MECHANICS

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AXIALLYSYMMETRIC PROBLEM ON A CYLINDRICAL GRYPHON

Abstract

The process of development of axiallysymmetric gryphons, moving along a casing in rock around a drilling well is considered. The field of a pressure at the neighbourhood of gryphon and coefficient of intensity of a singularity of pressure at the front a gryphon, criteria of growth of a gryphon are defined. The solution of a problem is necessary for the scientifically-grounded estimation of hazard of gas coning with a high formation pressure behind a casing.

By drilling-out collectors with a high formation pressure of gas a well sometimes temporarily is conserved to avoid gas blowout. However, often, despite of adopted precautions, gas breaks near to edge of a casing and washes out rock.

Here, the formed rock caverns are called "gryphons". This dangerous phenomenon sometimes precede to disastrous gas blowouts. It, obviously, relates to the category of hydro-erosive phenomena.

Problem statement. Let the homogeneous porous body take up the exterior of the cylinder, $r > a$, where a is radius of the cylinder in a cylindrical co-ordinates rx (fig.1.a).

At $x < 0$ surface of the cylinder is impermeable the remaining part of the surface $x > 0 \;\; r = a$ is the boundary of domain of the constant pressure P_0 of the cavern or "gryphons". At infinity as $r \to \infty$ the pressure is equal to $P_\infty \;\; (P_\infty >> P_r)$.

Such problem statement is obtained as a result of the following assumptions: a) "gryphon" is axiallysymmetric; b) the length of "gryphon" along to axis x is greater in comparison with its width in a radial direction; c) the front of "gryphon" moves along the axis with a speed, small in comparison with $V\infty$, so that we can consider the process of filtration as a quasistationary in coordinates, moving together with front $x = 0, \; r = a$; d) the inclination of a surface of "gryphon" to an axis x is small, and width of a gryphon in a radial direction is small in comparison with radius of the cylinder, so that boundary conditions from a surface of "gryphon" may be carried on a surface of the cylinder; e) the pressure gradient of a concavity of "gryphon" is small in comparison with a pressure gradient in a surrounding body; f) the length of "gryphon" is great in comparison with the radius of the cylinder. In this approach "gryphon" is represented by a cylindrical semi-infinite slit of zero along $r = a$, $x > 0$.

Let's consider a closed surface $\sum$, formed by a sphere $\sum_R$ of a large radius $R >> a$ by surface of the cylinder $\sum_c$ $(r = a)$ and by surface of a torus $\sum_t$, formed by rotation of a circle of small radius $r_t << a$ around the axis x at a distance a from the axis and from an origin of coordinates. The surface $\sum$ envelopes all porous body, except for front of "gryphon" and a point at infinity.

According to the theory of invariant Γ-integrals [6] the equality holds:

$$\frac{\rho}{2\varepsilon^2}\int\limits_{\Sigma}\left(v_i v_i n_1 - 2v_i n_i v_1\right)d\Sigma = 0$$
$$\left(x_1 = x,\ x_2^2 + x_3^2 = r^2,\quad i = 1,\ 2,\ 3\right) \tag{1.1}$$

Here n_i is a component of a unit vector of an exterior normal line to a surface Σ; v_i are components of a filtration speed; ε is a porosity, ρ is a density.

According to [2] integral (1.1) by surface Σ is equal to

$$\int\limits_{\Sigma_t} = 2\pi\alpha\Gamma \tag{1.2}$$

Here Γ is a density of configurational force of a filtration stream at the front of gryphon.

Integral (1.1) by a surface Σ_c is equal to zero, as on a surface of cylinder $n_1 = 0$ and besides equalities hold at $x < 0 \quad r = a \quad v_i n_i = 0$, at $x > 0 \quad r = a \quad v_1 = 0$ (since $v_i\tilde{\ }P,\ i$, and pressure p is constant at $x > 0 \quad r = a$).

Suppose, that the amount of gas, immersed by "gryphon" per a time unit, is finite, and is equal to Q, and speed of a motion of the "gryphon" is equal to v. In this case integral (1.1) by closed surface Σ_r at infinity will be equal to $\rho v Q \varepsilon^{-2}$.

