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ON EIGENVALUES AND EIGENFUNCTIONS OF ONE CLASS OF DIRAC OPERATORS WITH DISCONTINUOUS COEFFICIENTS

Abstract

In the paper we study properties of eigenvalues and eigenfunctions of the system of Dirac equations with discontinuous coefficients; completeness theorem and theorem on expansion in eigenfunctions are proved.

Let us consider the system of Dirac equations

$$By' + \Omega(x)\,y = \lambda\rho(x)\,y, \quad 0 < x < \pi. \tag{1}$$

Here

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix},$$

$$\rho(x) = \begin{cases} 1, & 0 \le x \le a \\ \alpha, & a < x \le \pi \end{cases}, \ y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Assume that $0 < \alpha \neq 1$, $p(x)$ and $q(x)$ are real-valued functions and $p(x) \in L_2(0,\pi)$; $q(x) \in L_2(0,\pi)$; λ is a complex parameter.

Let us join the following boundary conditions to equation (1)

$$y_1(0) = y_1(\pi) = 0 \tag{2}$$

$$y_1(0) = y_2(\pi) + Hy_1(\pi) = 0 \tag{3}$$

$$y_2(0) - hy_1(0) = y_2(\pi) + Hy_1(\pi) = 0 \tag{4}$$

In the given paper we study the asymptotic behavior of eigenvalues and eigenfunctions of boundary value problems (1),(2); (1),(3); (1),(4), and also we will prove the completeness theorem and the theorem on expansion in eigenfunctions.

In case of $\rho(x) \equiv 1$ solution of similar problems are well known (see, e.g., [1]-[4]).

1. In this point we investigate in details the asymptotics of eigenvalues, eigenfunctions and normalizing numbers of boundary value problem (1)-(2). Denote by $S(x,\lambda)$, $C(x,\lambda)$ solutions of system of equations (1) satisfying the boundary conditions

$$S(0,\lambda) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \quad C(0,\lambda) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

It is easy to show that eigenvalues of boundary value problems (1), (2); (1), (3) and (1), (4) are the roots of characteristic functions

$$\Delta_1(\lambda) = S_1(\pi,\lambda), \ \Delta_2(\lambda) = S_2(\pi,\lambda) + HS_1(\pi,\lambda)$$

$$\Delta_3(\lambda) = C_2\pi, \lambda - hS_2(\pi,\lambda) + H\left(C_1(\pi,\lambda) - hS_1(\pi,\lambda)\right), \qquad (5)$$

respectively. In case of $\Omega(x) \equiv 0$ characteristic functions of these boundary value problems will have the form

$$\Delta_{10}(\lambda) = \sin\lambda\mu(\pi), \quad \Delta_{20}(\lambda) = -\cos\lambda\mu(\pi) + H\sin\lambda\mu(\pi)$$

$$\Delta_{30}(\lambda) = \sin\lambda\mu(\pi) - h\cos\lambda\mu(\pi) + H\left(\cos\lambda\mu(\pi) + h\sin\lambda\mu(\pi)\right) \qquad (6)$$

respectively, where $\mu(\pi) = \alpha\pi - \alpha a + a$.

Theorem 1. *1) Boundary value problem (1)-(2) has a countable set of simple eigenvalues* $\{\lambda_n\}_{n=-\infty}^{\infty}$*, at that*

$$\lambda_n = \frac{n\pi}{\alpha\pi - \alpha a + a} + \varepsilon_n, \quad \{\varepsilon_n\} \in l_2,$$

