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BENDING OF RECTANGLE ANISOTROPIC PLATE WITH LOCAL CURVED STRUCTURES

Abstract

The bending of linear anisotropic plate made of laminary composite materials with local curved materials under normal load is studied in frames of continual theory. To solve the problem a small parameter method is used. Zero approximation is the solution of the problem for homogeneous limear orthotropic plate. Recurrent equations was obtained for corresponding approximation. The methods for finding the quantities of each approximation in an analytic form are stated.

It follows from the observation of the cut of different laminary composite materials that bending in the structure may be periodic and local [3].

In the papers [4.5] following the continual theory [1], a concrete problem was considered for the case when the bendings in the structure of laminary composites are periodic.

In the given paper, in the frames of continual theory [1] the bending of a rectangle anisotropic plate made of laminary composite materials with local curvatures in the structure under the action of normal load (whose y opposite sides are supported, and the order two sides are arbitrary fixed) is investigated.

We take the mean surface equation of the chosen curved layer in the following form [1]:

$$F(x_1, x_2) = \varepsilon f(x_1, x_2) \tag{1}$$

ε is a dimensionless small parameter $0 < \varepsilon < 1$, whose sense is determined for each concrete given function of the curvature form [2].

Let's assume that in the general case the plate is not orthotropic, but at each point it has a plane of elastic symmetry parallel to the mean surface [6].

Adopt the mean surface of undeformed plate for the plane x_1, x_2 arranging the origin in the middle of the supported side, and direct the axis x_1 along the supported side, but the axis x_2 perpendicular to it. Direct the axis x_3 to the side of unloaded external surface. By the made assumptions with respect to elastic properties the equations for a generalized Hook law with the cited elasticity modules in the form [1]

$$\begin{aligned} \sigma_{11} &= A_{11}\varepsilon_{11} + A_{12}\varepsilon_{22} + A_{16}\varepsilon_{12}, \\ \sigma_{22} &= A_{12}\varepsilon_{11} + A_{22}\varepsilon_{22} + A_{26}\varepsilon_{12}, \\ \sigma_{12} &= A_{16}\varepsilon_{11} + A_{26}\varepsilon_{22} + A_{66}\varepsilon_{12}, \end{aligned} \tag{2}$$

we'll assume valid.

Here

$$A_{sp}(x_1x_2)=\begin{cases} A_{sp_0}+\sum\limits_{k=1}^{\infty}\varepsilon^{2k}A_{sp_k} \text{ for } sp=11,12,22,66 \\ \sum\limits_{k=1}^{\infty}\varepsilon^{2k-1}A_{sp_k} \text{ for } sp=16,26 \end{cases}, \tag{3}$$

where A_{sp_0} is an elasticity modules of homogeneous linear anisotropic body, A_{sp_k} is determined also by A_{sp_0} and the parameters of the layer curvature [1], ε is a small parameter. Denote by h the thickness of the plate, by u_1, u_2 the permutations of any points in the direction of axis u_1 and u_2, and by $w(x_1, x_2)$ deflection of mean surface, by the form of the function w define the form of curved mean surface [7]. It follows from the conjecture of linear normals that

$$u_1=-x_3\frac{\partial w}{\partial x_1},\ u_2=-x_3\frac{\partial w}{\partial x_2}, \tag{4}$$

$$\varepsilon_{11}=-x_3\frac{\partial^2 w}{\partial x_1^2};\ \varepsilon=-x_3\frac{\partial^2 w}{\partial x_2^2};\ \varepsilon_{12}=--2x_3\frac{\partial^2 w}{\partial x_1\partial x_2}. \tag{5}$$

Represent the equilibrium equation in the form [8]:

$$\frac{\partial^2 M_{11}}{\partial x_1^2}+\frac{\partial^2 M_{22}}{\partial x_2^2}-2\frac{\partial^2 M_{12}}{\partial x_1\partial x_2}=-q, \tag{6}$$

q is a load distributed on the external surface per unit area. M_{11}; M_{22} is a bending moment, M_{12} is a torque:

$$M_{11}=\int\limits_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_{11}x_3dx_3;\ M_{22}=\int\limits_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_{22}x_3dx_3;\ M_{12}=\int\limits_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_{12}x_3dx_3. \tag{7}$$

It is assumed that the following conditions are fulfilled on the edges of the plate

$$w=0;\frac{\partial^2 w}{\partial x_2^2}=0 \text{ for } x_2=0;b,$$

$$w=0;\frac{\partial^2 w}{\partial x_1^2}=0 \text{ for } x_1=\pm\frac{a}{2}. \tag{8}$$

The problem on the elastic equilibrium of the plate bended by any forces is reduced to the definit.ion of the function $w(x_1, x_2)$ in the domain occupied by the plate.