Hence from (1.1) and (1.2), the quantity Γ is equal to

$$\Gamma = \frac{\rho v Q}{2\pi a \varepsilon^2}$$

This expression can be used for solving the question on development of "gryphon" in time if we use the experimental dependence $\frac{dl}{dt} = f(\Gamma)$ for $v = \frac{dl}{dt}$.

It is supposed, that $\frac{dv}{dt} << \frac{v}{\tau}$, where τ is the reference time of process.

The pressure in a porous body in the neighbourhood of a head of a gryphon is defined from the following boundary problem:

$$\Delta p^{\gamma} = 0 \qquad \gamma \geq 1 \qquad \left(\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial x^2}\right) \tag{1.3}$$

where constant γ is greater for a unit than the index of the gas polytrope

$$v_r = k\frac{\partial p}{\partial r}, \qquad v_x = k\frac{\partial p}{\partial x} \tag{1.4}$$

$$at \qquad r = 1 \qquad x > 0 \qquad p = \rho_0 \tag{1.5}$$

$$at \qquad r = 1 \qquad x < 0 \qquad \frac{\partial \rho}{\partial r} = 0 \tag{1.6}$$

$$at \qquad r \to \infty \qquad P = P_{\infty} \tag{1.7}$$

Here as a unit of length the radius of cylinder a is taken.

Let's differentiate (1.5) by x and denote $u = p^{\gamma}$. For function $u(r, x)$ we'll obtain the following linear boundary value problem.

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial x^2} = 0 \qquad (r \geq 1) \tag{1.8}$$

$$at \qquad r = 1 \qquad x > 0 \qquad \frac{\partial u}{\partial x} = 0 \tag{1.9}$$

$$at \qquad r = 1 \qquad x < 0 \qquad \frac{\partial u}{\partial r} = 0 \tag{1.10}$$

$$at \qquad r \to \infty \qquad u = P_{\infty}^{\gamma} \tag{1.11}$$

$$at \qquad x^2 + (r-1)^2 = \varepsilon^2 \to 0 \qquad u = P_0^{\gamma} + o\left(\varepsilon^{1/2}\right) \qquad (v \geq 1) \tag{1.12}$$

As a result, for a new function

$$\omega(r, x) = u - P_{\infty}^{\gamma} = P^{\gamma} - P_{\infty}^{\gamma}$$

We'll obtain the following homogeneous boundary problem

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \omega}{\partial r}\right) + \frac{\partial^2 \omega}{\partial x^2} = 0 \qquad (r \geq 1) \tag{1.13}$$

$$at \qquad r = 1 \qquad x > 0 \qquad \frac{\partial \omega}{\partial x} = 0 \tag{1.14}$$

$$at \qquad r = 1 \qquad x < 0 \qquad \frac{\partial \omega}{\partial r} = 0 \tag{1.15}$$

$$at \qquad r \to \infty \qquad \omega = 0 \tag{1.16}$$

$$at \qquad x^2 + (r-1)^2 = \varepsilon^2 \to 0 \qquad \omega = P_0^{\gamma} - P_{\infty}^{\gamma} + o\left(\varepsilon^{1/2}\right) \tag{1.17}$$

Let's take Fourier transformation by x to (1.13), denoting

$$\bar{\omega}(r, \lambda) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \omega(r, x)\, e^{-i\lambda x} dx \tag{1.18}$$

(λ is an arbitrary parameter).

We'll obtain

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\bar{\omega}}{dr}\right) - \lambda^2 \bar{\omega} = 0 \qquad (r \geq 1) \tag{1.19}$$

This is a modified Bessel equation [4]. The solution of this equation, according to (1.16) tending to zero as $r \to \infty$, has the form [4]

$$\begin{gathered} \bar{\omega}(r, \lambda) = A(\lambda) K_0(\lambda r) \\ \frac{d\bar{\omega}}{dr} = \bar{\omega}'(r, \lambda) = \lambda A(\lambda) K_0'(\lambda r) = -A(\lambda) K_1(\lambda r) \end{gathered} \tag{1.20}$$

