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ASYMPTOTIC ANALYSIS OF OSCOLLATION EIGENFREQUENCY OF ORTOTROPIC CYLINDRICAL SHELLS IN INFINITE ELASTIC MEDIUM FILLED WITH LIQUID

Abstract

The problem on free non-axially symmetrical oscillations of orthotropic cylindrical shell of infinite length contacting with infinite elastic medium and filled with liquid is considered.

The analysis of frequency and mode shape of shell was carried out under the assumption that variability of mode deformation in circular direction is great, material rigidity of shell much greater than material rigidity of filler, inertial influence of elastic medium on oscillation process has weak or essential effect.

System of equations of free oscillations of thin elastic orthotropic shell according to moment theory has the following from [1]:

$$a_1\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial \varphi^2}+(1+a_{12})\frac{\partial^2 \upsilon}{\partial x\partial \varphi}-a_{12}\frac{\partial \mathrm{w}}{\partial x}+\frac{xR^2}{G_{12}h}=0,$$

$$(1+a_{12})\frac{\partial^2 u}{\partial x\partial \varphi}+a_2\frac{\partial^2 u}{\partial \varphi^2}+\frac{\partial^2 \upsilon}{\partial \varphi^2}-a_2\frac{\partial^2 \mathrm{w}}{\partial \varphi^2}+\frac{yR^2}{G_{12}h}=0,$$

$$a_{12}\frac{\partial \varphi}{\partial x}+a_2\frac{\partial \upsilon}{\partial \varphi}+a_2\mathrm{w}- \tag{1}$$

$$-b^2\left[a_1\frac{\partial^4 \mathrm{w}}{\partial x^4}+2(a_{12}+2)\frac{\partial^4}{\partial x^2\partial \varphi^2}+a_2\frac{\partial^4 \mathrm{w}}{\partial x^4}\right]+\frac{zR^2}{G_{12}h}=0,$$

$$X=q_x-\rho h\frac{\partial^2 u}{\partial t^2};\ Y=q_4-\rho h\frac{\partial^2 \upsilon}{\partial t^2};\ z=q_2-\rho h\frac{\partial^2 \mathrm{w}}{\partial t^2}.$$

$q_x,\ q_\varphi,\ q_r$ are components of contact pressure of elastic medium and liquid on the shell, R,h are radius and shell thickness, respectively; $a_i=\dfrac{\mathrm{Ei}}{G_{12}(1-\nu_{12}\nu_{21})}$; $a_{12}=a_1\nu_{21}=a_2\nu_{12}$; u,υ,w are components of shifted median surface of the shell; $\nu_{12},\ \nu_{21}$ are the Poisson coefficients of orthotropic material of the shell; $E_i,\ G_{12}$ are the coefficients of longitudinal and shear elasticity; x,φ are longitudinal and circular coordinates; t is a time; ρ is density of the material of the shell. Vector equation of harmonic oscillations of isotropic elastic medium is the following [2]:

$$a_l^2\ grad\ div\ \vec{s}-a_t^2\ rot\ rot\vec{s}+\omega^2\vec{s}=0, \tag{2}$$

where $a_l^2=(\lambda_s+2\mu_s)/\rho_s,\ a_t^2=\mu_c/\rho_s$ are squared speeds of longitudinal and transverse waves propagation; $S=S(S_x,S_\varphi,S_r)$ is displacement vector, ρ_s is density, λ_s and μ_s are Lame constants ω, is the unknown frequency.

The linearized wave equation describing the propagation of small perturbations in ideal compressible liquid has the form [3]

$$\nabla^2\Phi-\frac{1}{a^2}\ \frac{\partial^2\Phi}{\partial t^2}=0, \tag{3}$$

where Φ is a liquid potential, a is speed of propagation of sound in the liquid.

Equations of motion of the shell (1), medium (2) and liquid (3) are supplemented with the contact conditions.

Conditions of equality of displacement components

$$S_x = u,\ S_\varphi = \upsilon,\ S_r = \mathrm{w} \quad (r = R) \tag{4}$$

and equality of pressures

$$\begin{gathered} x_1 = -\sigma_{rx} - \rho h \frac{\partial^2 u}{\partial t^2},\ Y_1 = -\tau_{r\varphi} - \rho h \frac{\partial^2 \upsilon}{\partial t^2}, \\ Z_1 = -\sigma_{rr} - \rho h \frac{\partial^2 \mathrm{w}}{\partial t^2} \quad (r = R) \end{gathered} \tag{5}$$

where X_1, Y_1, Z_1 are pressures of the shell on the medium, are imposed on medium and shells.