This function satisfies the fourth order differential equation with variable coefficients. Therefore a small parameter method is used for solving the problem.

Represent all the quantities in the form of series by parameter ε:

$$M_{ij}=\sum_{k=0}^{\infty}\varepsilon^k M_{ij}^{(k)};\ \sigma_{ij}=\sum_{k=0}^{\infty}\varepsilon^k\sigma_{ij}^{(k)};.\varepsilon_{ij}=\sum_{k=0}^{\infty}\varepsilon^k\varepsilon_{ij}^{(k)};w==\sum_{k=0}^{\infty}\varepsilon^k w^{(k)}. \tag{9}$$

Allowing for (2), (3), (9) in (6) and performing grouping by the same powers of ε, for each approximation, we get:

I. Zero approximation.

a) Equilibrium equation

$$\frac{\partial^2 M_{11}^{(0)}}{\partial x_1^2}+\frac{\partial^2 M_{22}^{(0)}}{\partial x_2^2}-2\frac{\partial^2 M_{12}^{(0)}}{\partial x_1\partial x_2}=-q. \tag{10}$$

b) Hook's law

$$\begin{aligned}\sigma_{11}^{(0)}&=A_{110}\varepsilon_{11}^{(0)}+A_{120}\varepsilon_{22}^{(0)},\\ \sigma_{22}^{(0)}&=A_{120}\varepsilon_{11}^{(0)}+A_{220}\varepsilon_{22}^{(0)},\\ \sigma_{12}^{(0)}&=2A_{660}\varepsilon_{12}^{(0)}.\end{aligned} \tag{10*}$$

c) Boundary conditions

$$w^{(0)}=0;\ \frac{\partial^2 w^{(0)}}{\partial x_2^2}=0 \text{ for } x_2=0;b,$$

$$w^{(0)}=0;\ \frac{\partial^2 w^{(0)}}{\partial x_1^2}=0 \text{ for } x_1=\pm\frac{a}{2}. \tag{10**}$$

II. The first approximation.

a) Equilibrium equation

$$\frac{\partial^2 M_{11}^{(1)}}{\partial x_1^2}+\frac{\partial^2 M_{22}^{(1)}}{\partial x_2^2}-2\frac{\partial^2 M_{12}^{(1)}}{\partial x_1\partial x_2}=-q. \tag{11}$$

b) Hook's law

$$\begin{aligned}\sigma_{11}^{(1)}&=A_{110}\varepsilon_{11}^{(1)}+A_{120}\varepsilon_{22}^{(1)}+2A_{161}\varepsilon_{12}^{(0)},\\ \sigma_{22}^{(1)}&=A_{120}\varepsilon_{11}^{(1)}+A_{220}\varepsilon_{22}^{(1)}+2A_{161}\varepsilon_{12}^{(0)},\\ \sigma_{12}^{(1)}&=2A_{660}\varepsilon_{12}^{(1)}+A_{161}\varepsilon_{22}^{(0)}+A_{261}\varepsilon_{11}^{(0)}.\end{aligned} \tag{11*}$$

c) Boundary conditions

$$w^{(1)}=0;\ \frac{\partial^2 w^{(1)}}{\partial x_2^2}=0 \text{ for } x_2=0;b,$$

$$w^{(1)}=0;\ \frac{\partial^2 w^{(1)}}{\partial x_1^2}=0 \text{ for } x_1=\pm\frac{a}{2}. \tag{11**}$$

