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FATIGUE FAILURE OF SUBMERGED HOLLOW THIN-WALLED WITH FIXED ENDS CYLINDRICAL ELEMENT FROM AIRY NAVES ACTION

Abstract

Time to fatigue failure of submerged definite depth hollow thin-walled cylindrical element, whose ends are fixed to units of construction of shelf for successive action of different Airy waves is defined. At that the criterion taking into account the stress history is used.

Let a hollow thin- walled cylindrical element with internal radius a, wall thickness δ, length l submerged into water be exposed to Airy waves. Ends of the cylindrical element are fixed for linear and rotational displacement. The element isn't filled with water. We'll use cylindrical coordinate system $(r\varphi z)$. We'll direct axis z along the cylinder's axis.

At that the cylinder is in all-round reduction state stipulated by external pressure $P = P_1 + P_2$, where P_1 is hydrostatic pressure, P_2 is pressure connected with the action of Airy wave. It's clear that pressure P_2 is cyclic with respect to time therefore the external pressure P is also cyclic one. Cyclically acting pressure P after some time reduces the cylindrical element to fatigue failure moreover, fatigue failure of the element occurs even at stresses not exceeding the yield point [1]. Hence, under the conditions of elastic deformation of the considered element let's define time to its fatigue failure- cyclic durability. The fatigue failures usually occur in those places where stresses have maximal values. According to monograph [1] maximal normal and tangential stresses in the considered element appear at its fixed end points. Following [1], maximal normal stresses appear on interior and exterior surface of the cylinder in fixation places and are expressed by the formulas:

$$\sigma_z = -\frac{P_a}{2\delta}\left(1 \mp \frac{\sqrt{3}\,(2-\nu)}{\sqrt{1-\nu^2}}\right); \qquad \sigma_\varphi = \nu\sigma_z. \tag{1}$$

Here sign "-" corresponds to the exterior surface, sign "+"- to the interior surface of the cylinder.

Tangential stresses on the cylinder's surface are absent; the maximal tangential stress appears on median surface of the cylinder and is represented by formula [1]

$$\tau = \frac{3}{4}\frac{(2-\nu)\,P}{\left[3\left(1-\nu^2\right)\right]^{\frac{1}{4}}}\sqrt{\frac{a}{\delta}}. \tag{2}$$

Note that formulas (1) and (2) hold provided $l > \frac{6}{\beta}$, where

$$\beta = \left[3\left(1-\nu^2\right)/a^2\delta^2\right]^{1/4}.$$

It's easy to make sure that the tangential stress τ is small in comparison with σ_z and σ_φ, therefore we shall not use it when calculating the fatigue failure of the element.

Let's calculate now the stress intensity σ_+ whose expression in conformity to our problem has the form:

$$\sigma_+ = \sqrt{\sigma_\varphi^2 - \sigma_\varphi \sigma_z + \sigma_z^2} = |\sigma_z| \sqrt{\nu^2 - \nu + 1} \tag{3}$$

or subject to (1)

$$\sigma_+ = \left| -\frac{Pa}{2\delta} \left(1 \mp \frac{\sqrt{3}\,(2-\nu)}{\sqrt{1-\nu^2}} \right) \right| \sqrt{\nu^2 - \nu + 1}. \tag{4}$$

Suppose non that the cylindrical element situates at a distance y from seabed with depth h. At that following the theory of Airy waves [2] the external pressure upon the cylindrical element is defined by the formula

$$P = \frac{\gamma H}{2} \frac{ch\ ky}{ch\ kh} \cos(kx - \omega t) + \gamma (h - y). \tag{5}$$

Here γ is specific gravity of water; H is height of wave; k is wave number; $k = 2\pi/\lambda$; λ is wave lemgth; t is time; x is an axis of coordinate system directed towards the wave propagation.

Formulas for Airy wave propagation velocity c and for wave period T are represented in the form:

$$c = \left(\frac{a}{k} th\ kh \right)^{1/1}, \qquad T = \frac{\lambda}{c}, \tag{6}$$

where g is gravitational accdleration.

