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STABILITY OF ELASTIC ROD WITH VARIABLE MODULUS OF ELASTICITY SITUATED ON NON-HOMOGENEOUS BASE

Abstract

In the paper it is assumed that a rod with rectangular cross-section lies on the Fuss-Winkler type base. Stability equation is the fourth order equation with variable coefficients,. Solution of the problem is constructed using a small parameter method. Two cases of built-in of end of rod: simple support and rigid are considered. The carried out calculations and the obtained results are represented in the form of graph of dependence between critical parameters and parameters which characterize non-homogeneities.

It is known that structural elements, whose physicomechanical properties are variable, and elements being on various bases, whose properties may obey the Winkler law and also nonuniform, nonlinear, anisotropic and nonelastic laws, are widely used in engineering. In most cases this question was well-investigated for Winkler base, moreover, when rod material is homogeneous [1].

Unlike the above-mentioned papers, in the given paper we assume that the modulus of elasticity is a function of coordinate of length and height of the rod, i.e.

$$E = E_0 \left(1 + \varepsilon f_1 (x)\right) f_2 (z) . \tag{1}$$

Here E_0 is a modulus of elasticity of homogeneous material, ε is a small parameter $(0 \le \varepsilon < 1)$.

The function $f_1 (x)$ with its derivatives $af_2 (z)$ is a continuous function, strength of the base is of Fuss-Winkler type

$$K (W) = K_0 \left(1 + \varepsilon \varphi (x)\right) W \tag{2}$$

where K_0 is Winkler coefficient, $\varphi (x)$ is some continuous function, which characterizes properties of the base, W is sag of central line at stability loss.

At central compression by the force P lies on non-homogeneous base (2).

Here we assume that at stability loss the central (neutral) line at stability loss moment doesn't change its state a little although we can avoid this assumption, because it is not difficult to ignore this condition.

In this case stability equation with respect to sag has the following form

$$\frac{d^2}{dx^2}\left[\left(1 + \varepsilon f_1 (x)\right) \frac{d^2 W}{dx^2}\right] + \tilde{K}_0 \left(1 + \varepsilon \varphi (x)\right) W + Pm^{-2} \frac{d^2 W}{dx^2} = 0. \tag{3}$$

Here we used the following denotation

$$K_0 = K_0 m^{-2}; \quad m^{-2} = (E_0 J_0 A)^{-1}; \quad A = \int_{-h}^{h} f(P)\,\rho^2 d\rho \tag{4}$$

$E_0 J_0$ is rigidity of homogeneous rod.

$$\left(\rho = zh^{-1}\right)$$

Equation (3) takes on the following form

$$\begin{gathered}(1 + \varepsilon f_1(x))\frac{d^4 W}{dx^4} + 2\varepsilon f_1'(x)\frac{d^3 W}{dx^3} + \varepsilon f_1''(x)\frac{d^2 W}{dx^2} + \\ + \tilde{K}(1 + \varepsilon\varphi(x))\,W - Pm^{-2}\frac{d^2 W}{dx^2} = 0\ .\end{gathered} \tag{5}$$

Note that solution of (5) may be found using one of approximate analytical methods. In the given case we apply the method

$$\begin{gathered}W = W_0 + \varepsilon W_1 + \varepsilon^2 W_2 + ... + \varepsilon^n W_n \\ P = P_0 + \varepsilon P_1 + \varepsilon^2 P_2 + ... + \varepsilon^n P_n\ .\end{gathered} \tag{6}$$

Substituting (6) into (5) and comparing with respect to powers of ε, we obtain the following system of equations

$$\frac{d^4 W_0}{dx^4} + m^{-2} P_0 \frac{d^2 W_0}{dx^2} + \tilde{K} W_0 = 0\ .$$

$$\begin{gathered}\frac{d^4 W_1}{dx^4} + m^{-2} P_0 \frac{d^2 W_1}{dx^2} + \tilde{K} W_1 = -f_1(x)\frac{d^4 W_0}{dx^4} - 2f'(x)\times \\ \times\frac{d^3 W_0}{dx^3} - f''(x)\frac{d^2 W_0}{dx^2} - \tilde{K}\varphi(x)\,W_0\end{gathered} \tag{7}$$

....