2) Eigen vector-functions of problem (1)-(2) can be represented in the form

$$S(x,\lambda_n) = \begin{pmatrix} \sin\frac{n\pi\mu(x)}{\alpha\pi-\alpha a+a} \\ -\cos\frac{n\pi\mu(x)}{\alpha\pi-\alpha a+a} \end{pmatrix} + \begin{pmatrix} \xi_n^{(1)}(x) \\ \xi_n^{(2)}(x) \end{pmatrix},$$

$$\sum_{n=-\infty}^{\infty}\left\{\left|\xi_n^{(1)}(x)\right|^2 + \left|\xi_n^{(2)}(x)\right|^2\right\} \le C; \quad \mu(x) = \begin{cases} x, & 0 \le x \le a \\ \alpha x - \alpha a + a, & a < x \le \pi \end{cases}$$

3) Normalizing numbers of problem (1)-(2) have the form

$$\alpha_n = \alpha\pi - \alpha a + a + \delta_n, \qquad \{\delta_n\} \in l_2$$

Proof. Using integral representation of solution $S(x,\lambda)$ (see [5])

$$S(x,\lambda) = \begin{pmatrix} \sin\lambda\mu(x) \\ -\cos\lambda\mu(x) \end{pmatrix} + \int_0^{\mu(x)} A(x,t)\begin{pmatrix} \sin\lambda t \\ -\cos\lambda t \end{pmatrix} dt \qquad (7)$$

where $A = (A_{ij})_{i,j=1}^{2}$ is a quadratic matrix function $A_{ij}(x,\cdot) \in L_2(0,\pi)$; for the characteristic function $\Delta_1(\lambda)$ we obtain the following representation

$$\Delta_1(\lambda) = \Delta_{10}(\lambda) + \int_0^{\mu(\pi)} A_{11}(\pi,t)\sin\lambda t dt - \int_0^{\mu(\pi)} A_{12}(\pi,t)\cos\lambda t dt \qquad (8)$$

Denote by $G_\delta = \left\{\lambda : \left|\lambda - \frac{n\pi}{\mu(\pi)}\right| \ge \delta\right\}$, where δ is a sufficiently small positive number. It is easy to show that there exists a positive number C_δ such that

$$|\Delta_{10}(\lambda)| = |\sin\lambda\mu(\pi)| \ge C_\delta e^{|Jm\lambda|\mu(\pi)}, \quad \lambda \in G_\delta$$

On the other hand, applying lemma 1.3.1 from [2] to relation (8), we obtain

$$\Delta_1(\lambda) - \Delta_{10}(\lambda) = 0\left(e^{|Jm\lambda|\mu(\pi)}\right), \quad |\lambda| \to \infty$$

Therefore on infinitely expanding contours $\Gamma_n = \left\{\lambda : |\lambda| = \frac{n\pi}{\mu(\pi)} + \frac{\pi}{2\mu(\pi)}\right\}$ for sufficiently large n we have

$$|\Delta_1(\lambda) - \Delta_{10}(\lambda)| < |\Delta_{10}(\lambda)|$$

Then by Rouche theorem number of zeros of function $\{\Delta_1(\lambda) - \Delta_{10}(\lambda)\} + \Delta_{10}(\lambda) = \Delta_1(\lambda)$ inside the contour Γ_n coincides with the number of zeros of the function $\Delta_{10}(\lambda)$. Function $\Delta_{10}(\lambda) = \sin\lambda\mu(\pi)$ has $(2n+1)$ zeros in Γ_n, therefore for sufficiently large n the function $\Delta_1(\lambda)$ has the same number of zeros. Denote them by $\lambda_{-n}, \lambda_{-(n-1)}, ..., \lambda_0, \lambda_1, ..., \lambda_n$.