Here $K_0(\lambda r)$ is a modified Hankel function (or McDonald function) of zero order; $A(\lambda)$ is an arbitrary function; $K_1(\lambda r)$ is a first order McDonald function: $K_1(\lambda) = -K_0'(\lambda)$

According to boundary condition (1.14) on the base of (1.18) and (1.20) we have

$$i\lambda\bar{\omega}\left(1,\lambda\right)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{0}\left.\frac{\partial\bar{\omega}}{\partial x}\right|_{r=1}e^{-i\lambda x}dx=i\lambda A\left(\lambda\right)K_0\left(\lambda\right)=\Phi^{+}\left(\lambda\right)\qquad(1.21)$$

Here $\Phi^{+}(\lambda)$ is an analytical function of the complex variable λ in a upper half-plane $\operatorname{Im}\lambda>0$ (fig.1b). Formula (1.21) is based on the following transformation

$$\int\limits_{-\infty}^{+\infty}\frac{\partial\bar{\omega}}{\partial x}e^{-i\lambda x}dx=\left.\bar{\omega}e^{-i\lambda x}\right|_{-\infty}^{+\infty}+i\lambda\int\limits_{-\infty}^{+\infty}\bar{\omega}e^{-i\lambda x}dx\qquad(r\geq 1)$$

To equate to zero the first addend in the right-hand side of this equation, it is necessary to consider, that $\lambda=\operatorname{Re}\lambda-io$ in formula (1.18) and that $\lim\limits_{x\to\infty}\left(\bar{\omega}e^{-i\lambda x}\right)=0$ $(r=1)$. According to boundary condition (1.15) on the base of (1.18) and (1.20) we have

$$\bar{\omega}'\left(1,\lambda\right)=\frac{1}{\sqrt{2\pi}}\int\limits_{0}^{\infty}\left.\frac{\partial\bar{\omega}}{\partial r}\right|_{r=1}e^{-i\lambda x}dx=\lambda A\left(\lambda\right)K_0\left(\lambda\right)=\Phi^{-}\left(\lambda\right)\qquad(1.22)$$

Here $\Phi^{-}(\lambda)$ is analytical function λ in the lower half-plane $Jm\lambda<0$ (fig.1b). We'll exclude from two correlations (1.21) and (1.22) the function $A(\lambda)$ and we'll find

$$\Phi^{+}\left(\lambda\right)=-iG\left(\lambda\right)\Phi^{-}\left(\lambda\right)\qquad(1.23)$$

where

$$G\left(\lambda\right)=-\frac{K_0\left(\lambda\right)}{K_0'\left(\lambda\right)}=\frac{K_0\left(\lambda\right)}{K_1\left(\lambda\right)}\,.$$

This is a homogeneous functional Wiener-Hopf equation connecting the limiting values of the piecewise - analytic function in opposite points of its break line-real axis $Jm\lambda=0$.

Points $x=0$ and $\lambda=\infty$ are unique singular points of this equation. Let's study them. First of all let's note the following properties

a) the function $K_0(\lambda)$ is an even function λ, monotonically decreasing with growth λ at $\lambda>0$;

b) function $K_0(\lambda)$ has no zeros

c) at singular points the function $K_0(\lambda)$ behaves so:

At $\lambda\to+0$ $\quad K_0\left(\lambda\right)=-\ln\lambda\left[1+0\left(\lambda^2\right)\right]$

At $\lambda\to+\infty$

$$K_0\left(\lambda\right)=\sqrt{\frac{\pi}{2\lambda}}e^{-\lambda}\left[1+0\left(\frac{1}{\lambda}\right)\right]\qquad(1.24)$$

On base of (1.23) and (1.24) the function $G(\lambda)$ on the real axis is a real odd function λ, which at singular points behaves so (fig.1c):

At

$$\lambda\to+0\qquad G\left(\lambda\right)=-\lambda\ln\lambda\left[1+0\left(\lambda^2\ln\lambda\right)\right]\qquad(1.25)$$

At $\lambda \to +\infty \qquad G(\lambda) = 1 + 0\left(\frac{1}{\lambda}\right).$

It is visible, that at $\lambda > 0$ function $G(\lambda)$ monotonically increases from zero up to unity with growth of λ.