III. The second approximation

a) Equilibrium equation

$$\frac{\partial^2 M_{11}^{(2)}}{\partial x_1^2}+\frac{\partial^2 M_{22}^{(2)}}{\partial x_2^2}-2\frac{\partial^2 M_{12}^{(2)}}{\partial x_1\partial x_2}=0. \tag{12}$$

b) Hook's law

$$\sigma_{11}^{(2)} = A_{110}\varepsilon_{11}^{(2)} + A_{120}\varepsilon_{22}^{(2)} + 2A_{161}\varepsilon_{12}^{(1)} + A_{112}\varepsilon_{11}^{(0)} + A_{122}\varepsilon_{22}^{(0)},$$

$$\sigma_{22}^{(2)} = A_{120}\varepsilon_{11}^{(2)} + A_{220}\varepsilon_{22}^{(2)} + 2A_{261}\varepsilon_{12}^{(1)} + A_{212}\varepsilon_{11}^{(0)} + A_{22}\varepsilon_{22}^{(0)}, \qquad (12^*)$$

$$\sigma_{12}^{(2)} = 2A_{660}\varepsilon_{12}^{(2)} + A_{161}\varepsilon_{22}^{(1)} + A_{261}\varepsilon_{22}^{(1)} + A_{662}\varepsilon_{12}^{(0)}.$$

c) Boundary conditions

$$w^{(2)} = 0; \ \frac{\partial^2 w^{(2)}}{\partial x_2^2} = 0 \text{ for } x_2 = 0; b,$$

$$w^{(2)} = 0; \ \frac{\partial^2 w^{(2)}}{\partial x_1^2} = 0 \text{ for } x_1 = \pm\frac{a}{2}. \qquad (12^{**})$$

Now, using Cauchy relation and Hook's law we get corresponding equation for w in the form:

$$B_{110}\frac{\partial^4 w^{(k)}}{\partial x_1^4} + (2B_{120} + 4B_{660})\frac{\partial^4 w^{(k)}}{\partial x_1^2 \partial x_2^2} + B_{220}\frac{\partial^4 w^{(k)}}{\partial x_2^4} = D^{(k)}(x_1 x_2), \qquad (13)$$

where

$$D^k(x_1 x_2).$$

I. Zero approximation. For $k = 0$, $D^0 = q$.

II. The first approximation. For $k = 1$,

$$D^{(1)} = -2\left[\frac{\partial^2}{\partial x_1^2}\left(B_{161}\frac{\partial^2 w^{(0)}}{\partial x_1 \partial x_2}\right) + \frac{\partial^2}{\partial x_2^2}\left(B_{261}\frac{\partial^2 w^{(0)}}{\partial x_1 \partial x_2}\right) + \right.$$

$$\left. + \frac{\partial^2}{\partial x_1 \partial x_2}\left(B_{161}\frac{\partial^2 w^{(0)}}{\partial x_1^2} + B_{261}\frac{\partial^2 w^{(0)}}{\partial x_2^2}\right)\right]. \qquad (13^*)$$

III. The second approximation. For $k = 2$.

$$D^{(2)} = -\left\{\frac{\partial^2}{\partial x_1^2}\left[2B_{161}\frac{\partial^2 w^{(1)}}{\partial x_1 \partial x_2} + B_{112}\frac{\partial^2 w^{(0)}}{\partial x_1^2} + B_{122}\frac{\partial^2 w^{(0)}}{\partial x_2^2}\right] + \right.$$

$$+ \frac{\partial^2}{\partial x_2^2}\left[2B_{261}\frac{\partial^2 w^{(1)}}{\partial x_1 \partial x_2} + B_{212}\frac{\partial^2 w^{(0)}}{\partial x_1^2} + B_{220}\frac{\partial^2 w^{(0)}}{\partial x_2^2}\right] +$$

$$+2\frac{\partial^2}{\partial x_1 \partial x_2}\left[B_{161}\frac{\partial^2 w^{(1)}}{\partial x_1^2} + B_{261}\frac{\partial^2 w^{(1)}}{\partial x_2^2} + 2B_{662}\frac{\partial^2 w^{(0)}}{\partial x_1 \partial x_2}\right], \qquad (13^{**})$$

where B_{spk} are connected with A_{spk}; $B_{spk}(x_1 x) = A_{spk}(x_1, x_2)\dfrac{h^3}{12}$.

It should be noted that the zero approximation is the solution of the problem for a homogeneous linear orthotropic plate. Therefore, the influence of the curvature on the bending in the structure of the plate will be characterized by the quantities of the first, second and subsequent approximations.

Now state the methods for finding quantities of each approximation.

