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THE TRANSVERSE VIBRATION OF A PILE IN VISCO-ELASTIC MEDIUM

Abstract

The given paper is devoted to the investigation of transverse vibration of a pile in visco-elastic medium. The pile is modeled as a rectilinear rod. The dependence of a deflection of a pile from vibration frequency is obtained. It is shown, that with increasing of frequency of amplitude, the deflection of pile decreases.

The given paper is devoted to the investigation of transverse vibration of a pile in visco-elastic medium. The pile is modeled as a rectilinear rod of length L and with transverse section F. It is supposed, that the surface of a pile has a full linking with ground, i.e. at deformation of a pile detachment of a pile doesn't occur. In this case the ground action on a pile we substitute by a force constant in the line of cross-section of a pile and acting against its motion.

The equation of motion of points of a pile we'll take in the following form:

$$EJ\frac{\partial^4 w}{\partial x^4}+pF\frac{\partial^2 w}{\partial x^2}+q+\rho F\frac{\partial^2 w}{\partial t^2}=0, \tag{1}$$

where E-is a n elasticity modulus of a material of a rod, ρ- is a density, w-is a displacement, F-is a area of a cross-section of rod, J-is a moment of inertia of cross-section relative to neutral rod axis, q-is a loading intensity which operates on a pile from the side of soil, p-is an intensity of the loading uniformly distributed on butt end. Supposing that the lower butt end of a pile is hard-mounted and the upper butt end of a pile is under the influence of uniformly distributed on butt end, variable on time t, moments of intensity M, the boundary conditions we'll represent as follows:

$$at\ x=0\quad w=0;\quad \frac{\partial^2 w}{\partial x^2}=0;\quad at\quad x=L\qquad w=0$$

$$\frac{\partial^2 w}{\partial x^2}=M=M_0\sin\omega_0 t. \tag{2}$$

Note, that for clay soils the elasticity appears in the first moments of loading, then characteristic properties appear.

Suppose, that the intensity of loading, acting on a pile from the side of a soil we can write in the following form:

$$q=k_c\cdot w-\int\limits_{-\infty}^{t}\Gamma\left(t-\tau\right)w\left(\tau\right)d\tau, \tag{3}$$

where Γ-is a kernel of relaxation [1], and k_c-is a modulus of subgrade reaction. Then motion equation (1) of a pile points will take the form:

$$EJ\frac{\partial^4 w}{\partial x^4}+pF\frac{\partial^2 w}{\partial x^2}+k_c\cdot w-\int\limits_{-\infty}^{t}\Gamma\left(t-\tau\right)w\left(\tau\right)d\tau+\rho F\frac{\partial^2 w}{\partial t^2}=0. \tag{4}$$

The solution (4) we'll find in the form:

$$w = w_0(x)\, e^{i\omega_0 t}. \tag{5}$$

Substituting (5) in (4) we'll get:

$$EJ\frac{d^4 w_0(x)}{dx^4} + pF\frac{d^2 w_0(x)}{dx^2} + k_c \cdot w_0(x) -$$

$$-w_0(x)\, e^{-i\omega_0 t}\int\limits_{-\infty}^{t}\Gamma(t-\tau)\, e^{-i\omega_0\tau}d\tau + \rho F\omega_0^2 w_0(x) = 0, \tag{6}$$

We transform the integral $\int\limits_{-\infty}^{t}\Gamma(t-\tau)\, e^{-i\omega_0\tau}d\tau$. Introduce the substitution $y = t-\tau$. Then

$$\int\limits_{-\infty}^{t}\Gamma(t-\tau)\, e^{-i\omega_0\tau}d\tau = \int\limits_{0}^{\infty}\Gamma(y)\, e^{i\omega_0(t-y)}dy = e^{i\omega_0 t}\int\limits_{0}^{\infty}\Gamma(y)\, e^{i\omega_0 y}dy =$$

$$= e^{i\omega_0 t}\int\limits_{0}^{\infty}\Gamma(y)\left(\cos\omega_0 y - i\sin\omega_0 y\right)dy =$$

$$= e^{i\omega_0 t}\left[\int\limits_{0}^{\infty}\Gamma(y)\cos\omega_0 y dy - i\int\limits_{0}^{\infty}\Gamma(y)\sin\omega_0 y dy\right].$$