Let's use now the following numerical data: a =0,3m, $\delta = 6,012$m, $\nu = 0,3$. At that $\beta = 21,4$ m^{-5} and in order that solution (1) exists in our case, i.e. in the case when both elements of the cylindrical elements of the cylindrical element are fixed, assume length l of the element be $l > 6/\beta = 0,25$m.

Moreover assume h =90m, y =30m, $\gamma = 10$ kN/m^3. Besides we'll suppose that in construction exploitation area Airy waves with different height H and length λ act in order indicated in table in columns H and λ. By formulas (6) subject to the represented numerical data we'll define correspondent velocities and periods of these waves and we'll tabulate them (columns C and T). Terheafter in conformity with formula (5) the expression for pressure correspondent to each roughness level becomes known

$$P_n = P_2 + P_2^{(n)} \cos(k_n x - \omega_n t) \quad \left(n = \overline{2;91} \right). \tag{7}$$

Here $P_1 = \gamma (b - y) = 0,6$MPa; $P_2^{(1)} = P_2^{(11)} \approx 0$; $P_1^{(2)} = P_2^{(10)} = 2 \cdot 10^{-5}$MPa; $P_5^{(3)} = P_2^{(9)} = 0,00048$ MPa; $P_2^{(4)} = P_2^{(8)} = 5,00486$ MPa; $P_2^{(5)} = P_2^{(7)} = 6,0350$ MPa; $P_2^{(6)} = 0,0375$ MPa; $k_n = 2\pi/\lambda_n$; $\omega_n = 2\pi/T_n$.

Stress intensities $\sigma_+(n)$ possess the maximal values on interior surface of the fixed ends of the cylindrical element. Their expressions corrrespondent to the each step of roughness ($n=\overline{1,11}$) according to formulas (4), (7) and represented numerical data are represented in the form

$$\sigma_{+}^{(n)}=27,24\,(\mathrm{MPa})+45,9P_2^{(n)}\cos\left(k_nx+\omega_nt\right). \tag{8}$$

Let's define now the fatigue failure of the fixed end of the cylindrical element, which is successively exposed to stress (8). Previously let's define maximal stresses $\sigma_{\max}^{(n)}$ and ratio of the minimal and maximal stresses $r_n=\sigma_{+\min}^{(n)}/\sigma_{+\max}^{(n)}$. We obtain

$$\sigma_{+\max}^{(n)}=27,24+49,4P_2^{(n)}, \tag{9}$$

$$r_n=\frac{27,24-45,4P_2^{(n)}}{27,24+45,4P_2^{(n)}}, \tag{10}$$

the values of the quantities $\sigma_{+\max}^{(n)}$ and r_n for each number, and let's tabulate them (column $\sigma_{+\max},r$).

Let now the dependence of number of cycles before the pattern failure on values $\sigma_{+\max}$ and r : $N=N\left(\sigma_{+\max},r\right)$ be defined from experiment. Results of such experiments are represented, e.g., in paper [2].

We'll approximate dependence $N=N\left(\sigma_{+\max},r\right)$ in the form

$$N=A\exp\left[\gamma\left(1-\sigma_{+\max}/\sigma_s\right)-æ r\right], \tag{11}$$

where A is number of cycles at $r=0$ and $\sigma_{+\max}=\sigma_s$, σ_s is reduction stress for which fatigue limit σ_{-1} can be chosen; γ and æ are experimentally defined material constants. For steel usually used for constructions of continental shelf the fatigue limit is $\sigma_{-1}=\sigma_s=7$ MPa. The rest of constants using fatigue curve for the noted steel [1] have the following values: $\gamma=0,75$, æ$=8,43$; $A=4,6\cdot 10^3$ cycles.

