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MAXIMUM PRINCIPLE FOR ONE PROBLEM OF STOCHASTIC OPTIMAL CONTROL WITH VARIABLE DELAY

Abstract

The stochastic problem of optimal control with variable delay is considered. At first the necessary condition is obtained for stochastic systems without phase limitation. Further using Ekland variational principle the maximum principle is proved for stochastic control systems with delay of phase limitations.

Let (Ω, F, P) be a complete probability space with defined on it non-decreasing flow of σ-algebras $\{F^t, t \in [t_0, t_1]\}$ embedded in $F = \bar{\sigma}(w_s, t_0 \leq s \leq t)$. w_t is a one-dimensional Wiener process defined on a probability space (Ω, F, P) and on $[t_0, t_1]$. $L_F^2(t_0, t_1, R^n)$ is a space of measurable by (t, ω) and associated processes $x : [t_0, t_1] \times \Omega \to R^n$ such that $E\int\limits_{t_0}^{t_1} |x_t|^2 dt < \infty$. $L_F^2 C(t_0, t_1, R^n)$ is a space of random functions $x_t \in L_F^2(t_0, t_1, R^n)$ with almost sure (a.s.) continuous trajectories.

Consider the following stochastic delay system.

$$dx_1 = g\left(x_t, x_{t-h(t)}, u_t, t\right) dt + \sigma\left(x_t, x_{t-h(t)}, t\right) dw_t; t \in (t_0, t_1] \tag{1}$$

$$x_t = \Phi(t), \quad t \in [t_0 - h(t_0); t_0], \quad h(t) > 0 \tag{2}$$

$$u_t \in U_\partial \equiv \left\{u \in L_F^2(t_0, t_1; R^m) \,|u_t \in U \subset R^m, \ a.s.\right\} \tag{3}$$

Here

$$x(\cdot) \in L_F^2(t_0, t_1, R^n), \quad \Phi(\cdot) \in L_F^2([t_0 - h(t_0); t_0]; R^n), \quad \frac{dh(t)}{dt} < 1$$

Let it be required to minimize the functional

$$J(u) = E\left\{p(x_{t_1}) + \int\limits_{t_0}^{t_1} l(x_t, u_t, t)\, dt\right\} \tag{4}$$

on the set of admissible controls U_∂ under the condition

$$Eq(x_{t_1}) \in G \subset R^k \ . \tag{5}$$

Assume that the following conditions are fulfilled:

I. The function (l, g, σ) is continuous on totality of arguments

$$l : R^n \times R^m \times [t_0, t_1] \to R$$

$$g : R^n \times R^n \times R^m \times [t_0, t_1] \to R^n$$

$$\sigma : R^n \times R^n \times [t_0, t_1] \to R^{n\times n}$$

II. The functions l, g, σ are continuously differentiable by (x, y) and satisfy the condition of linear growth.

III. The function $p(x) : R^n \to R$ is continuously differentiable and satisfies the condition:

$$|p(x)| + |p_x(x)| \le N(1 + |x|)$$

IV. The function $q(x) : R^n \to R^k$ is continuously differentiable and satisfies the condition:

$$|q(x)| + |q_x(x)| \le N(1 + |x|)$$

At first the stochastic problem of optimal control (1)-(4) is considered. The following result is obtained:

Theorem 1. *Let the conditions I-III be fulfilled and* $\left(x_t^0, u_t^0\right)$ *be a solution of problem (1)-(4). If the random processes* $\psi_t \in L_F^2 C(t_0, t_1; R^n)$ *and* $\beta_t \in L_F^2 C(t_0, t_1; R^{n\times n})$, *are the solutions of the adjoint equation:*

$$\begin{cases} d\psi_t = -\left[H_x\left(\psi_t, x_t^0, y_t^0, u_t^0, t\right) + H_y\left(\psi_z, x_z^0, y_z^0, u_z^0, z\right)|_{z=s(t)}\dfrac{ds(t)}{dt}\right]dt+ \\ +\beta_t dw_t,\ t_0 \le t < t_1 - h(t_1) \\ d\psi_t = -H_x\left(\psi_t, x_t^0, y_t^0, u_t^0, t\right)dt + \beta_t dw_t,\ t_1 - h(t_1) \le t < t_1 \\ \psi_{t_1} = -p_x\left(x_{t_1}^0\right) \end{cases} \tag{6}$$

then

$$\max_{u\in U} H\left(\psi_t, x_t^0, y_t^0, u, t\right) = H\left(\psi_t, x_t^0, y_t^0, u^0, t\right)\ \ a.s. \tag{7}$$