Introduce the substitution

$$\int\limits_{0}^{\infty}\Gamma(y)\cos\omega_0 y dy = \Phi_c;\quad \int\limits_{0}^{\infty}\Gamma(y)\sin\omega_0 y dy = \Phi_s,$$

where Φ_c and Φ_s are Fourier images of the function $\Gamma(t)$ [1,2]. Then, we'll get [1]:

$$\int\limits_{0}^{\infty}\Gamma(t-\tau)\, e^{i\omega_0\tau}d\tau = e^{i\omega_0\tau}\left(\Phi_c - i\Phi_s\right). \tag{7}$$

Allowing for (7) in (6) for determination of amplitude of moving at large values of t we'll get:

$$EJ\frac{d^4 w_0(x)}{dx^4} + pF\frac{d^2 w_0(x)}{dx^2} + w_0(x)\left(k_c - \Phi_c + i\Phi_s - \rho F\omega_0^2\right) \tag{8}$$

$$at x = 0\ w_0 = 0;\ \frac{d^2 w}{dx^2} = 0;\ at\ x = L\ \ w_0 = 0;\ \frac{d^2 w}{dx^2} = M = M_0$$

The roots of characteristic equation have the form:

$$\alpha_{1,2}^2 = \frac{1}{2}\left[-\frac{pF}{EF} - \sqrt{\left(\frac{pF}{EF}\right)^2 - 4\frac{k_c - \Phi_c + i\Phi_s - \rho F\omega_0^2}{EJ}}\right]$$

The solution of the equation (8) we'll write in the following form:

$$w_0(x) = C_1 sh\beta_1 x + C_2 ch\beta_1 x + C_3 sh\beta_2 x + C_4 ch\beta_2 x$$

where

$$\beta_1 = \left\{\frac{1}{2}\left[\frac{pF}{EF} - \sqrt{\left(\frac{pF}{EF}\right)^2 - 4\frac{k_c - \Phi_c + i\Phi_s - \rho F\omega_0^2}{EJ}}\right]\right\}^{\frac{1}{2}}$$

$$\beta_2 = \left\{\frac{1}{2}\left[\frac{pF}{EF} - \sqrt{\left(\frac{pF}{EF}\right)^2 - 4\frac{k_c - \Phi_c + i\Phi_s - \rho F\omega_0^2}{EJ}}\right]\right\}^{\frac{1}{2}}.$$

Based on form of the boundary conditions (2) we have:

$$w_0(x) = \frac{M_0}{\beta_2^2 - \beta_1^2}\left(sh\beta_1 L sh\beta_1 x + sh\beta_2 L sh\beta_2 x\right). \tag{9}$$

The obtained expression can have a feature, if $\beta_1 = \beta_2$. For fulfilment of this condition it is necessary, that $\Phi_s = 0$, i.e. β_1 and β_2 were real numbers.

In the given formulation it is impossible, i.e. the natural frequency misses, the resonance is impossible.

Consider the example. Suppose, that the kernel of relaxation is an exponent of the form:

$$\Gamma(t) = Ae^{-\beta t},$$

where A- is a dimensional parameter, β-is a parameter of an exponent. Then

$$\Phi_c = A\int_0^\infty e^{-\beta t}\cos\omega_0 t dt = A\frac{\beta}{\beta^2 + \omega_0^2},$$

$$\Phi_s = A\int_0^\infty e^{-\beta t}\sin\omega_0 t dt = A\frac{\beta}{\beta^2 + \omega_0^2}, \tag{10}$$