I.Zero approximation. Deflection equations will be of the form

$$B_{110}\frac{\partial^4 w^{(0)}}{\partial x_1^4}+(2B_{120}+4B_{660})\frac{\partial^4 w^{(0)}}{\partial x_1^2\partial x_2^2}+B_{220}\frac{\partial^4 w^{0}}{\partial x_2^4}=q, \tag{14}$$

where $q(x_2)$ is a given function. Here we'll consider the case when the load doesn't change along the supported sides. For the indicated plate we can get the solution in the form of prime series which is the generalization of the known Morris Levy solution for an orthotropic plate case [7].

We'll search solution (14) in the form of the sum

$$w^{(0)}=w_0(x_2)+w_1^{(0)}(x_1x_2). \tag{15}$$

$w_0(x_2)$ is a partial solution (14) and it is of the form:

$$w_0=\frac{b}{B_{220}\pi^4}\sum_{n=1}^{\infty}\frac{a_n}{n^4}\sin\frac{n\pi x_2}{b}, \tag{16}$$

where

$$a_n=\frac{2}{b}\int_0^b q\sin\frac{n\pi x_2}{b}dx_2 \tag{17}$$

are the coefficients of the expansion in Fourier series of the functions $q(x_2)$ representing the load distribution law.

$w_1^{(0)}$ solution of homogenous equation (14) we search in the form:

$$w_1^{(0)}=\sum_{n=1}^{\infty}X_n^{(0)}(x_1)\sin\frac{n\pi x_2}{b}. \tag{18}$$

Then, for the function $X_n(x_1)$ we get the equation

$$B_{110}X_n^{(0)^{IV}}(x_1)-2\left(B_{120}+2B_{660}\right)\left(\frac{n\pi}{b}\right)^2X_n^{(0)^{II}}(x_1)+B_{220}\left(\frac{n\pi}{b}\right)^4X_n^{(0)}(x_1)=0. \tag{19}$$

We look for the solution of homogenous equation in the form:

$$X_n^{(0)}(x_1)=\exp(\lambda_n\beta x_1);\ \lambda_n=\frac{n\pi}{b}.$$

Then, for determination of the quantity β we have the characteristic equation

$$B_{110}\beta^4-2\left(B_{120}+2B_{660}\right)\beta^2+B_{220}=0. \tag{20}$$

Hence

$$\beta_{1,\ldots,4} = \pm\sqrt{\left[(B_{120}+2B_{660}) \pm \sqrt{(B_{120}+2B_{660})^2 - 4B_{110}B_{220}}\right]/B_{110}}.$$

Thus, we get the following expression for total deflection

$$w^{(0)} = \sum_{n=1}^{\infty}\left(\frac{a_n b^4}{B_{220}\pi n^4} + C_{1n}^{(0)} e^{\frac{n\pi}{b}\beta_1^{(0)}x_1} + C_{2n}^{(0)} e^{\frac{n\pi}{b}\beta_1^{(0)}x_1} + \right.$$

$$\left. + C_{3n}^{(0)} e^{\frac{n\pi}{b}\beta_2^{(0)}x_1} + C_{4n} e^{-\frac{n\pi}{b}\beta_2^{(0)}x_1}\right)\sin\frac{n\pi x_2}{b}. \tag{21}$$

Substituting (21) into (10**) we get a system of in homogenous linear algebraic equations with respect to the constants $C_{1n}^{(0)} \ldots C_{4n}^{(0)}$. Having defined the unknown constants we find the quantity of the deflection in zero approximation. In sequel with regard to zero approximation quantity we define the quantities of subsequent approximations.

The first approximation.

Consider the case when the curvature in the structure of the considered plate is only in the plane $x_2 \bigcirc x_3$. Accept the functions characterizing the curvature form in the plate structure in the following form [2]:

$$F(x_2) = Le^{-\left(\frac{2x_2-b}{2l_2}\right)^2}. \tag{22}$$

Here L is a maximal value of the rise, l_2 is the introduced geometric which is shown in figure. Assume that $L < L_2$, as a small parameter ε we take L/l_2 i.e. $\varepsilon = L/l_2$, then we have from (1) and (22):

$$f(x_2) = l_2 e^{-\left(\frac{2x_2-b}{2l_0}\right)^2}. \tag{23}$$

Now, taking into account that $B_{161} = A(x_2)B'_{161}, B_{261} = A(x_2)B'_{261}$ (1) and (13*) in (13), for the deflection we obtain the following inhomogenous differential equation