The function $G(\lambda)$ is a value of a main branch of multivalued analytic function on a complex plane λ with a slit along the ray $(0, \infty)$ of a real axis, taken on the upper coast of a slit and on prolongation of a slit. The indicated analytic function has the infinite number of zeros and poles, all of them are located on imaginary axis symmetrically with respect to the origin; zeros and poles alternate.

For definition of asymptotics of function $\Phi^+(\lambda)$ and $\Phi^-(\lambda)$ at $\lambda \to +0$ and $\lambda \to \infty$ we use the following Abelian theorem, connecting this asymptotics with the asymptotics of the corresponding integrand functions at $x \to \infty$ and $x \to 0$ respectively

Let

$$F^+(\lambda) = \frac{1}{\sqrt{2\pi}} \int_0^\infty f(x)\, e^{i\lambda x} dx\ , \qquad f(x) \approx Ax^n \tag{1.26}$$

$$(0 > \eta > -1, \quad x \to +0, \quad x \to \infty)$$

Then

$$F^+(\lambda) \approx \frac{A}{\sqrt{2\pi}} \Gamma(\eta+1)\, \lambda^{-(\eta+1)} \exp \frac{\pi i (\eta+1)}{2} \qquad (\lambda \to \infty, \quad \lambda \to +0)$$

Where everywhere we have to take either upper, or lower limit passages, $\Gamma(\eta)$ is gamma function of Euler. Here it is considered, that λ tends both to zero, and to the infinity, remaining in upper half-plane $Jm\lambda > 0$ (A is some constant).

An asymptotics of desired functions at the front of "gryphon" according to [2]

$$f(z) = -iK\sqrt{z-l}$$

$$v_x + iv_y = \frac{kK}{2\sqrt{r_0}} \left(-\sin\frac{\theta_0}{2} + i\cos\frac{\theta_0}{2}\right) \qquad r_0 = |z-l|,\ \theta = \arg(z-l)\,, \quad K > 0$$

will be the following

$$at \quad r = 1 \qquad x \to +0 \qquad \frac{\partial \omega}{\partial r} = \frac{1}{2} K_* x^{-1/2} \tag{1.27}$$

$$at \quad r = 1 \qquad x \to -0 \qquad \frac{\partial \omega}{\partial x} = -\frac{1}{2} K_* (-x)^{-1/2} \tag{1.28}$$

$$K_* = \gamma P_0^{\gamma-1} K$$

Let's transform integral (1.21) to form (1.26) by substitution $x_1 = -x$; use the Abelian type theorem and formula (1.28) we'll obtain

$$at \quad \lambda \to \infty \quad \Phi^+(\lambda) = \frac{1}{4} K_* (1+i)\, \lambda^{-1/2}, \quad as \quad \Gamma(1/2) = \sqrt{\pi} \tag{1.29}$$

Integral in (1.22) is reduced to the form (1.26) by substitution $\lambda_1 = -\lambda$. With the help of Abelian type theorem and formula (1.27) we'll obtain at

$$at \quad x \to \infty \quad \Phi^-(x) = \frac{1}{4} K_* (1+i)(-\lambda)^{-1/2} \tag{1.30}$$

The asymptotics of desired functions $r^2 + x^2 \to \infty$ is required to define in the process of solution.

We use the following fact: Function $\pi\lambda cth\pi\lambda$ on the real axis can be represented (factorized) by the following way: [6]

$$\lambda \quad cth \quad \pi\lambda = K^+(\lambda) K^-(\lambda)$$

$$K^+(\lambda) = \frac{\Gamma(1-i\lambda)}{\Gamma(1/2-i\lambda)}, \quad K^-(\lambda) = K^+(\lambda) \tag{1.31}$$

Here $K^+(\lambda)$ is an analytical and absencing zeros function at $Jm\lambda > -1/2$ (respectively, $K^-(\lambda)$ is analytical an has no zeros at $Jm\lambda < 1/2$).