Here $\tau(t) = t - h(t)$; $y_t = x_{t-h(t)}$; $\tau = s(\tau)$ *is a solution of the equation* $\tau = \tau(t)$ and

$$H(\psi_t, x_t, y_t, u, t) = \psi_t^* g(x_t, y_t, u, t) + \beta_t^* \sigma(x_t, y_t, t) - l(x_t, u_t, t).$$

Then using the obtained result the following theorem is proved for the problem of optimal control with phase limitations (5).

Theorem 2. *Let conditions I-IV be fulfilled and* $\left(x_t^0, u_t^0\right)$ *be a solution of problem (1)-(5). If exist the constants* $(\lambda_0, \lambda_1) \in R^{k+1}$ *and the random processes* $\psi_t \in L_F^2 C(t_0, t_1, R^n)$, $\beta_t \in L_F^2 C(t_0, t_1, R^{n\times n})$ *such that*

a) $\lambda_0 \ge 0$, λ_1 *is normal to the set* G *at the point* $Eq\left(x_{t_1}^0\right)$, $\lambda_0^2 + |\lambda_1|^2 = 1$

b)

$$\begin{cases} d\psi_t = -\left[H_x\left(\psi_t, x_t^0, y_t^0, u_t^0, t\right) + H_y\left(\psi_z, x_z^0, y_z^0, u_z^0, z\right)|_{z=s(t)}\dfrac{ds(t)}{dt}\right]dt+ \\ +\beta_t dw_t,\ t_0 \le t < t_1 - h(t_1) \\ d\psi_t = -H_x\left(\psi_t, x_t^0, y_t^0, u_t^0, t\right)dt + \beta_t dw_t,\ t_1 - h(t_1) \le t < t_1 \\ \psi_{t_1} = -\lambda_0 p_x\left(x_{t_1}^0\right) - \lambda_1 q_x\left(x_{t_1}^0\right) \end{cases} \tag{8}$$

then

$$\max_{u\in U} H\left(\psi_t, x_t^0, y_t^0, u, t\right) = H\left(\psi_t, x_t^0, y_t^0, u^0, t\right) \ a.s. \tag{9}$$

Here

$$H\left(\psi_t, x_t, y_t, u, t\right) = \psi_t^* g\left(x_t, y_t, u, t\right) + \beta_t^* \sigma\left(x_t, y_t, t\right) - \lambda_0 l\left(x_t, u_t, t\right).$$

The proof of theorem 1. Let $\bar{u}_t = u_t^0 + \Delta u_t$ be some admissible control and $\bar{x}_t = x_t^0 + \Delta x_t$ be its corresponding trajectory. For calculations of the increase of functional (4) we'll use the equalities:

$$\begin{cases} d\Delta x_t = d\left(\bar{x}_t - x_t^0\right) = \left[g\left(\bar{x}_t, \bar{x}_{t-h(t)}, \bar{u}_t, t\right) - g\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right)\right] dt + \\ + \left[\sigma\left(\bar{x}_t, \bar{x}_{t-h(t)}, t\right) - \sigma\left(x_t^0, x_{t-h(t)}^0, t\right)\right] dw_t = \\ = \{\Delta_{\bar{u}} g\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right) + g_x\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right)\Delta x_t + \\ + g_y\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right)\Delta x_{t-h(t)}\} dt + \{\sigma_x\left(x_t^0, x_{t-h(t)}^0, t\right)\Delta x_t + \\ + \sigma_y\left(x_t^0, x_{t-h(t)}^0, t\right)\Delta x_{t-h(t)}\} dw_t + d\eta_t^1, \quad t \in (0, t_1], \\ \Delta x_t = 0, \quad t \in [-h(t_0); t_0] \end{cases} \tag{10}$$