$$B_{110}\frac{\partial^4 w^{(1)}}{\partial x_1^4} + 2\left(B_{120}+2B_{660}\right)\frac{\partial^4 w^{(1)}}{\partial x_1^2 \partial x_2^2} + B_{220}\frac{\partial^4 w^{(1)}}{\partial x_2^4} =$$

$$= -2\left[B'_{261}\frac{\partial^2 w^{(0)}}{\partial x_1 \partial x_2}\frac{\partial^2 A}{\partial x_2^2} + \left(B'_{161}\frac{\partial^3 w^{(0)}}{\partial x_1^3} + 3B'_{261}\frac{\partial^3 w^{(1)}}{\partial x_1 \partial x_2^2}\right)\frac{\partial A}{\partial x_2} + \right.$$

$$\left. +2\left(B'_{161}\frac{\partial^4 w^{(0)}}{\partial x_1^3 \partial x_2} + B'_{261}\frac{\partial^4 w^{(0)}}{\partial x_1 \partial x_2^3}\right)A\right]. \tag{24}$$

Here B'_{161}; B'_{261} are the unknown constants [1]:

$A(x_2) = \dfrac{\partial f}{\partial x_2}$ from (23) we have:

$$A(x_2) = -2\left(\frac{2x_2-b}{2l_2}\right)e^{-\left(\frac{2x_2-b}{2l_0}\right)^2}. \tag{25}$$

Putting (21), (25) into (24) we get the following inhomogenous differential equation for determining $w^{(1)}(x_1;x_2)$.

$$B_{110}\frac{\partial^4 w^{(1)}}{\partial x_1^4}+2\left(B_{120}+2B_{660}\right)\frac{\partial^4 w^{(1)}}{\partial x_1^2\partial x_2^2}+B_{220}\frac{\partial^4 w^{(1)}}{\partial x_2^2}=$$

$$=\sum_{n=1}^{\infty}e^{-\left(\frac{2x_2-b}{2l_0}\right)^2}\left\{\left[Y_{n_0}(x_1)+Y_{n_1}(x_1)\left(\frac{2x_2-b}{2l_2}\right)\right]\sin\frac{n\pi x_2}{b}+\left[Y_{n_2}(x_1)\left(\frac{2x_2-b}{2l_2}\right)+\right.\right.$$

$$\left.\left.+Y_{n_3}(x_1)\left(\frac{2x_2-b}{2l_2}\right)^3\right]\cos\frac{n\pi x_2}{b}\right\}. \quad (26)$$

Here

$$Y_{n_0}(x_1)=-\frac{12}{l_2}\left(\frac{n\pi}{b}\right)^2B'_{261}X_n^{(0)'}(x_1);\ Y_{n_1}(x_1)=-2Y_{n_0}(x_1),$$

$$Y_{n_2}(x_1)=8\left(\frac{n\pi}{b}\right)B_{161}X_n^{(0)'''}(x_1)-8\left[\frac{3}{l_2^2}\frac{n\pi}{b}+\left(\frac{n\pi}{b}\right)^3\right]B'_{261}X_n^{(0)'}(x_1),$$

$$Y_{n_3}(x_1)=16\frac{1}{l_2^2}\frac{n\pi}{b}B'_{261}X_n^{(0)'}(x_1).$$

We choose the solution of equation (2) as:

$$w^{(1)}(x_1x_2)=\sum_{n=1}^{\infty}X_n^{(1)}(x_1)e^{-\left(\frac{2x_2-b}{2l_2}\right)^2}\sin\frac{n\pi x_2}{b}. \quad (27)$$