Further applying Rouche theorem to the circle $\gamma_n(\delta) = \left\{\lambda : \left|\lambda - \frac{n\pi}{\mu(\pi)}\right| < \delta\right\}$ we conclude that for sufficiently large $|n|$ in $\gamma_n(\delta)$ lies only one root of the function $\Delta_1(\lambda) : \lambda_n$. By virtue of the arbitrariness of $\delta > 0$ we have

$$\lambda_n = \frac{n\pi}{\mu(\pi)} + \varepsilon_n, \quad \lim_{n\to\pm\infty} \varepsilon_n = 0 \tag{9}$$

Substituting (9) into (8) and taking into account $\Delta_1(\lambda_n) = 0$, we have

$$0 = \sin\left(\frac{n\pi}{\mu(\pi)} + \varepsilon_n\right)\mu(\pi) + \int_0^{\mu(\pi)} A_{11}(\pi,t)\sin\left(\frac{n\pi}{\mu(\pi)} + \varepsilon_n\right) tdt -$$

$$- \int_0^{\mu(\pi)} A_{12}(\pi,t)\cos\left(\frac{n\pi}{\mu(\pi)} + \varepsilon_n\right) tdt$$

or

$$(-1)^n \sin\varepsilon_n\mu(\pi) + \int_0^{\mu(\pi)} A_{11}(\pi,t)\sin\left(\frac{n\pi}{\mu(\pi)} + \varepsilon_n\right) tdt -$$

$$- \int_0^{\mu(\pi)} A_{12}(\pi,t)\cos\left(\frac{n\pi}{\mu(\pi)} + \varepsilon_n\right) tdt = 0 \tag{9'}$$

On the other hand, since $A_{11}(\pi,\cdot) \in L_2(0,\pi)$, $A_{12}(\pi,\cdot) \in L_2(0,\pi)$, according to [2, p.67] we have

$$\left\{\int_0^{\mu(\pi)} A_{11}(\pi,t)\sin\left(\frac{n\pi}{\mu(\pi)} + \varepsilon_n\right) tdt\right\} \in l_2,$$

$$\left\{\int_0^{\mu(\pi)} A_{12}(\pi,t)\cos\left(\frac{n\pi}{\mu(\pi)} + \varepsilon_n\right) tdt\right\} \in l_2,$$

consequently, it follows from $(9')$ that $\sum_{n=-\infty}^{\infty} |\varepsilon_n|^2 < +\infty$, i.e., $\{\varepsilon_n\} \in l_2$.

Thus assertion 1) of theorem 1 is proved.

2) It is obvious that vector-functions $S(x,\lambda_n)$ are eigenfunctions of problem (1)-(2). Using representation (7) we can write $S(x,\lambda_n)$ in the form

$$S(x,\lambda_n)=\begin{pmatrix} \sin\frac{n\pi\mu(x)}{\mu(\pi)} \\ -\cos\frac{n\pi\mu(x)}{\mu(\pi)} \end{pmatrix}+\begin{pmatrix} \xi_n^{(1)}(x) \\ \xi_n^{(2)}(x) \end{pmatrix},$$

where

$$\xi_n^{(1)}(x)=\sin\frac{n\pi\mu(x)}{\mu(\pi)}\left[\cos\varepsilon_n\mu(x)-1\right]+\cos\frac{n\pi\mu(x)}{\mu(\pi)}\sin\varepsilon_n\mu(x)+$$

$$+\int_0^x A_{11}(x,t)\sin\left(\frac{n\pi}{\mu(\pi)}+\varepsilon_n\right)tdt-\int_0^x A_{12}(x,t)\cos\left(\frac{n\pi}{\mu(\pi)}+\varepsilon_n\right)tdt;$$

$$\xi_n^{(2)}(x)=-\cos\frac{n\pi\mu(x)}{\mu(\pi)}\left[\cos\varepsilon_n\mu(x)-1\right]+\sin\frac{n\pi\mu(x)}{\mu(\pi)}\sin\varepsilon_n\mu(x)+$$

$$+\int_0^x A_{21}(x,t)\sin\left(\frac{n\pi}{\mu(\pi)}+\varepsilon_n\right)tdt-\int_0^x A_{22}(x,t)\cos\left(\frac{n\pi}{\mu(\pi)}+\varepsilon_n\right)tdt.$$

Hence $\sup\limits_{0\le x\le\pi}\sum\left\{\left|\xi_n^{(1)}(x)\right|^2+\left|\xi_n^{2}(x)\right|^2\right\}<+\infty$, since $\{\varepsilon_n\}\in l_2$.