On the contact surface shell-liquid the continuity of range rates and pressures and absence of tangential stresses,

$$\upsilon_2 = \frac{\partial \omega}{\partial t},\ z_2 = -P,\ X_2 = Y_2 = 0 \quad (r = R) \tag{6}$$

is observed, where X_2, Y_2, Z_2 are pressures of the shell on the liquid, P is pressure.

Components of surface force X, Y, Z per a unit area is defined in the following way:

$$X = X_1 + X_2,\ Y = Y_1 + Y_2,\ Z = Z_1 - Z_2. \tag{7}$$

Besides, it is required that the components of vector of shifted medium $S_\lambda, S_\varphi, S_n$ as $r \to \infty$ and liquid potential Φ for $r = 0$ be finite.

If we add contact conditions (4)-(6) to equations of motion of the shell (6), medium (2) and liquid (3), we obtain the contact problem of free oscillations of the shell with the liquid in finite elastic medium. In other words, the problem of free oscillations of the shell with the liquid in elastic medium is reduced to simultaneous integration of equations of the theory of shells, medium and liquid at fulfilment of the mentioned conditions on the contact surfaces.

Solutions for the shell has the following form [2];

$$\begin{gathered} u = A \cos n\varphi \cos kx \exp(i\omega t) \\ \upsilon = B \sin n\varphi \sin kx \exp(i\omega t) \\ \omega = C \cos n\varphi \sin kx \exp(i\omega t). \end{gathered} \tag{8}$$

Solution of motion equation of the medium will be considered in two variants:

a) it is assumed that the influence of the medium inertia on the oscillation process is small;

b) it is assumed that the influence of inertia of medium motion on the oscillation processes is essential and it cannot be ignored.

Let's cite the solution for the medium motion equation [2] in the case a):

$$\begin{gathered} S_x = \left[\left(-kr \frac{\partial K_n(kr)}{\partial r} - \varphi(1 - \nu_s) k\ K_n(kr) \right) As + \right. \\ \left. + kKn(kr)Bs \right] \cos n\varphi \cos kx \exp(i\omega t) \\ S_\varphi = \left[-\frac{n}{r} K_n(kr) Bs - \frac{\partial K_n(kr)}{\partial r} c_s \right] \sin n\varphi \sin nx \exp(i\omega t) \\ S_r = \left[-k^2 r k_n(kr) As + \frac{\partial k_n(kr)}{\partial r} Bs + \frac{n}{r} k_n(kr) Cs \right] \end{gathered} \tag{9}$$

in the case b)

$$S_x = \left[\bar{A}_s k k_n(\gamma_e r) - \tilde{c}_s \frac{\gamma_t^2}{\mu_t} k_n(\gamma_t r)\right] \cos n\varphi \cos kx \exp(i\omega t)$$

$$S_\varphi = \left[-\frac{\tilde{A}_s n}{r} k_n - (\gamma_l r) - \frac{\tilde{c}_s nk}{r\mu_t} k_n(\gamma_t r) - \frac{\tilde{B}_s}{n} \frac{\partial k_n(\gamma_l^r)}{\partial r}\right] \times \sin n\varphi \sin nx \exp(i\omega t) \tag{10}$$

$$S_r = \left[\frac{\partial k_n(\gamma_l^2)}{\partial r} \tilde{A}s - \frac{\tilde{c}_s k}{\mu_t} \frac{\partial k_n(\gamma_t r)}{\partial r} + \frac{\tilde{B}_s n}{r} k_n(\gamma_t r)\right] \times \cos n\varphi \sin kx \exp(i\omega t)$$

where k_n is the modified n-th order (10) Bessel function of the second kind [3], n -is amount of half –waves in circular direction, $\frac{\Pi}{k}$ is the length of half –waves along the cylinder generator, $A_s, B_s, C_s, \tilde{A}_s, \tilde{B}_s, \tilde{C}_s$ are unknown constants, $\mu_l = \dfrac{\omega}{a_t}$, $\gamma_t^2 = k^2 - \dfrac{\omega^2}{a_t^2}$; $\gamma_l^2 = k^2 - \dfrac{\omega^2}{a_l^2}$.