Allowing for (27) in (26) and partitioning each side to $\exp\left[-\left(\frac{2x_2-b}{2l_2}\right)^2\right]$ we get:

$$B_{0_n}(x_2)X_n^{(1)^{IV}}(x_1)+B_{1_n}(x_2)X_n^{(1)''}(x_1)+B_{2n}(x_2)X_n^{(1)}(x_1)=Y_n^{(1)}(x_1x_2), \quad (28)$$

where

$$B_{0_n}(x_2)=B_{110}\sin\frac{n\pi x_2}{b},$$

$$B_{1n}(x_2)=$$

$$=2\left(B_{120}+2B_{660}\right)\left\{\left[k_{o_n}+k_{1n}\left(\frac{2x_2-b}{2l_2}\right)^2\right]\sin\frac{n\pi x_2}{b}+k_{2n}\frac{2x_2-b}{2l_2}\cos\frac{n\pi x_2}{b}\right\},$$

$$B_{2n}(x_2)=B_{220}\left\{\left[k_{3n}+k_{4n}\left(\frac{2x_2-b}{2l_2}\right)^2+k_5\left(\frac{2x_2-b}{2l_2}\right)^4\right]\sin\frac{n\pi x_2}{b}+\right.$$

$$\left.+\left[k_{6n}\frac{2x_2-b}{2l_2}+k_{7n}\left(\frac{2x_2-b}{2l_2}\right)^3\right]\cos\frac{n\pi x_2}{b}\right\},$$

$$Y_n^{(1)}(x_1x_2)=\left[Y_{n0}(x_1)+Y_{n1}(x_1)\frac{2x_2-b}{2l_2}\right]\sin\frac{n\pi x_2}{b}+$$

$$+\left[Y_{n_2}(x_1)\frac{2x_2-b}{2l_2}+Y_{n_3}(x_1)\left(\frac{2x_2-b}{2l_2}\right)^3\right]\cos\frac{n\pi x_2}{b},$$

$$k_{on}=-\left(\frac{n\pi}{b}+\frac{2}{l_2^2}\right):k_{1n}=4/l_2^2,\ k_{2n}=-4\left(\frac{n\pi}{b}\right)\frac{1}{l_2},$$

$$k_{3n}=\left(\frac{n\pi}{b}\right)^4-12\frac{1}{l_2^2}\left(\frac{n\pi}{b}\right)^2+12/l_2^4;\ k_{4n}=-24\frac{1}{l_2^2}\left[\left(\frac{n\pi}{b}\right)^2-\frac{1}{l_2^2}\right],$$

$$k_{5n}=16/l_2^4;\ k_{6n}=8\frac{1}{l_2}\left[\left(\frac{n\pi}{b}\right)^3-\frac{1}{l_2^2}\right];\ k_{7n}=32/l_2^3.$$

In sequel, applying the Bubnov-Galerkin method having excluding the variable x_2, with respect to the variable x_1 we get the following ordinary differential equation:

$$D_{11}X_n^{(1)^{IV}}(x_1)+D_{12}X_n^{(1)''}(x_1)+D_{13}X_n^{(1)}(x_1)=Y_n^{(1)}(x_1). \tag{29}$$

Here

$$D_{11}=\int_0^b B_{0n}(x_2)\sin\frac{n\pi x_2}{b}dx_2;\ D_{12}=\int_0^b B_{1n}(x_2)\sin\frac{n\pi x_2}{b}dx_2,$$

$$D_{13}=\int_0^b B_{2n}(x_2)\sin\frac{n\pi x_2}{b}dx_2;\ Y_n^{(1)}(x_1)\int_0^b Y_{nn}(x_1x_2)\sin\frac{n\pi x_2}{b}dx_2=$$

$$=b_{1n}e^{\frac{n\pi}{b}\beta_1^{(0)}x_1}+b_{2n}e^{-\frac{n\pi}{b}\beta_1^{(0)}x_1}+b_{3n}e^{\frac{n\pi}{b}\beta_2^{(0)}x_1}+b_{4n}e^{-\frac{n\pi}{b}\beta_2^{(0)}x_1},$$

where $b_{1n};\ b_{2n};\ b_{3n};\ b_{4n}$ are the known constants.

The homogeneous part of equation (24) is solved as equation (19). Assuming

$$X_n^{(1)}(x_1)=\exp\left(\beta_i^{(1)}x_1\right)$$

for determining the quantity $\beta^{(1)}$ we have the characteristic equation

$$D_{11}\beta^{(1)^4}+D_{12}\beta^{(1)^2}+D_{13}=0. \tag{30}$$

Then, fundamental solution of homogeneous equation will be

$$X_{n_{gen}}^{(1)}(x_1)=C_{1n}^{(1)}e^{\beta_1^{(1)}x_1}+C_{2n}^{(1)}e^{-\beta_1^{(1)}x_1}+C_{3n}e^{\beta_2^{(1)}x_1}+C_{4n}e^{-\beta_2^{(1)}x_1}. \tag{31}$$

Particular solution (29) is determined by simple algebraic methods, since the right hand side has a special form [9].