3) For normalizing numbers of problem (1)-(2) we have

$$\alpha_n^{(1)}=\int_0^\pi\rho(x)\left\{\left|S_1(x,\lambda_n)\right|^2+\left|S_2(x,\lambda_n)\right|^2\right\}dx=$$

$$=\int_0^\pi\rho(x)\left\{\sin^2\frac{n\pi\mu(x)}{\mu(\pi)}+\cos^2\frac{n\pi\mu(x)}{\mu(\pi)}\right\}dx+\delta_n=\mu(\pi)+\delta_n,$$

where

$$\delta_n=2\int_0^\pi\rho(x)\sin\frac{n\pi\mu(x)}{\mu(\pi)}\xi_n^{(1)}(x)\,dx-2\int_0^\pi\rho(x)\cos\frac{n\pi\mu(x)}{\mu(\pi)}\times$$

$$\times\xi_n^{(2)}(x)\,dx+\int_0^\pi\rho(x)\left(\xi_n^{(1)}(x)\right)^2dx+\int_0^\pi\rho(x)\left(\xi_n^{(2)}(x)\right)^2dx.$$

Hence $\{\delta_n\}\in l_2$. Finally note that the simplicity of eigenvalues follows from the equality

$$\alpha_n=-\mathring{\Delta}_1(\lambda_n)\,S_2(\pi,\lambda_n).$$

Theorem 1 is proved.

Denote by L_1 and L_2 boundary value problems (1), (3) and (1), (4), respectively. One can analogously prove the following theorem on asymptotics of eigenvalues and eigenfunctions for the boundary value problem $L_i,\ i=1,2$.

Theorem 2. *1) Boundary value problems L_i have a countable number of simple eigenvalues $\{\lambda_{ni}\}_{n=-\infty}^{\infty}$ which can be represented in the form*

$$\lambda_{ni} = \lambda_{ni}^{\circ} + \varepsilon_{ni}, \ \{\varepsilon_{ni}\} \in l_2, \quad i = 1, 2.$$

where λ_{ni}° are zeros of functions $\Delta_{i+1,0}(\lambda)$.

2) Eigen vector-functions $\varphi_{n1}(x) = S(x, \lambda_{n1})$ and $\varphi_{n2}(x) = C(x, \lambda_{n2}) - hS(x, \lambda_{n2})$ of problems L_1 and L_2, respectively, have the form

$$\varphi_{n1}(x) = \begin{pmatrix} \sin \lambda_{n1}^{\circ} \mu(x) \\ -\cos \lambda_{n1}^{\circ} \mu(x) \end{pmatrix} + \begin{pmatrix} \xi_{n1}^{(1)}(x) \\ \xi_{n1}^{(2)}(x) \end{pmatrix},$$

$$\varphi_{n2}(x) = \begin{pmatrix} \cos \lambda_{n2}^{\circ} \mu(x) - h \sin \lambda_{n2}^{\circ} \mu(x) \\ \sin \lambda_{n2}^{\circ} \mu(x) + h \cos \lambda_{n2}^{\circ} \mu(x) \end{pmatrix} + \begin{pmatrix} \xi_{n2}^{(1)}(x) \\ \xi_{n2}^{(2)}(x) \end{pmatrix},$$

where $\sup\limits_{0 \le x \le \pi} \sum \left\{ \left| \xi_{ni}^{(1)}(x) \right|^2 + \left| \xi_{ni}^{(2)}(x) \right|^2 \right\} < +\infty.$