On infinity these functions behave so: [6]

$$at \quad \lambda \to \infty \qquad K^+(\lambda) = e^{-\frac{i\pi}{4}} \sqrt{\lambda} \left[1 + 0\left(\frac{1}{\lambda}\right)\right] \tag{1.32}$$

$$at \quad \lambda \to \infty \qquad K^-(\lambda) = -e^{\frac{i\pi}{4}} \sqrt{-\lambda} \left[1 + 0\left(\frac{1}{\lambda}\right)\right]$$

(the sign minus in the last correlation is taken subject to modification of a slit for functions $\sqrt{\lambda}$ and $\sqrt{-\lambda}$).

Let's transform the coefficient $G(\lambda)$ in functional Wiener-Holf equations by the following way

$$G(\lambda) = G_0(\lambda) \frac{\lambda cth\pi\lambda}{\lambda} \tag{1.33}$$

where $C_0(\lambda) = \dfrac{G(\lambda)}{cth\pi\lambda}$.

Function $C_0(\lambda)$ on the real axis is a real, non-negative, even function, which is as $\lambda \to -\infty$ tends to unit, but as $\lambda \to 0$ behaves itself as a $-\lambda^2 \ln \lambda$. At $\lambda > 0$ this function monotonically increases from zero up to unit by increasing λ. Index of this function is equal to zero and it can be factorized by the following way

$$G_0(\lambda) = \frac{\chi^+(\lambda)}{\chi^-(\lambda)} \tag{1.34}$$

Here $\chi^+(\lambda)$ and $\chi^-(\lambda)$ are analytical and not vanishing functions in upper and lower half-planes, respectively.

Taking the logarithm (1.34) we find

$$\ln \chi^+(\lambda) - \ln \chi^-(\lambda) = \ln G_0(\lambda) \tag{1.35}$$

Hence

$$\ln \chi^+(\lambda) = \frac{1}{2\pi i} \int\limits_{-\infty}^{+\infty} \frac{\ln G_0(\lambda)}{\lambda_0 - \lambda} d\lambda_0 \tag{1.36}$$

Consequently, recalling (1.23) and (1.33) we have

$$\exp\left\{\frac{1}{2\pi i}\int\limits_{-\infty}^{+\infty}\ln\left[\frac{K_0(\lambda_0)}{K_1(\lambda_0)\,cth\pi\lambda_0}\right]\frac{d\lambda_0}{\lambda_0-\lambda}\right.=\left\{\begin{array}{lll}\chi^+(\lambda) & at & Jm\lambda>0\\ \chi^-(\lambda) & at & Jm\lambda<0\end{array}\right. \qquad (1.37)$$

(at $\lambda\to\infty \quad \chi^{\pm}(\lambda)\to 1$).

Gathering formulas (1.23), (1.31), (1.33), (1.34) we obtain the Wiener-Holf equation

$$\frac{\lambda\Phi^+(\lambda)}{K^+(\lambda)\,\chi^+(\lambda)}=-\frac{i\Phi^-(\lambda)\,K^-(\lambda)}{\chi^-(\lambda)} \qquad (Jm\lambda=0) \qquad (1.38)$$

The left-hand-side of this equation is a function, analytical in the upper half-plane $Jm\lambda>0$, but the right hand-side is a function, analytical in the lower half-plane $Jm\lambda<0$. With the help of formulas (1.29), (1.30), (1.32), (1.37) it is easy to establish that these functions behave itself at infinity so:

$$\begin{array}{llll} at & \lambda\to\infty & Jm\lambda>0 & \dfrac{\lambda\Phi^+(\lambda)}{K^-(\lambda)\,\chi^+(\lambda)}=\dfrac{1}{2\sqrt{2}}K_*i\\ at & \lambda\to\infty & Jm\lambda<0 & \dfrac{-i\Phi^-(\lambda)\,K^-(\lambda)}{\chi^-(\lambda)}=\dfrac{1}{2\sqrt{2}}K_*i \end{array} \qquad (1.39)$$

In the base of continuous continuation of the theory of analytical functions of a complex variable the left and right hand-sides of equality (1.38) is single analytical function on all the plane λ, on the base of (1.39) and Liouville theorem it is equal to the constant $iK_*2^{-3/2}$.