Using the operator writing

$$P_4(D)X_n^{(1)}(x_1)=Y_n^{(1)}(x_1)\text{ or}$$

$$\left(D_{11}D^4+D_{12}D^2+D_{13}\right)X_n^{(1)}(x_1)=Y^{(1)}(x_1).$$

D is a differentiation operator.

$$DX_n^{(1)} = \frac{dX_n^{(1)}(x_1)}{dx_1}$$

and knowing that

$$P_4\left(\frac{n\pi}{b}\beta_1^{(0)}\right); P_4\left(-\frac{n\pi}{b}\beta_1^{(0)}\right);\ P_4\left(\frac{n\pi}{b}\beta_2^{(0)}\right);\ P_4\left(-\frac{n\pi}{b}\beta_2^{(0)}\right) \neq 0.$$

Since $\frac{n\pi}{b}\beta_{1,2}^{(0)} \neq \beta_{1,2}^{(1)}$ i.e. $\frac{n\pi}{b}\beta_{1,2}^{(0)}$ is not a root of characteristic equation (30). Moreover, particular solution (29) is

$$X_{n_{priv}}^{(1)}(x_1) = \frac{b_{1n}e^{\frac{n\pi}{b}\beta_1^{(0)}x_1}}{P_4(\lambda\beta_1^{(0)})} + \frac{b_{2n}e^{-\frac{n\pi}{b}\beta_1^{(0)}x_1}}{P_4(-\lambda\beta_1^{(0)})} + \frac{b_{3n}e^{\frac{n\pi}{b}\beta_2^{(0)}x_1}}{P_4(\lambda\beta_1^{(0)})} + \frac{b_{4n}e^{-\frac{n\pi}{b}\beta_2^{(0)}x_1}}{P_4(-\lambda\beta_2^{(0)})}. \quad (32)$$

Then, a general solution of in homogeneous equation (26) will be

$$w^{(1)}(x_1, x_2) = \sum_{n=1}^{\infty}\left(X_{n_g}^{(1)}(x_1) + X_{n_p}^{(1)}(x_1)\right) e^{-\left(\frac{2x_2 - b}{2l_0}\right)} \sin\frac{n\pi x_2}{b}. \quad (33)$$

Satisfying the boundary conditions (11**) with regard to (33) we get a system of four inhomogeneous linear algebraic equations for determining the unknown constants $C_{1n}^{(1)}...C_{4n}^{(1)}$ contained in the expression $w^{(1)}(x_1x_2)$. So, we completely define the quantity of the first approximation. Continuing the stated procedure with regard to (12*) in (12) we can define the quantities of the second approximation and etc.

Thus, in the frames of continual theory the methods for studying the bending of inhomogeneous anisotropic plate with local curved layers are developed.

The methods for finding the quantities of each approximation are stated in an analytical form.

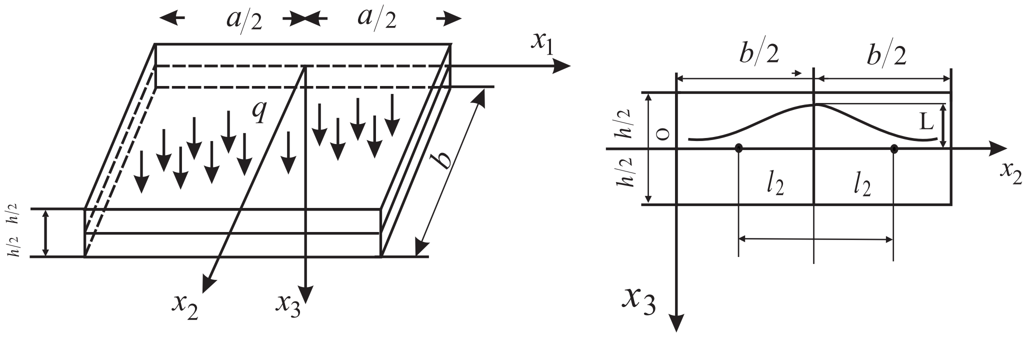

Fig.1.

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