3) *Normalizing numbers* $\alpha_{ni} = \int\limits_0^{\pi} \rho(x) \left\{ \left(\varphi_{ni}^{(1)}(x) \right)^2 + \left(\varphi_{ni}^{(2)}(x) \right)^2 \right\} dx$ *of the problem L_i can be represented in the form*

$$\alpha_{ni} = \alpha_{ni}^{\circ} + \delta_{ni}, \quad \{\delta_{ni}\} \in l_2,$$

where $\alpha_{n1}^{\circ} = \mu(\pi) = \alpha\pi - \alpha a + a, \ \alpha_{n2}^{\circ} = \left(1 + h^2\right) \mu(\pi)$

2. In this point we prove the completeness theorem and theorem on expansion in eigenfunctions. Denote by $L_{2,\rho}\left(0, \pi; \ \mathbb{C}^2\right)$ Hilbert space of measurable complex-valued vector-functions $f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix}$ such that

$$\|f\|^2 = \int\limits_0^{\pi} \rho(x) \left\{ |f_1(x)|^2 + |f_2(x)|^2 \right\} dx < +\infty.$$

Scalar product in this space is defined by the following formula

$$< f, g >= \int\limits_0^{\pi} \rho(x) \left\{ f_1(x) \overline{g_1(x)} + f_2(x) \overline{g_2(x)} \right\} dx$$

Theorem 3. *a) The system of eigen vector-functions $\{S(x, \lambda_n)\}_{n=-\infty}^{+\infty}$ of problem (1)-(2) is complete in space $L_{2,\rho}\left(0, \pi; \mathbb{C}^2\right)$;*

b) Let $f(x)$ be an absolutely continuous vector-function on the segment $[0, \pi]$ and $f_1(0) = f_1(\pi) = 0$. Then

$$f(x) = \sum_{n=-\infty}^{+\infty} a_n S(x, \lambda_n), \tag{10}$$

$$a_n = \frac{1}{\alpha_n} < f(x), S(x, \lambda_n) >,$$

moreover, the series converges uniformly with respect to $x \in [0, \pi]$*;*

c) For $f(x) \in L_{2,\rho}\left(0, \pi; \mathbb{C}^2\right)$ *series (10) converges in* $L_{2,\rho}\left(0, \pi; \mathbb{C}^2\right)$*; moreover, the Parseval equality holds*

$$\|f\|^2 = \sum_{n=-\infty}^{+\infty} \alpha_n |a_n|^2 .$$

Proof. Let $\psi(x, \lambda)$ be a solution of equation (1) under the boundary conditions $\psi_1(\pi, \lambda) = 0,\ \psi_2(\pi, \lambda) = -1$. Denote

$$G(x, t, \lambda) = \frac{1}{\Delta_1(\lambda)} \begin{cases} \psi(x, \lambda) \widetilde{S}(t, \lambda), & x \geq t, \\ S(x, \lambda) \widetilde{\psi}(t, \lambda), & x \leq t \end{cases}$$

($\widetilde{y}$ denotes the conjugate of vector y) and consider the function

$$Y(x, \lambda) = \int_0^\pi G(x, t, \lambda) f(t) \rho(t)\, dt, \tag{11}$$

which gives solution of the boundary value problem

$$BY' + \Omega(x) Y = \lambda \rho(x) Y + f(x) \rho(x), \quad Y_1(0, \lambda) = Y_1(\pi, \lambda) = 0 \tag{12}$$

It is easy to show that

$$\psi(x, \lambda_n) = \frac{\mathring{\Delta}_1(\lambda_n)}{\alpha_n} S(x, \lambda_n),$$

therefore

$$\operatorname*{Re}_{\lambda=\lambda_n} sY(x, \lambda) = \frac{1}{\alpha_n} S(x, \lambda_n) \int_0^\pi \widetilde{S}(t, \lambda_n) f(t) \rho(t)\, dt \tag{13}$$

Let $f(x) \in L_{2,\rho}\left(0, \pi; \mathbb{C}^2\right)$ be such that

$$< f(x), S(x, \lambda_n) >= \int_0^\pi \widetilde{S}(t, \lambda_n) f(t) \rho(t)\, dt = 0, \quad n = 0, \pm 1, \pm 2, ...$$