where

$$\eta_t^1 = \left\{\int_0^1 \left[g_x\left(x_t^0 + \mu_1 \Delta x_t, \bar{x}_{t-h(t)}, \bar{u}_t, t\right) - g_x\left(x_t^0, x_{t-h(t)}^0, \bar{u}, t\right)\right]\Delta x_t d\mu_1 + \right.$$

$$\left. + \int_0^1 \left[g_y\left(x_t^0, x_{t-h(t)}^0 + \mu_2 \Delta x_{t-h(t)}, \bar{u}_t, t\right) - g_y\left(x_t^0, x_{t-h(t)}^0, \bar{u}, t\right)\right]\Delta x_{t-h(t)}^0 d\mu_2 \right\} dt +$$

$$+ \left\{\int_0^1 \left[\sigma_x\left(x_t^0 + \mu_1 \Delta x_t, \bar{x}_{t-h(t)}, t\right) - \sigma_x\left(x_t^0, x_{t-h(t)}^0, t\right)\right] d\mu_1 + \right.$$

$$\left. + \int_0^1 \left[\sigma_y\left(x_t^0, x_{t-h(t)}^0 + \mu_2 \Delta x_{t-h(t)}, t\right) - \sigma_y\left(x_t^0, x_{t-h(t)}^0, t\right)\right] d\mu_2 \right\} dw_t \tag{11}$$

and

$$d\left(\psi_t^* \Delta x_t\right) = d\psi_t^* \Delta_{x_t} + \psi_t^* d\Delta x_t + \left\{\beta_t^* \sigma_x\left(x_t^0, x_{t-h(t)}^0, t\right)\Delta x_t + \right.$$

$$+ \beta_t^* \sigma_y\left(x_t^0, x_{t-h(t)}^0, t\right)\Delta x_{t-h(t)} +$$

$$+ \beta_t^* \int_0^1 \left[\sigma_x\left(x_t^0 + \mu_1 \Delta x_t, \bar{x}_{t-h(t)}, t\right) - \sigma_x\left(x_t^0, x_{t-h(t)}^0, t\right)\Delta x_t\right] d\mu_1 +$$

$$\left. + \beta_t^* \int_0^1 \left[\sigma_y\left(x_t^0, x_{t-h(t)}^0 + \Delta x_{t-h(t)}, t\right) - \sigma_y\left(x_t^0, x_{t-h(t)}^0, t\right)\right]\Delta x_{t-h(t)} d\mu_2 \right\} dt, \tag{12}$$

which is obtained by means of the Ito formula.

The increase of functional (4) along the admissible control has the form:

$$\Delta_{\bar{u}} J(u) = J(\bar{u}) - J\left(u^0\right) = E\left\{p(\bar{x}_{t_1}) - p(x_0) + \int_{t_0}^{t_1} \left[l(\bar{x}_t, \bar{u}_t, t) - l\left(x_t^0, u_t^0, t\right)\right] dt\right\} =$$

$$= E\left\{p_x(\bar{x}_{t_1})\,\Delta x_{t_1} + \int_{t_0}^{t_1} \left[\Delta_u l\left(x_t^0, u_t^0, t\right) + l_x\left(x_t^0, u_t^0, t\right)\Delta x_t\right] dt\right\} + d\eta_1^2 \qquad (13)$$

where

$$\eta_1^2 = E\int_0^1 \left[p_x^*\left(x_{t_1}^0 + \mu_1 \Delta x_{t_1}\right) - p_x^*\left(x_{t_1}^0\right)\right]\Delta x_{t_1} d\mu_1 +$$

$$+E\left\{\int_{t_0}^{t_1} \left[l_x^*\left(x_{t_1}^0 + \mu_1 \Delta x_t, \bar{u}_t, t\right) + l_x^*\left(x_t^0, u_t^0, t\right)\right]\Delta x_t d\mu_1\right\} dt\ .$$

