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NUMERICAL METHOD FOR CALCULATION OF DYNAMICAL PROCESSES IN BORING ELECTRIC DRIVER WITH DISTRIBUTED PARAMETERS ALLOWING FOR BIT JUMPING LOSSES

Abstract

It is represented a numerical method for calculation of dynamical processes in boring electric drive including a unit with distributed parameters described by telegraph equation in the process of bit jumping at rotor boring of oil wells.

The peculiarity of dynamic properties of boring electric drive at rotor drilling is that its mechanical part – columns of drill pipes is an object with distributed parameters [1-7].

At present, computer technology is widely applied to engineering practice. Therefore, a great attention is given to creation of numerical processes in systems with distributed parameters.

There are various numerical methods for solving dynamics problems in the indicated systems.

Among them a method of finite differences, both on "noevidence" and on "evident" schemes, method of characteristic this [7,8] are widely used in engineering practice.

Analysis shows that a general deficiency of finite difference method is that at chosen step by the form of the curve and by the accuracy of calculation it gives the worst coincidence with the analytic solution with respect to the method of characteristics [7,9].

A numerical method [1] based on the theory of impulse systems and mathematical device of discrete Laplace transform is one of effective numerical methods in calculating dynamical processes in objects with distributed parameter described by partial equations of hyperbolic type.

Such an approach allows to find dynamical processes proceeding in objects with distributed parameters without finding the roots of characteristic equation, expansion of operator coefficients of wave propagation and operator wave resistance in series that significantly simplifies mathematical computations and increases the accuracy of calculations.

In the paper [1] on the bases of impulse systems and mathematical device it is represented an analytical method for solving telegraph equations for the initial point ($x = 0$) of the object with distributed parameters for $0,05 \leq \alpha T \leq 1,5, 0,05 \leq$ $\leq \beta T \leq 1,5$(α is a wave attenuation coefficient; β is wave distortion coefficient; $T = 2\tau, \tau$ is the time of wave run to one end of the object with distributed parameters).

Approximation by lattice functions was performed for infinite sum containing Bessel functions. The functions represented by sufficiently prime expressions, coinciding well with approximated functions in the time intervals we are interested in, were chosen for the approximation. In the paper [2] the further development of the analytic method [1] is given for numerical determination of transient processes in electric drives with distributed parameters described by telegraph equations under sufficiently complicated boundary conditions.

In the given paper it given a generalized numerical method for calculating dynamical processes in drilling electric drive with distributed parameters described by telegraph equations at bit jumping.

The essence of the suggested numerical method is in using discrete analogy of integral convolution equation [5-6].

The advantage of the suggested numerical method is that it allows to find dynamical processes in the initial system with distributed parameters for arbitrary values of αT and βT not going over to the domain of discrete images, expansion of operator coefficients of wave propagation and operator wave resistance in series, and also to realize the transition from Laplace images of the desired functions to the pre-image domain without finding the roots of the characteristic equation. This signicuntly simplifies mathematical calculations and extends the circle of practical solutions.

The property of the given condition is the following. The column of the pipe rotates with angular speed $\omega_{init.}$. Moment $M_{init.}$ initial transmitted by a rotary table to drill pipes is determined by the resistance moment of the rock. In consequence of this the pipes undergo constant stress σ. If the column of pipes rotates with angular speed $\omega_{init.}$ initial and the bit is jammed then in the system dynamic process corresponding to the case jammed shaft end begins.

The motion equation of the drilling electric drive at linear mechanical characteristics of an engine in the lattice form is as follows:

$$I\left(\omega_H[n+1]-\omega_H[n]\right)/T/\lambda = a_1 - b_1\omega_H[n] - M_c[n], \tag{1}$$

where I is inertia moment of the electric drive; a_1 and b_1 are mechanical characteristics parameters of the engine; $\omega_H[n]$, $M_c[n]$ are the values of rotation frequency and loading moment of the electric drive in a lattice form for the initial point unit with distributed parameters; λ is any integer.

The error of calculations is connected with the choice of the quantity λ.

In equation (1) the values of the loading moment $M_c[n]$ is determined by the following method.

Dynamic processes proceeding in the columns of drill pipes as an object with distributed parameters by calculating the friction between the strings and drilling mud are described by telegraph equations [1, 2, 5];

$$-\frac{\partial \omega}{\partial x} = k_1\frac{\partial M}{\partial t} + k_3 M,$$

$$-\frac{\partial M}{\partial x} = k_2\frac{\partial \omega}{\partial t} + k_4\omega,\ \ 0 \le x \le l, \tag{2}$$

where $\omega = \omega(x,t)$ is angular speed; $M = M(x,t)$ is a torque; K_1, K_2, K_3, K_4 are the coefficients characterizing the elasticity, inertia, pliability and friction loss of elementary part of drill pipe string; l is the length of drill pipe strings.

Initial conditions

$$\omega(x,t)_{t=0} = \omega_{init.},\ M(x,t)_{t=0} = M_{init}.$$

Boundary conditions

$$\omega(x,t)_{x=0} = \omega_n(t),\ \omega(x,t)_{x=l} = 0,$$

where $\omega_H(t)$ is an arbitrary law of angular speed alternation at the beginning of strings.