Then subject to (13) we obtain $\operatorname*{Re}_{\lambda=\lambda_n} sY(x, \lambda) = 0$ and consequently for each fixed $x \in [0, \pi]$ function $Y(x, \lambda)$ is entire with respect to λ. Now we use estimate $|\Delta_1(\lambda)| \geq C_\delta e^{|Jm\lambda|\mu(\pi)}$, which is valid in domain $G_\delta = \left\{\lambda : \left|\lambda - \frac{n\pi}{\mu(\pi)}\right| \geq \delta\right\}$, where δ is sufficiently small positive number, and the following lemma whose proof is analogous to the proof of lemma 1.3.1 [2, p.36].

Lemma. *For all vector-functions* $f(x) \in L_{2,\rho}\left(0, \pi; \mathbb{C}^2\right)$ *the following equality is valid*

$$\lim_{|\lambda| \to \infty} \max_{0 \leq x \leq \pi} e^{-|Jm\lambda|\mu(x)} \left| \int_0^x \widetilde{S}(t, \lambda) f(t) \rho(t)\, dt \right| =$$

$$= \lim_{|\lambda|\to\infty} \max_{0\le x\le\pi} e^{-|Jm\lambda|(\mu(\pi)-\mu(x))} \left| \int_x^\pi \widetilde{\psi}(t,\lambda) f(t) \rho(t) dt \right| = 0$$

Applying these facts, from (11) we have

$$\lim_{\substack{|\lambda|\to\infty \\ \lambda\in G_\delta}} \max_{0\le x\le\pi} |Y(x,\lambda)| = 0.$$

Thus $Y(x,\lambda) \equiv 0$. From here and (12) it follows that $f(x) = 0$ a.e. on $(0,\pi)$. Statement a) is proved.

b) Let now $f(x)$ be an arbitrary absolutely continuous vector-function on $[0,\pi]$ and $f_1(0) = f_1(\pi) = 0$. Since $S(x,\lambda)$ and $\psi(x,\lambda)$ are solutions of equation (1), then vector-function $Y(x,\lambda)$ can be transformed to the form

$$Y(x,\lambda) = \frac{1}{\lambda\Delta_1(\lambda)} \left\{ \psi(x,\lambda) \int_0^x \frac{1}{\rho(t)} \left(\widetilde{BS'}(t,\lambda) + \widetilde{\Omega(t)S}(t,\lambda) \right) \times \right.$$

$$\left. \times f(t)\rho(t)dt + S(x,\lambda) \int_x^\pi \frac{1}{\rho(t)} \left(\widetilde{B\psi'}(t,\lambda) + \widetilde{\Omega(t)\psi}(t,\lambda) \right) f(t)\rho(t)dt \right\} =$$

$$= -\frac{1}{\lambda\Delta_1(\lambda)} \left(\psi(x,\lambda) \int_0^x \widetilde{S}'(t,\lambda) Bf(t) dt + S(x,\lambda) \int_x^\pi \widetilde{\psi}'(t,\lambda) Bf(t) dt \right) +$$

$$+\frac{1}{\lambda\Delta_1(\lambda)} \left(\psi(x,\lambda) \int_0^x \widetilde{S}(t,\lambda)\Omega(t) f(t) dt + S(x,\lambda) \int_x^\pi \widetilde{\psi}(t,\lambda)\Omega(t) f(t) dt \right)$$

