* + \Delta t_s^*)$ is remaining modeling time.

If we take into account $t^* << T$, then passing to the limit as $T \to \infty$ we obtain

$$\lim_{T\to\infty} W_s^* = W_k.$$

If we sum up above mentioned ones, then we obtain following theorem 3.

Theorem 3. *In the systems with external action the condition (AES2) is the best condition of moving*

$$W_2^* \le W_k^* \le W_s^*.$$

Thus, unlike conditions *(AES2)* and $AES\,(k_1, k_2)$, saturated condition appeared to be unstable to noise. It is enough only one delay in order to the best condition for the determinate system appears to be nearly worst condition of moving. The essence of theorem 3 is led to the fact that the system should not be saturated with server since although they are the best for ideal systems, but in practice they do not stand the tests. The balance between facilities is easily broken and the system doesn't return to ideal steady state no more.

Thus we obtain a unique answer to the stated experiment in the paper [10] where the paradoxical modeling case was described.

This unexpected effect at first is obtained by modeling on computer and the picture of states before and after effect of external noise in a saturated system is led in Fig.3 below.

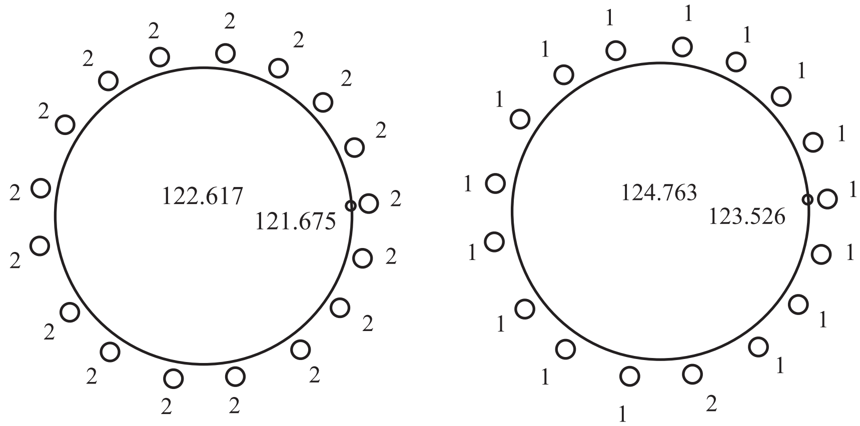

Fig 3. The state of saturated system before and after delay during modeling (1 and 2 are particles rates)

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