Hence we obtain

$$\Phi^+(\lambda)=\frac{K_*i}{2\sqrt{2}\lambda}K^+(\lambda)\,K^-(\lambda) \qquad (1.40)$$

$$\Phi^-(\lambda)=-\frac{K_*}{2\sqrt{2}\lambda}\frac{\chi^-(\lambda)}{K^-(\lambda)} \qquad (1.41)$$

With the help of (1.21), (1.40) we find $A(\lambda)$, then using (1.20) we define $\omega(r,\lambda)$ and by formula of the inverse Fourier transformation $\omega(r,x)$ finally we'll obtain the following expression for the desired field of pressure in the porous body:

$$P^\gamma=P_\infty^\gamma+\frac{K_*}{4\sqrt{\pi}}\int\limits_{-\infty}^{+\infty}\frac{K^+(\lambda)\,\chi^+(\lambda)}{\lambda^2K_0(\lambda)}K_0(\lambda r)\,e^{i\lambda x}dx \qquad (1.42)$$

Let's recall that the functions $K^+(\lambda)$ and $\chi^+(\lambda)$ are defined by expressions (1.31), (1.37).

For definition of coefficient of intensity K_* at the front of "gryphon" we use condition (1.17) according to which $P=P_0$ at $r=1 \quad x=0$ (pressure in a concavity of "gryphon"). Hence with the help of (1.42) we obtain

$$K_*=-4\sqrt{\pi}\,(P_\infty^\gamma-P_0^\gamma)\left[\int\limits_{-\infty}^{+\infty}\lambda^{-2}K^+(\lambda)\,\chi^+(\lambda)\,d\lambda\right]^{-1} \qquad (1.43)$$

Let's study a singular point $\lambda = 0$. According to (1.24) we have at real λ

$$At \quad \lambda \to \pm 0 \quad \frac{K_0(\lambda)}{K_1(\lambda)\, cth\pi\lambda} = -\pi\lambda^2 \ln|\lambda| \; [1 + 0(\lambda)] \tag{1.44}$$

Substituting (1.44) in (1.37) we arrive at the following common question: what is the behaviour of Cauchy type integral

$$\begin{gathered} F_1(z) = \frac{1}{2\pi i}\int_{-1}^{1} \frac{\ln|x|}{x-z}dx \\ F_2(z) = \frac{1}{2\pi i}\int_{-1}^{1} \frac{\ln(-\ln|x|)}{x-z}dx \end{gathered} \tag{1.45}$$

at $z > 0$ in the upper half-plane?

Let's consider the following auxiliary function

$$\omega(z) = -\frac{1}{4\pi i}\ln^2 z + \frac{1}{2}\ln z - \frac{1}{4\pi i}\ln^2(-z) + \frac{1}{2}\ln(-z) \tag{1.46}$$

Here under the functions $\ln\ z\ and\ \ln(-z)$ we perceive the unique branches of the logarithm on a plane z with semi-infinite linear slits along $(0,\infty)$ and $(-\infty, 0)$ of a real axis, respectively, at that these branches take real values on the upper coasts of the corresponding slits. The function $\omega(z)$ represents the unique piecewise-analytic function in a plane z with a cutting line of a real axis: with direct calculations it is possible to show, that the saltus of this function at passage through a real axis is equal to

$$\omega^+(x) - \omega^-(x) = \ln|x| \tag{1.47}$$

According to the Sohotskii formula from (1.45) we have [3]

$$F_1^+(x) - F_1^-(x) = \ln|x| \tag{1.48}$$

Subtracting from (1.46) the expression we get

$$[F_1(x) - \omega(x)]^+ = [F_1(x) - \omega(x)]^- \tag{1.49}$$

So, the function $F_1(z) - \omega(z)$ by Liouville theorem is unique analytical at the same neighbourhood of the origin; analogously using inequality $|\ln|z|| < |z|^\alpha$

$(\alpha < 1)$, we can show, that $|F_1 - \omega| < 0(|z|^\alpha)$, i.e. poles of function $F_1(z) - \omega(z)$ at the point $z = 0$ are excluded.