$$\begin{gathered}\frac{d^4 W_n}{dx^4} + m^{-2} P_n \frac{d^2 W_n}{dx^2} + \tilde{K} W_n = -f_1(x)\frac{d^4 W_{n-1}}{dx^4} - \\ -2f'(x)\frac{d^3 W_{n-1}}{dx^3} - f''(x)\frac{d^2 W_{n-1}}{dx^2} - \tilde{K}\varphi(x)\,W_{n-1}\end{gathered}$$

Note that to obtain the concrete results we must set, generally speaking, concrete values of $f_1(x)$ and $\varphi(x)$. Solution of the problem is sufficiently simplified in the case, when $f_1(x) = xe^{-1}$; $\varphi(x) = xl^{-1}$,(l is a length of rod).

In this case (7) takes on the following form

$$\frac{d^4W_0}{dx^4}+m^{-2}P_0\frac{d^2W_0}{dx^2}+\tilde{K}W_0=0$$

$$\frac{d^4W_1}{dx^4}+m^{-2}P_1\frac{d^2W_1}{dx^2}+\tilde{K}W_1=-f_1^{-1}\left(x\right)\frac{d^4W_0}{dx^4}-2l^{-1}\frac{d^3W_0}{dx^3}-\tilde{K}xW_0 \qquad (8)$$

....

$$\frac{d^4W_n}{dx^4}+m^{-2}\frac{d^2W_n}{dx^2}+\tilde{K}\ W_n=-xl^{-1}\frac{d^4W_{n-1}}{dx^4}-2l^{-1}\frac{d^3W_{n-1}}{dx^3}-\tilde{K}xW_{n-1}.$$

At first consider the posed problem without resistance. Then system (8) is simplified and it takes on the following form

$$\frac{d^4W_0}{dx^4}+m^{-2}P_0\frac{d^2W_0}{dx^2}=0\ .$$

$$\frac{d^4W_1}{dx^4}+m^{-2}P_1\frac{d^2W_1}{dx^2}=-f_1\left(x\right)\frac{d^4W_0}{dx^4}-2f'\left(x\right)\frac{d^3W_0}{dx^3}-f''\left(x\right)\frac{d^2W_0}{dx^2} \qquad (9)$$

....

$$\frac{d^4W_n}{dx^4}+m^{-2}P_n\frac{d^2W_n}{dx^2}=-f_1'\left(x\right)\frac{d^4W_{n-1}}{dx^4}-2f'\left(x\right)\frac{d^3W_{n-1}}{dx^3}-f''\left(x\right)\frac{d^2W_{n-1}}{dx^2}\ .$$

In the case $f_1=xl^{-1}$ (9) is simplified and it assumes the form

$$\frac{d^4W_0}{dx^4}+m^{-2}P_0\frac{d^2W_0}{dx^2}=0. \qquad (10)$$

$$\frac{d^4W_1}{dx^4}+m^{-2}P_1\frac{d^2W_1}{dx^2}=-xl^{-1}\frac{d^4W_0}{dx^4}-2l^{-1}\frac{d^3W_0}{dx^3}\ .$$

....

$$\frac{d^4W_n}{dx^4}+m^{-2}P_n\frac{d^2W_n}{dx^2}=-xl^{-1}\frac{d^4W_{n-1}}{dx^4}-2l^{-1}\frac{d^3W_{n-1}}{dx^3}\ .$$

We will construct solution of (10) for the cases when the rod is bilateral simply supported and bilateral rigidly built-in. In the first case the following conditions must be satisfied

$$\left.\begin{array}{l} W=0 \\ \dfrac{d^2W}{dx^2}=0 \end{array}\right\} \quad at \quad x=0;\quad x=l \qquad (11)$$

In the second case the following conditions must be fulfilled

$$\frac{dW}{dx}=0 \quad at \quad x=0; \quad x=l \tag{12}$$

In the first case if we choose $W_0 = f\sin\frac{n\pi}{l}x$, then we can establish that

$$\frac{P_0}{P_l}=A^{-1} \tag{13}$$

(P_l is Euler critical load).

This formula corresponds to the case, when a rod is non-homogeneous only in width.

For different types of non-homogeneity graphs of dependence between characteristic parameters. $A=1$ corresponds to the homogeneous case (Fig.1).