Subject to (11) and (12) expression (13) takes the following form:

$$\Delta J\left(u^0\right) = -E\int_{t_0}^{t_1} d\psi_t^* \Delta x_t - E\int_{t_0}^{t_1} d\psi_t^* \left\{\left[\Delta_{\bar{u}} g\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right) + \right.\right.$$

$$\left. +g_x\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right)\Delta x_t + g_y\left(x^0, x_{t-h(t)}^0, u_t^0, t\right) x_{t-h(t)}^0 \Delta x_{t-h(t)}\right] dt +$$

$$+\left[\sigma_x\left(x_t^0, x_{t-h(t)}^0, t\right)\Delta x_t + \sigma_y\left(x_t^0, x_{t-h(t)}^0, t\right)\Delta x_{t-h(t)}\right] dw_t -$$

$$-E\int_{t_0}^{t_1} \beta_t^* \left[\sigma_x\left(x_t^0, x_{t-h(t)}^0, t\right)\Delta x_t + \sigma_y\left(x_t^0, x_{t-h(t)}^0, t\right)\Delta x_{t-h(t)}\right] dt +$$

$$+E\int_{t_0}^{t_1} \left[\Delta_{\bar{u}} l\left(x_t^0, u_t^0, t\right) + l_x\left(x_t^0, u_t^0, t\right)\Delta x_t\right] dt + \eta_{t_0,t_1} \qquad (14)$$

where

$$\eta_{t_0,t_1} = \eta_t^2 +$$

$$+E\int_{t_0}^{t_1}\left\{\int_0^1 \psi_t^*\left(g_x\left(x_t^0 + \mu_1 \Delta x_t, \bar{x}_{t-h(t)}, u_t^0, t\right) - g_x\left(x_t^0, \bar{x}_{t-h(t)}, u_t^0, t\right)\right)\Delta x_t d\mu_1 +\right.$$

$$\left.+\int_0^1 \psi_t^*\left(g_y\left(x_t^0, \bar{x}_{t-h(t)} + \mu_2 \Delta x_{t-h(t)}, u_t^0, t\right) - g_y\left(x_t^0, \bar{x}_{t-h(t)}, u_t^0, t\right)\right)\Delta x_{t-h(t)} d\mu_2\right\} dt +$$

$$+E\int_{t_0}^{t_1}\left\{\int_0^1 \beta_t^*\left(\sigma_x\left(x_t^0 + \mu_1 \Delta x_t, \bar{x}_{t-h(t)}, t\right) - \sigma_x\left(x_t^0, \bar{x}_{t-h(t)}, t\right)\right)\Delta x_t d\mu_1 +\right.$$

$$+\int_0^1 \beta_0^* \left(\sigma_y\left(x_t^0, \bar{x}_{t-h(t)} + \mu_1 \Delta x_{t-h(t)}, t\right) - \sigma_y\left(x_t^0, \bar{x}_{t-h(t)}, t\right)\right) \Delta x_{t-h(t)} d\mu_2 \Bigg\} dt \quad (15)$$

By means of simple transformations expression (14) takes the form:

$$\Delta_{\bar{u}} J\left(u^0\right) = E\int_{t_0}^{t_1} \left[\psi_t^* \Delta_{\bar{u}} g\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right) - \Delta_{\bar{u}} l\left(x_t^0, u_t^0, t\right)\right] dt-$$

$$-E\int_{t_0}^{t_1} \left[d\psi_t^* + \psi_t^* g_x\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right) + \beta_t^* \sigma_x\left(x_t^0, u_t^0, t\right) - l_x'\left(x_t^0, u_t^0, t\right)\right] \Delta x\left(t\right) dt-$$

$$-E\int_{t_0}^{t_1} \left[\psi_t^* g_y\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right) + \beta_t^* \sigma_y\left(x_t^0, u_t^0, t\right)\right] \Delta x_{t-h(t)} dt + \eta_{t_0, t_1} .$$

Subject to (6) the increase functional formula takes the form:

$$\Delta J\left(u^0\right) = -E\int_{t_0}^{t_1} \left[\psi_t^* \Delta_{\bar{u}} g\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right) - \Delta_{\bar{u}} l\left(x_t^0, u_t^0, t\right)\right] dt + \eta_{t_0, t_1} . \quad (16)$$