Hence, it implies

$$F_1(z) = \omega(z) + 0(1) \qquad at \quad z \to 0 \tag{1.50}$$

The behaviour of integral $F_2(z)$ at $z \to 0$ is more difficult, however for our purposes it is enough the following estimation

$$F_2(z) = 0(\omega(z)) \qquad at \quad z \to 0 \tag{1.51}$$

Here $0\left(\omega\left(z\right)\right)-$ is a quantity, infinitesmall with respect to $\omega\left(z\right)$ at small z.

Really, this estimation is on the base of the fact, that $\ln\left(-\ln\left|x\right|\right) << -\ln x$ at $x\rightarrow 0$.

On the base of (1.46), (1.49), (1.51) by formula (1.37) we find

$$z\rightarrow 0_{\chi}^{+}\left(\lambda\right)=\lambda^{2}\ln\left(\lambda\right)\left[1+0\left(\lambda\right)\right] \tag{1.52}$$

Calculation on a computer of integral (1.43) has given the following result

$$\eta=\left[\int\limits_{-\infty}^{+\infty}\lambda^{-2}K^{+}\left(\lambda\right)\chi^{+}\left(\lambda\right)d\lambda\right]^{-1}=2,3604$$

The resultant expression for the coefficient K according to (1.28) and (1.43) takes the following simple form (in dimensional variables).

$$K=\eta 4\sqrt{\pi}\frac{P_{0}}{\pi\sqrt{a}}\left[\left(\frac{P_{\infty}}{P_{0}}\right)^{\gamma}-1\right] \tag{1.53}$$

On the base of the reasons explained in [2], "gryphon" doesn't develop, if the inequality

$$\frac{P_{0}}{\gamma\sqrt{a}}\left[\left(\frac{P_{\infty}}{P_{0}}\right)^{\gamma}-1\right]<\frac{K_{c}}{4\eta\sqrt{\pi}} \tag{1.54}$$

is fulfilled.

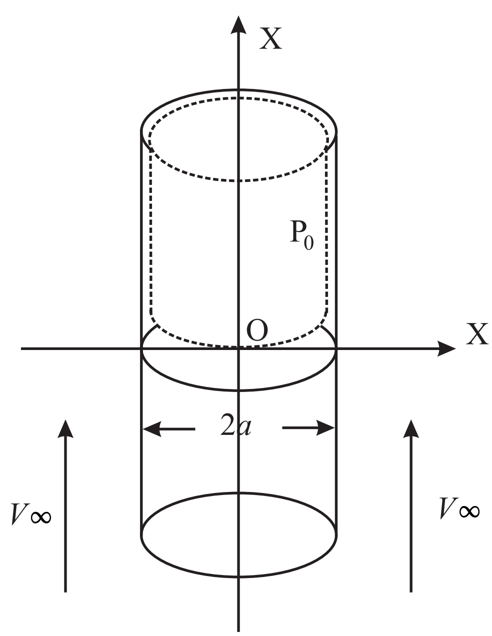

Fig. 1. ***a*** **.**

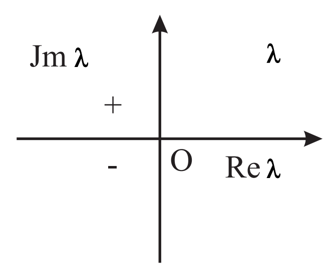

Fig. 1. ***b*** **.**

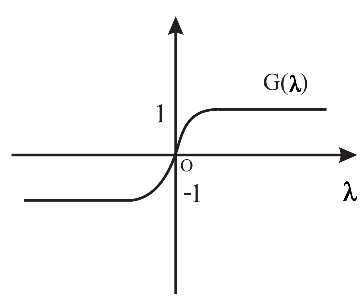

Fig. 1. ***c*** **.**

Gryphon, developing along a casing of a drill site well.

At

$$\frac{P_0}{\gamma\sqrt{a}}\left[\left(\frac{P_\infty}{P_0}\right)^\gamma - 1\right] > \frac{K_c}{4\eta\sqrt{\pi}} \tag{1.54}$$

"Gryphon" non-stop develops, reducing finally to gas blowout.

(K_c is some constant system "rock-gas", defined in independent experiment).

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