Solution of the first approximation is reduced to the following equation

$$\frac{d^4\bar{W}_1}{dx^4}+m^{-2}P_1\frac{d^2\bar{W}_1}{dx^2}=\left(-xl^{-1}\sin\frac{n\pi}{l}x+2l^{-1}\frac{n\pi}{l}\right)\left(\frac{n\pi}{l}\right)^3. \tag{14}$$

Solution of (14) is a sum of general solution of corresponding homogeneous equation and particular solution, i.e.,

$$\bar{W}_1=W_1f_0^{-1}; \quad \bar{W}_1=c_1\sin k_1x+c_2\cos k_1x+c_3x+c_4+\bar{W}_r$$

For the case (12) solution can be found in the form corresponding to homogeneous equation

$$\bar{W}=x^4-2x^3l+x^2l^2. \tag{15}$$

Then in this case solution is reduced to the following

$$\frac{d^4\bar{W}_1}{dx^4}+m^{-2}P_1\frac{d^2\bar{W}_1}{dx^2}=-24\left(xl^{-1}-1\right) \tag{16}$$

One can see from (16) and (15) that in the second case equation is simpler than in the first case. Later on for simplicity of analysis, to illustrate and carry out calculations we will study influence process for the second case.

It is easy to show that partial solution of (16) has the following form

$$W_r=-\frac{4}{k^2}\left(x^3l^{-1}-3x^2\right). \tag{17}$$

Then general solution has the following form

$$W_1=c_1\sin k_1x+c_2\cos kx+c_3x+c_4+W_1^r \tag{18}$$

Defining W^r and using boundary conditions we find P_1. Thus, we obtain that

$$P = P_0 + \varepsilon P_1 \quad or \quad \frac{P}{P_0} = 1 + \varepsilon \frac{P_1}{P_0} \ .$$

Results of calculations are represented in fig.1.

In conclusion we note that if we take into account resistance, we can take finished solution of the first approximation from the monograph [2].

In the same way one can construct solution subject to resistance of environment because it creates no problems.

Note that the analogous problem is solved for the case of nonlinear base.

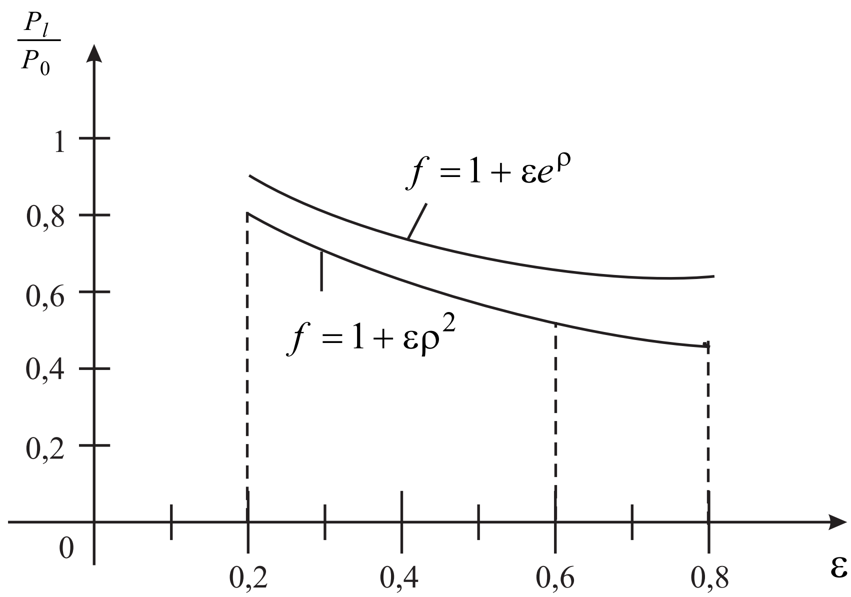

Fig. 1.

References

[1]. Lomakin V.A. *Theory elasticity of heterogeneous bodies.* M.: Izd. MGU, 1976, 376 p. (Russian)

[2]. Rzhanitsyn R.A. *Stability of equilibrium of elastic systems.* M.: 1955, 475 p. (Russian)

[3]. Gutag V. *Effect of nonhomogeneity on elastic plastic transition in a thin rotating disk.* Indian Pure and Appl. Math., 1994, 281 p.

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