At the given statement the peculiarity of the solution for the posed problem is that at boundary conditions the value of the functions $\omega_H(t)$ at the beginning of the beginning solution of the stated problem is unknown. Its value is determined is the course of the solution of the problem.

At adopted initial and boundary conditions, from the solution of the system of differential equations (2) for the torsion moment we can represent the following expression in operator form for the initial point ($x = 0$) of strings

$$M_H(s) = \frac{1}{\rho(s)}\frac{ch\gamma l}{sh\gamma l}\left(\omega_H(s) - \frac{\omega_{init.}}{s}\right) + \frac{1}{\rho(s)}\frac{1}{sh\gamma l} + \frac{M_{init.}}{s}, \tag{3}$$

where $\gamma = \gamma(s) = \frac{1}{v}\sqrt{(s+\alpha)^2 - \beta^2}$ is an operator constant of wave propagation; $\rho(s) = \sqrt{\frac{sk_1 + k_3}{sk_2 + k_4}}$ is an operator wave resistance of drill pipe strings ; s - is an operator of Laplace transform; $M_H(s),\ \omega_H(s)$ is Laplacian image of functions $M_H(t),\ \omega_H(t);\ v = 1/\sqrt{k_1 k_2}$ is wave propagation speed.

By the method suggested in [3-6] we can represent expression (3) as:

$$M_H(s)\left[\frac{1}{s} - k_1(s)\right] = \frac{1}{\rho}\left[k_2(s) + \frac{k_4}{k_2}k_3(s) + k_4(s)\right]\left(\omega_H(s) - \frac{\omega_{init.}}{s}\right) +$$

$$+\frac{1}{s}\frac{M_{init.}}{s} - \frac{M_{init.}}{s}k_1(s) + \frac{M_{init.}}{s}\frac{2}{\rho}\left(k_3(s) + \frac{k_4}{k_2}k_7(s)\right), \tag{4}$$

$$k_1(s) = \frac{e^{-2\gamma l}}{s};\ k_2(s) = \frac{1}{\sqrt{(s+\alpha)^2 - \beta^2}},\ k_3(s) = \frac{1}{s}\frac{1}{\sqrt{(s+\alpha)^2 - \beta^2}},$$

$$k_4(s) = \frac{e^{-2\gamma l}}{\sqrt{(s+\alpha)^2 - \beta^2}},\ k_5(s) = \frac{1}{s}\frac{e^{-2\gamma l}}{\sqrt{(s+\alpha)^2 - \beta^2}},$$

$$k_6(s) = \frac{e^{-\gamma l}}{\sqrt{(s+\alpha)^2 - \beta^2}},\ k_7(s) = \frac{1}{s}\frac{e^{-\gamma l}}{\sqrt{(s+\alpha)^2 - \beta^2}}\rho = \sqrt{\frac{k_1}{k_2}} -$$

wave resistance of drill pipe string ignoring losses; $\alpha = \frac{1}{2}\left(\frac{k_3}{k_1} + \frac{k_4}{k_2}\right),\ \beta = \frac{1}{2}\left(\frac{k_3}{k_1} + \frac{k_4}{k_2}\right).$

In special case, if $K_4 = 0$ then $\alpha = \beta$. For $\beta = 0$ in so called balanced units where the following ration of parameters holds.

For the balanced unit ($\beta = 0$) coefficient ρ becomes equal to the same value that for the unit without loss.

On the basis of the convolution theorem [4-6] going over from equation (4) with respect to the images to the equation with pre-images, we get:

$$\int_0^t M_H(t-\theta)\,1(\theta)\,d\theta-\int_0^t M_H(t-\theta)\,k_1(\theta)d\theta=\frac{1}{\rho}\left[\int_0^t k_2(\theta)\left(\omega_H(t-\theta)-\omega_{init.}\right)d\theta+\right.$$

$$+\frac{k_4}{k_2}\int_0^t k_3(\theta)\left(\omega_H(t-\theta)-\omega_{init.}\right)d\theta+\int_{\frac{2l}{v}}^t k_4(\theta)(\omega t-\theta\Bigg)-\omega_{init.})d\theta+$$

$$\left.+\frac{k_4}{k_2}\int_{\frac{2l}{v}}^t k_5(\theta)\left(\omega_H(t-\theta)-\omega_{init.}\right)d\theta\right]+$$

$$+M_{init.}\left[\int_0^t 1(\theta)k_1(t-\theta)d\theta-\int_{\frac{2l}{v}}^t 1(\theta)\,k_1(t-\theta)\,d\theta-\right.$$

$$\left.-2\frac{\omega_{init.}}{\rho}\left[\int_{\frac{l}{v}}^t 1(\theta)k_6(t-\theta)\,d\theta\right]+\frac{k_4}{k_2}\int_{\frac{l}{v}}^t 1(\theta)k_7(t-\theta)\,d\theta\right], \tag{5}$$

where $K_1(t),...,K_7(t)$ are the known pre-images of transfer functions $K_1(s),...,K_7(s)$.