Consider the following needle-shaped variation:

$$\Delta u_t = \Delta u_t^\theta \ , \quad \varepsilon = \begin{cases} 0, \quad t\bar{\in}[\theta, \theta+\varepsilon), \quad \varepsilon > 0 \ ; \quad \theta \in [t_0, t_1) \\ \nu - u_t^0, \quad t \in [\theta, \theta+\varepsilon) \ , \quad \nu \in L^2\left(\Omega, F^\theta, P; R^m\right) \end{cases} \quad (17)$$

Let $x_{t,\varepsilon}^\theta$ be a trajectory corresponding the control $u_{t,\varepsilon}^\theta = u_t^0 + \Delta u_{t,\varepsilon}^\theta$. Then expression (16) takes the form

$$\Delta_\theta J\left(u^0\right) = -E\int_{\theta}^{\theta+\varepsilon} \left[\psi_t^* \Delta_\nu g\left(x_t^0, x_{t-h(t)}^0, u_t^0, t\right) - \Delta_\nu l\left(x_t^0, u_t^0, t\right)\right] dt + \eta_{\theta, \theta+\varepsilon}$$

We'll use the following lemma:

Lemma. *Let conditions I-III be fulfilled. Then*

$$E\left|\frac{x_{t,\varepsilon}^\theta - x_t^0}{\varepsilon}\right|^2 \le N \ , \quad if \quad \varepsilon \to 0$$

According to lemma and (15) we obtain the estimation

$$\eta_{\theta, \theta+\varepsilon} = o\left(\varepsilon\right)$$

and

$$\Delta_\theta J\left(u^0\right) = -E\left[\psi_t^* \Delta_\nu g\left(x_\theta^0, x_{\theta-h(\theta)}^0, u_\theta^0, \theta\right) - \Delta_\nu l\left(x_\theta^0, u_\theta^0, \theta\right)\right] \varepsilon + o\left(\varepsilon\right) \ge 0 \quad (18)$$

Whence from the sufficient smallness of ε and from (15) we have

$$E\left[\psi_t^* \Delta_\nu g\left(x_\theta^0, x_{\theta-h(\theta)}^0, u_\theta^0, \theta\right) - \Delta_\nu l\left(x_\theta^0, u_\theta^0, \theta\right)\right] \leq 0 \ .$$

From the arbitrariness $\theta \in [t_0, t_1]$ we obtain the fulfilment (7).

Theorem 1 is proved.

Proof of theorem 2. Introduce the following functional for any natural j:

$$J_j(u) = S_j\left(Ep(x_{t_1}) + E\int_{t_0}^{t_1} l(x_t, u_t, t)\,dt, Eq(x_{t_1})\right) =$$

$$= \min_{(c,y)\in\varepsilon}\sqrt{\left|c - \frac{1}{j} - Ep(x_{t_1}) - E\int_{t_0}^{t_1} l(x_t, u_t, t)\,dt\right|^2 + \|y - Eq(x_{t_1})\|^2}$$

where $\varepsilon = \left\{(c, y) : c \leq J^0, \quad y \in G\right\}$

Here J^0 is a minimum value of functional in problem (1)-(5). V is a space of control with the metric:

$$d(u, \nu) = Emes\{t_0 \leq t \leq t_1; \quad \nu_t \neq u_t\}$$

Use the following results:

Theorem (Ekland variational principle). *K is a complete metric space $f : K \to R \cup \{+\infty\}$ is a lower semi-continuous function, $\varepsilon, \lambda > 0$,* and *x_0 is some point for which the condition*

$$f(x_0) \leq \inf f(x) + \varepsilon\lambda$$

is fulfilled.

Then there exists the point $\bar{x} \in K$ for which

1) $f(\bar{x}) \leq f(x_0)$,

2) $d(x_0, \bar{x}) \leq \lambda$,

3) $\forall x \in K$, $f(\bar{x}) \leq f(x) + \varepsilon d(x, \bar{x})$

are fulfilled.