Integration by parts of the terms with the first derivatives gives

$$Y(x,\lambda) = \frac{1}{\lambda} f(x) + \frac{1}{\lambda} Z(x,\lambda) \tag{14}$$

where

$$Z(x,\lambda) = \frac{1}{\Delta_1(\lambda)} \left\{ \psi(x,\lambda) \int_0^x \widetilde{S}(t,\lambda) Bf'(t) dt + S(x,\lambda) \int_x^\pi \widetilde{\psi}(t,\lambda) \times \right.$$

$$\left. \times Bf'(t) dt + \psi(x,\lambda) \int_0^x \widetilde{S}(t,\lambda)\Omega(t) f(t) dt + S(x,\lambda) \int_x^\pi \widetilde{\psi}(t,\lambda)\Omega(t) f(t) dt \right\}$$

By means of above mentioned lemma we have

$$\lim_{\substack{|\lambda|\to\infty \\ \lambda\in G_\delta}} \max_{0\le x\le\pi} |Z(x,\lambda)| = 0 \tag{15}$$

Let us consider the contour integral

$$I_N(x) = \frac{1}{2\pi i}\oint_{\Gamma_N} Y(x,\lambda)\, d\lambda,$$

where $\Gamma_N = \left\{\lambda : |\lambda| = \frac{N\pi}{\mu(\pi)} + \frac{\pi}{2\mu(\pi)}\right\}$ is a oriented counter-clockwise, N is sufficiently large natural number. By means of residues theorem we have

$$I_N(x) = \sum_{n=-N}^{N} \operatorname*{Res}_{\lambda=\lambda_n} Y(x,\lambda) = \sum_{n=-N}^{N} a_n S(x,\lambda_n),$$

where

$$a_n = \frac{1}{\alpha_n}\int_0^\pi \widetilde{S}(t,\lambda_n)\,\rho(t)\, f(t)\, dt,$$

On the other hand, taking into account (14) we have

$$f(x) = \sum_{n=-N}^{N} a_n S(x,\lambda_n) + \varepsilon_N(x)$$

where $\varepsilon_N(x) = -\frac{1}{2\pi i}\oint_{\Gamma_N} \frac{1}{\lambda} Z(x,\lambda)\, d\lambda$

Further from (15) it follows that $\lim\limits_{N\to\infty}\max\limits_{0\le x\le\pi}|\varepsilon_N(x)| = 0$,consequently, statement b) of theorem 3 is proved.

c) System $\{S(x,\lambda_n)\}$ is complete and orthogonal in $L_{2,\rho}\left(0,\pi;\mathbb{C}^2\right)$. Therefore it forms orthogonal basis in $L_{2,\rho}\left(0,\pi;\mathbb{C}^2\right)$, and the Parseval equality holds. Theorem 3 is completely proved. The following theorem can be proved in a similar manner:

Theorem 4. *a) System of eigen vector-functions* $\{\varphi_{ni}(x)\}$ *of boundary problem* L_i *is complete in the space* $L_{2,\rho}\left(0,\pi;\mathbb{C}^2\right)$*;*

b) Let $f(x)$ *and* $g(x)$ *be arbitrary absolutely continuous vector-functions and* $f_1(0) = 0$*. Then*

$$f(x) = \sum_{n=-\infty}^{\infty} a_{n1}\varphi_{n1}(x), \quad g(x) = \sum_{n=-\infty}^{\infty} a_{n2}\varphi_{n2}(x),$$

$$a_{ni} = \frac{1}{\alpha_{ni}} < f, \varphi_{ni} >,$$

moreover, the series uniformly converge with respect to $x \in [0,\pi]$*.*

c) For $f(x) \in L_{2,\rho}\left(0,\pi;\mathbb{C}^2\right)$ *and* $g(x) \in L_{2,\rho}\left(0,\pi;\mathbb{C}^2\right)$ *series from point b) converge in* $L_{2,\rho}\left(0,\pi;\mathbb{C}^2\right)$*, and also the Parseval equality holds*

$$\|f\|^2 = \sum_{n=-\infty}^{\infty} |a_{n1}|^2\,\alpha_{n1}, \quad \|g\|^2 = \sum_{n=-\infty}^{\infty} |a_{n2}|^2\,\alpha_{n2}.$$

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