We represent the liquid potential Φ which is a solution of Helmholtz equation [3], in the form:

$$\Phi = A_p I_n(\gamma l) \sin nx \exp(i\omega t) \tag{11}$$

where A_p is a constant, $\gamma^2 = k^2 - \dfrac{\omega^2}{a^2}$, I_n is the modifined n-th ordes Bessel function of the first kind [3].

Acoustic pressure P and range rate υ_r in the liquid are defined in the following way [3];

$$P = -\rho_0 \frac{\partial \Phi}{\partial t}$$
$$\upsilon_r = \frac{\partial \Phi}{\partial r}. \tag{12}$$

Using motion equation of the shell (1) and its solution and also solution of motion equation of the medium (9) and (10), liquid (11), we obtain the system of homogeneous algebraic equations with respect to the constants $A, B, C, A_s, B_s, C_s, A_p$ and $A, B, C, \tilde{A}_s, \tilde{B}_s, \tilde{C}_s, A_p$; from the condition of existence of nontrivial solutions of this equation, putting the principal determinant of the system to zero, we find a special equation with respect to the frequency of oscillations of the shell-medium-liquid system;

$$\det \|a_{ij}\| = 0 \ \text{ in case of a)}$$
$$(i, j = 1, ..., 7) \tag{13}$$

$$\det \|\tilde{a}_{ij}\| = 0 \ (i, j = 1, .., 7) \ \text{ in case of b)} \tag{14}$$

We don't give here elements a_{ij}; $\tilde{a}_{ij}$, because of the awkwardness. Note that they depend on physical and mechanical parameters of the shell, medium and liquid.

Frequency equations (13) and (14) are numerically solved. For initial data we assume

$$\frac{E_1}{E_2} = 2;\ \nu_s = 0,3;\ E_s = 2 \cdot 10^4 \frac{H}{\mu^2};\ \rho_s = 6,4^2/cm^3;$$

$$k = 2;\ n = 4; 5; 6; 7; 8; ; \rho_0 = 1q/cm^3;\ a = 1800m/c.$$

The results are shown in fig.1.

The dependence of frequency of natural oscillations of system ω on wave making in circular direction are shown in it. From the fig. one can see that frequencies increase in proportion to E_1/E_2.

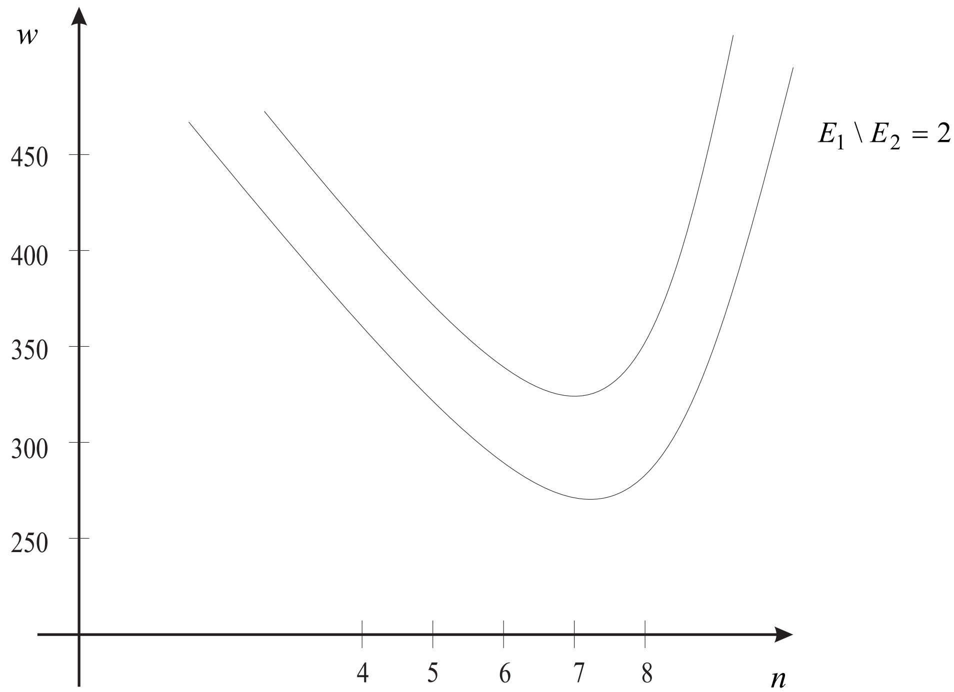

Fig. 1.

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