In accordance with the statement of the considered problem we'll represent formula (11) in the form

$$N_n=A\exp\left[\gamma\left(1-\frac{\sigma_{+\max}^{(n)}}{\sigma_s}\right)-æ r_n\right]\;,\quad (n=1,2,...). \tag{12}$$

Suppose that the considered cylindrical element is connected to some unit of construction of shelf. At that we exclude the failure of the unit by depression and suppose that only fatigue failure of the unit occurs. Now in order to define time to the fatigue failure of the considered cylindrical element we'll use the formula derived in [3]:

$$\left[\frac{\left(\frac{d_1}{T_1}+\frac{d_2}{T_2}+\cdots+\frac{d_n}{T_n}\right)^q-\left(\frac{d_2}{T_2}+\frac{d_3}{T_3}+\cdots+\frac{d_n}{T_n}\right)^q}{N_1^q}+\right.$$

$$\frac{\left(\frac{d_2}{T_2}+\frac{d_3}{T_3}+\cdots+\frac{d_n}{T_n}\right)^q-\left(\frac{d_3}{T_3}+\frac{d_4}{T_4}+\cdots+\frac{d_n}{T_n}\right)^q}{N_2^q}+\cdots+$$

$$+\frac{\left(\frac{d_{n-1}}{T_{n-1}}+\frac{d_n}{T_n}\right)^q-\left(\frac{d_n}{T_n}\right)^q}{N_{n-1}^q}+\frac{\left(\frac{d_n}{T_n}\right)^q}{N_n^q}\Bigg] T_*^q=1. \qquad (13)$$

Here T_* is time to the fatigue failure. Periods T_i $\left(i=\overline{1,n}\right)$ are equal to periods of the corresponding wave periods. Constant for each material q is defined from experiment in conformity with the technique represented in [3]. For metals usually $q>1$. Existence of the parameter q in formula (13) allows to say that it takes into account physically important phenomenon-stress history. Quantities N_i $\left(i=\overline{1,n}\right)$ are numbers of cycles to failure defined by formula (12) in which $\sigma_{+\max}^{(n)}$ are had to be increased on stress concetration coefficient k_σ. Usually such increasing of amplitude is taken into consideration with the purpose of accounting stress concentration and in connection with dynamic property of the considered process. Coefficient k_σ is accepted in the following range $2\le k_\sigma\le 3$. Quantities d_i $\left(i=\overline{1,n}\right)$ are given by time parts (durability) within which waves with the same distribution lows of height and period act [1]. Assumed values of d_i we'll also represent in the table (column d).

Table

No	H,m	λ,m	c, $\frac{m}{c}$	T,c	$\sigma_{+\max}$,MPa	r	d,%
1	1	15	4.89	3	27.24	1	44.3
2	2	60	10	6	27.241	1	5.13
3	4	100	12.6	8	27.26	0.998	0.5
4	8	170	16.37	10.4	27.46	0.98	0.05
5	16	250	19.7	12.7	28.8	0.86	0.015
6	21	300	21.3	14	29	0.87	0.01
7	16	250	19.7	12.7	28.8	0.89	0.015
8	8	170	16.37	10.4	27.46	0.98	0.05
9	4	100	12.6	8	27.26	0.998	0.5
10	2	60	10	6	27.241	1	5.13
11	1	15	4.89	3	27.24	1	44.3

Assuming stress concentration coefficient k_σ being equal 2 by formula (12) at changing $\sigma_{+\max}^{(n)}$ onto $2\sigma_{+\max}^{(n)}$ we find N_n $\left(n=\overline{1;11}\right)$. After this assuming for material of the cylindrical element $q=1,6$ and using the represented table by formula (13) we define life duration- time to fatigue failure T_* connected by the ends to the unit of the cylindrical element.

Results of calculations by formula (13) in the case of the considered stress programmer shows that $T_*=2,4\cdot 10^8 c=7,6$ years.

References

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[3]. Talybly L.Kh., Gulieva S.Yu. *Refer to calculation of fatigue failure of nodal joint of marine structures. Izvestiya NAN Azerb.* 2002, No2-3, p.141-148. (Russian)

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