Lemma. *Assume that the conditions I-IV are fulfilled and*

$$d\left(u_t^{(n)}, u_t\right) \to 0 \ , \quad n \to \infty$$

Then

$$\lim_{n\to\infty}\left\{\sup_{t_0\leq t\leq t_1} E\left|x_t^{(n)} - x_t\right|^2\right\} = 0$$

Since the functional $J_j : V \to R^n$ is continuous, according to the Ekland variational principle we have: $\exists u_t^j : d\left(u_t^j, u_t^0\right) \leq \sqrt{\varepsilon_j}$ and $\forall u \in V$ is fulfilled:

$$J_j\left(u^j\right) \leq J_j(u) + \sqrt{\varepsilon_j} d\left(u^j, u\right), \ \varepsilon_j = \frac{1}{j} \ .$$

This inequality means that $\left(x_t^j, u_t^j\right)$ is a solution of the following problem:

$$J_j\left(u^j\right) = J_j\left(u\right) + \sqrt{\varepsilon_j}E\int_{t_0}^{t_1}\delta\left(u_t, u_t^j\right)dt \to \min \tag{19}$$

$$dx_t = g\left(x_t, x_{t-h(t)}, u_t, t\right)dt + \sigma\left(x_t, x_{t-h(t)}, t\right)dw_t \tag{20}$$

$$x_t = \Phi\left(t\right), \quad t \in \left[-h\left(t_0\right), t_0\right], \; h\left(t\right) > 0 \tag{21}$$

$$u_t \in U_\partial \tag{22}$$

where $\delta\left(u,\nu\right) = \begin{cases} 0, & u = \nu \\ 1, & u \neq \nu \end{cases}$.

Since $\left(x_t^j, u_t^j\right)$ is a solution of problem (19)-(22) then according to theorem 1 there exists random processes $\psi_t^j \in L_F^2C\left(t_0, t_1; R^n\right)$, $\beta_t^j \in L_F^2C\left(t_0, t_1; R^{n\times n}\right)$ and the constants $\left(\lambda_0^j, \lambda_1^j\right) \in R^{k+1}$ such that

$$\begin{cases} d\psi_t = -\left[H_x\left(\psi_t^j, x_t^j, y_t^j, u_t^j, t\right) + H_y\left(\psi_z^j, x_z^j, x_z^j, u_z^j, z\right)|_{z=s(t)}\frac{ds(t)}{dt}\right]dt + \\ +\beta_t^j dw_t, \quad t_0 \leq t \leq t_1 - h\left(t_1\right) \\ d\psi_t^j = -H_x\left(\psi_t^j, x_t^j, y_t^j, u_t^j, t\right)dt + B_t^j dw_t\;, \quad t_1 - h\left(t_1\right) \leq t < t_1 \\ \psi_{t_1}^j = -\lambda_0^j p_x\left(x_{t_1}^j\right) - \lambda_1^j q_x\left(x_{t_1}^j\right) \end{cases} \tag{23}$$

$$\left(\lambda_0^j, \lambda_1^j\right) = \left(\frac{-c_j + \frac{1}{j} + Ep\left(x_{t_1}^j\right) + E\int_{t_0}^{t_1} l\left(x_t^j, u_t^j, t\right)dt}{J_j^0}, \; \frac{-y_j + Eq\left(x_{t_1}^j\right)}{J_j^0}\right) \tag{24}$$

and

$$\max_{u\in\nu} H\left(\psi_t^j, x_t^j, y_t^j, u, t\right) = H\left(\psi_t^j, x_t^j, y_t^j, u^j, t\right). \tag{25}$$

Here

$$H\left(\psi_t^j, x_t^j, y_t^j, u_t^j, t\right) = \psi_t^{j^*}g\left(x_t^j, y_t^j, u_t^j, t\right) + \beta_t^{j^*}\sigma\left(x_t^j, y_t^j, t\right) - \lambda_0 l\left(x_t^j, u_t^j, t\right)$$

Hence by virtue of assumptions I-III passing to the limit in equalities (23)-(25), we get the proof of theorem 2.

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