We can solve integral equations (5) numerically if we substitute the integrals by sums.

In this connection using the relation between continuous time t and discrete n in the form $t=nT/\lambda[1-6]$ $(n=0,1,2,...)$ we construct the discretization of equation (5) at the chosen interval T/λ by changing the continuous integration operation by the summation using the rectangles formula.

Here, instead of (5) we get the following expression in the lattice form:

$$\sum_{m=0}^{n}M_H[n-m]1[m]+\sum_{m=\lambda}^{n}k_1[m]M_H[n-m]=\frac{1}{\rho}\left[\sum_{m=0}^{n}(k_2[m]+\right.$$

$$\left.+\frac{k_4}{k_2}\frac{T}{\lambda}k_3[m])+\sum_{m=\lambda}^{n}(k_4[m]+\frac{k_4}{k_2}\frac{T}{\lambda}k_5[m])\right](\omega_H[n-m]-\omega_{init.})-$$

$$-M_{init.}\sum_{m=\lambda}^{n}1[m]k_1[n-m]+2\frac{\omega_{init.}}{\rho}\sum_{m=0,5\lambda}^{n}(k_6[n-m]+$$

$$+\frac{k_4}{k_2}\frac{T}{\lambda}k_7[n-m])1[m]+M_{init.}(n+1), \tag{6}$$

where $n<\lambda$ for

$$k_1[n] = \left\{ \begin{array}{c} 0 \\ e^{-\alpha T} + \alpha T \sum\limits_{m=\lambda+1}^{n} e^{-\frac{\alpha T}{\lambda}m} \dfrac{I_1\left(\dfrac{\beta T}{\lambda}\sqrt{m^2-\lambda^2}\right)}{\sqrt{m^2-\lambda^2}} \end{array} \right\},$$

where $n > \lambda$

$$k_2[n] = e^{-\frac{\alpha T}{\lambda}n} I_0\left(\beta\frac{T}{\lambda}n\right),\ k_3[n] = \sum_{m=0}^{n} k_2[m],\ k_4[n] = e^{-\frac{\alpha T}{\lambda}n} I_0\left(\beta\frac{T}{\lambda}\sqrt{n^2-\lambda^2}\right),$$

$$k_5[n] = \sum_{m=\lambda}^{n} k_4[m],\ k_6[n] = e^{-\frac{\alpha T}{\lambda}n} I_0\left(\beta\frac{T}{\lambda}\sqrt{n^2-(0.5\lambda)^2}\right),\ k_7[n] = \sum_{m=0.5\lambda}^{n} k_6[m].$$

Here

$$\sum_{m=0}^{n} 1[m] M_H[n-m] = M_H[n] + \sum_{m=0}^{n} M_H[m]. \qquad (7)$$

Allowing for (7) in expression (6) we get the following recurrent relation for the lattice function $M_H[n]$:

$$M_H[m] = \frac{1}{\rho}\left[\sum_{m=0}^{n}\left(k_2[m] + \frac{k_4}{k_2}\frac{T}{\lambda}k_3[m]\right) + \sum_{m=\lambda}^{n}(k_4[m] + \right.$$

$$\left. + \frac{k_4}{k_2}\frac{T}{\lambda}k_5[m])\right](\omega_H[n-m] - \omega_{init.}) - M_{init.}\sum_{m=\lambda}^{n} 1[m]k_1[n-m] +$$

$$+2\frac{\omega_{init.}}{\rho}\sum_{m=0.5\lambda}^{n}\left(k_6[n-m] + \frac{k_4}{k_2}\frac{T}{\lambda}k_7[n-m]\right)1[m] + M_{init.}(n+1) -$$

$$-\sum_{m=1}^{n} M_H[n-m]1[m] + \sum_{m=\lambda}^{n} k_1[m]M_H[n-m]. \qquad (8)$$

With regard to $Mc[n] = M_H[n]$ we can represent expression (1) as:

$$\omega_H[n+1] = (T/\lambda I)\, a_1 + (1 - Tb/\lambda I)\, \omega_H[n] - (T/\lambda I)\, M_H[n]. \qquad (9)$$

Thus, we obtain two connected among themselves one type simple recurrent relations (8), (9) which allow for each moment of time $n = 0, 1, 2, ...$ to find alternation of angular speed of the shaft $\omega_H[n]$ (12) and torsion moment $M_H[n]$ [4] of the initial point ($x = 0$) of the system of drilling elliptic driver with distributed parameters allowing for bit jamming losses that is indisputable advantage of the suggested numerical method.

Moreover, one-type property of algorithms (8) and (9) significantly facilitates the calculations by the suggested method in comparison with the method stated in [1].

Various practical calculations were carried out by the developed algorithms in systems of drilling electric drives with distributed parameters allowing for the bit jamming losses and it was detected that the suggested numerical method as the chosen step by the form of the curve and by the accuracy of the calculation gives well coincidence with analytic solution in comparison with difference method. Moreover, the number of iterations by the suggested numerical method is significantly less in comparison with difference method.

Thus, the suggested numerical method may be widely used for solving a wide class of problems of dynamics in systems of drilling electric drives with distributed parameters allowing for losses.

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