It should be noted that the expression (8) for moving of a pile is complex. For determination its real and imaginary part let's accept the notation: $\beta_1 = \beta_{11} + i\beta_{12}$; $\beta_2 = \beta_{21} + i\beta_{22}$. As a result for $\operatorname{Re} w_0$ and $\operatorname{Im} w_0$ we'll get:

$$\operatorname{Re} w_0 = \frac{M_0}{a_1^2 + a_2^2}\left(a_1 T_1 + a_2 T_2\right)$$

$$\operatorname{Im} w_0 = \frac{M_0}{a_1^2 + a_2^2}\left(a_1 T_2 - a_2 T_1\right), \tag{11}$$

where

$$a_1 = \beta_{11}\beta_{21} - \beta_{12}\beta_{22}; \qquad a_2 = \beta_{11}\beta_{22} + \beta_{21}\beta_{12},$$

$$T_1 = sh\beta_{11} L\cos\beta_{12} L sh\beta_{11} x\cos\beta_{12} x - ch\beta_{11} L\sin\beta_{12} L\times$$

$$\times ch\beta_{11} x\sin\beta_{12} x + sh\beta_{21} L\cos\beta_{22} L sh\beta_{21} x\cos\beta_{22} x-$$

$$-ch\beta_{21} L\sin\beta_{22} L ch\beta_{21} x\sin\beta_{22} x,$$

$$T_2 = ch\beta_{11}L\cos\beta_{12}Lsh\beta_{11}x\cos\beta_{12}x + sh\beta_{11}L\sin\beta_{12}L\times$$
$$\times ch\beta_{11}x\sin\beta_{12}x + ch\beta_{21}L\sin\beta_{22}Lsh\beta_{21}x\cos\beta_{22}x-$$
$$-sh\beta_{21}L\cos\beta_{22}Lch\beta_{21}x\sin\beta_{22}x.$$

The value $w_0 = \sqrt{\mathrm{Re}^2\, w_0 + \mathrm{Im}^2\, w_0}$ was found numerically. For initial data were accepted [3,4].

$$A = 0,1615; \quad \beta = 0,05; \quad E = 2\cdot 10^6 kg/cm^2 \qquad \rho = 7,8g/cm^2;$$
$$k_c = 24000n/m^4; \quad L = 5m; \quad M_0 = 0,1.$$

In figure 1are shown the dependence w_0 from the frequency ω_0/ω_{0c} $(\omega_{0c} = \pi c/L)$. From the figure it is seen, that with increase of ω_0 the amplitude of moving of a pile decreases.

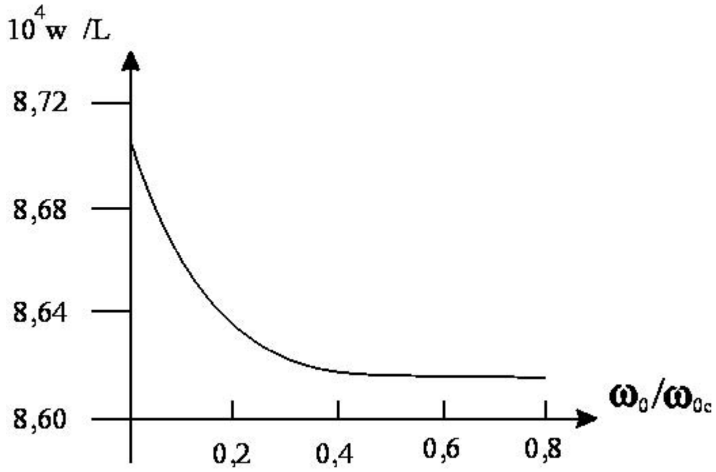

Fig. 1. Dependence of moving of a pile from frequency.

References

[1]. Robotnov Yu.N. *The elements of hereditary mechanics of solids.* M., Nauka, 1977. (Russian)

[2]. Robotnov Yu.N. *Mechanics of deformable solid.* M., Nauka, 1988. (Russian)

[3]. Koltunov M.A. *Creeping and relaxation.* M., Visshaya shkola, 1976. (Russian)

[4]. *Reference book on designing of trunk tubing.* L., Nedra, 1977. (Russian)

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